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Numerical investigation of the Korteweg–de Vries–Burgers equation using quintic B-spline differential quadrature and Crank–Nicolson integration

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ABSTRACT

The Korteweg–de Vries–Burgers equation describes the competition between nonlinear steepening, dispersion and viscous dissipation. This study examines a quintic B-spline differential quadrature formulation combined with fully nonlinear Crank–Nicolson time integration. Spatial derivatives are obtained from spline interpolation, and the resulting nonlinear systems are solved by Newton iteration using an analytical Jacobian. The numerical experiments consider an exact travelling shock, the effect of varying dissipation on a dispersive front, and the interaction of two unequal pulses. The finest reported shock calculation gives $L_2 = 2.941 \times 10^{-5}$ and $L_\infty = 3.895 \times 10^{-5}$. Increasing the dissipation coefficient from 0.05 to 0.50 reduces front overshoot and increases the reported transition thickness from 1.232459 to 4.096420. The pulse experiment reports a maximum relative mass drift of approximately 3×10^{-5} and an energy decrease of approximately 17.2%. These trends are consistent with dispersive and dissipative wave dynamics. The results provide a preliminary computational assessment; complete boundary closure, refinement records and diagnostic histories are needed to establish reproducibility and quantify accuracy or efficiency advantages.

Keywords: Korteweg–de Vries–Burgers equation, Quintic B-spline, Differential quadrature, Crank–Nicolson method, Travelling shock, Pulse interaction

1 Introduction

The Korteweg–de Vries (KdV) equation describes nonlinear dispersive waves (Korteweg *et al.*, 1895) [1], whereas Burgers' equation combines nonlinear convection with viscous smoothing (Burgers, 1948) [2]. Their combined form, the Korteweg–de Vries–Burgers (KdVB) equation, arises in long-wave approximations (Su *et al.*, 1969) [3] and supports travelling fronts shaped by both damping and dispersion (Johnson, 1970; Canosa *et al.*, 1977) [4, 5]. Numerical simulations must resolve this competition accurately, particularly when dispersive oscillations occur near a steep transition. Inadequate spatial or temporal resolution can otherwise make physical oscillations difficult to distinguish from numerical errors.

Differential quadrature approximates spatial derivatives by weighted sums of nodal values (Bellman *et al.*, 1972) [6]. Its connection with spline interpolation provides a convenient construction of derivative matrices (Bellman *et al.*, 1975; de Boor, 2001) [7, 8], and quintic B-spline differential quadrature has already been applied to the KdVB equation (Başhan *et al.*, 2015) [9]. Other approaches include compact finite differences (Koroche *et al.*, 2022) [10], B-spline collocation (Parumasur *et al.*, 2023) [11] and cubic Hermite collocation (Ari, 2025) [12]. The present contribution therefore concerns the assessment of a fully nonlinear implementation and its wave diagnostics, rather than the first application of quintic B-spline differential quadrature to this equation.

Crank–Nicolson integration is formally second-order accurate in time for sufficiently smooth solutions when the nonlinear system is solved to an appropriate tolerance (Crank *et al.*, 1947) [13]. Known travelling waves supply an analytical benchmark (Kudryashov, 2009) [14]. Here, the spatial operator and Newton Jacobian are stated explicitly, the reported shock errors are examined, and the front and pulse results are interpreted using travelling-wave analysis and integral diagnostics. The formulation is developed in Section 2, the numerical results are presented in Section 3, and their interpretation and limitations are discussed in Section 4.

2 Materials and methods

2.1 Governing equation

The nondimensional model is
 $u_t + Au u_x + Bu_{xxx} = Cu_{xx}$, $x \in \Omega$, $t > 0$. (1)

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Here, $u(x,t)$ denotes the wave amplitude. The constants $A > 0$, $B > 0$ and $C \geq 0$ represent nonlinear steepening, dispersion and viscous dissipation, respectively. Subscripts indicate partial differentiation. The computational domain Ω is either a finite interval with a specified boundary closure or a periodic interval. The exact shock considered below requires $C > 0$.

2.2 Exact travelling shock

Introduce the travelling coordinate $\xi = x - Vt - x_0$ and write $u(x,t) = U(\xi)$, where V is the wave speed and x_0 is a translation parameter. Substitution into Eq. (1) gives $-VU' + AUU' + BU''' = CU''$.

$$U(\xi) = V/A + [3C^2/(25AB)] \left[1 - 2 \tanh(\kappa\xi) - \tanh^2(\kappa\xi) \right], \kappa = C/(10B) \quad (3)$$

The upstream and downstream limits, $U_- = \lim U(\xi)$ as $\xi \rightarrow -\infty$ and $U_+ = \lim U(\xi)$ as $\xi \rightarrow +\infty$, are

$$U_- = V/A + 6C^2/(25AB), U_+ = V/A - 6C^2/(25AB) \quad (4)$$

These limits determine K through $K = (A/2)U_+^2 - VU_+ = (A/2)U_-^2 - VU_-$. Choosing $U_+ = 0$ gives $K = 0$ and the equivalent exact solution $u_{ex}(x,t) = [3C^2/(25AB)][\text{sech}^2(z) + 2(1 - \tanh(z))]$, (5) $z = [C/(10B)](x - Vt - x_0)$, $V = 6C^2/(25B)$, $U_- = 12C^2/(25AB)$. (6) The shock has unit height when $A = 12C^2/(25B)$, in which case $V = A/2$. The reported benchmark uses $A = 1$, $B = 0.02$, $C = \sqrt{(25B/12)} \approx 0.2041241452$, $V = 0.5$ and $x_0 = -2$. These choices fix the exact solution used for the initial data and endpoint values, and ensure consistency between its amplitude, speed and far-field states.

2.3 Criterion for an oscillatory tail

For a travelling wave connecting $U_- = 1$ to $U_+ = 0$, Eq. (2) gives $K = 0$ and $V = A/2$. The profile equation becomes $BU'' - CU' + \frac{A}{2}U(U-1) = 0$. Linearization near the upstream state $U = 1$ gives $B\lambda^2 - C\lambda + A/2 = 0$, $\lambda_{1,2} = [C \pm \sqrt{(C^2 - 2AB)}]/(2B)$, $\chi = C/\sqrt{(2AB)}$. (7)

The upstream spatial eigenvalues are complex when $\chi < 1$, real and distinct when $\chi > 1$, and repeated when $\chi = 1$. Thus, χ identifies the local transition between oscillatory and non-oscillatory upstream behaviour. This travelling-wave criterion guides the interpretation of the dissipation sweep, but it does not establish global monotonicity of a transient periodic solution.

2.4 Quintic B-spline spatial discretization

Let N denote the number of stored nodal values. On a finite interval $[a,b]$, the nodes are $x_i = a + ih$ for $i = 0, \dots, N-1$, with $h = (b-a)/(N-1)$. On a periodic interval $[a,b]$, the duplicate right endpoint is omitted and $h = (b-a)/N$. These conventions distinguish the finite shock grid from the periodic front and pulse grids.

A centred cardinal quintic B-spline with support $[-3,3]$ is defined by $M_6(s) = (1/120) \sum_{j=0}^6 (-1)^j \text{binom}(6,j)(s+3-j)_+^5$, $(y)_+ = \max(y, 0)$. (8)

Here, $\text{binom}(6,j)$ denotes the binomial coefficient. Set $\psi_j(x) = M_6((x-x_i)/h)$. At the offsets $s = -2, -1, 0, 1, 2$, the value stencil is $(1, 26, 66, 26, 1)/120$. The first-, second- and third-derivative stencils with respect to s are $(1, 10, 0, -10, -1)/24$, $(1, 2, -6, 2, 1)/6$ and $(1, -2, 0, 2, -1)/2$, respectively. Differentiation with respect to x introduces the factor h^{-r} for derivative order r . All four stencils follow directly from Eq. (8).

The spline approximation and interpolation matrices are

$$u_n(x,t) = \sum_j c_j(t) \psi_j(x), \Phi_{ij} = \psi_j(x_i), \Phi_{ij}^{(r)} = \psi_j^{(r)}(x_i). \quad (9)$$

The summation in Eq. (9) extends over the retained basis functions, and $\psi_j^{(r)}$ denotes the r th derivative with respect to x . For a square,

Integrating once with respect to ξ yields

$$BU'' - CU' - VU + (A/2)U^2 = K \quad (2)$$

Primes denote differentiation with respect to ξ , and K is an integration constant. A known quadratic hyperbolic-tangent profile (Kudryashov, 2009) [14] can be written as

nonsingular interpolation matrix obtained after a specified boundary treatment, the nodal and coefficient vectors satisfy

$$u = \Phi c, D_r = \Phi^{(r)} \Phi^{-1}, r = 1, 2, 3. \quad (10)$$

Thus, $D_r u$ approximates the r th spatial derivative at the nodes. In implementation, D_r should be assembled by solving $D_r \Phi = \Phi^{(r)}$, rather than explicitly computing Φ^{-1} . The coefficient matrices are banded before elimination; the resulting nodal differential quadrature matrices may be dense. The global accuracy order depends on the boundary closure and resolution, as well as the spline basis.

2.5 Boundary treatment

2.5.1 Periodic experiments

Cyclic wrapping defines the spline basis for the periodic front and pulse experiments. With N distinct nodes, the wrapped basis functions are

$$\psi_j^{\text{per}}(x) = \sum_l M_6((x-x_i)/h + lN), j = 0, \dots, N-1. \quad (11)$$

The summation is over all integers $l \in \mathbb{Z}$. Only finitely many terms are nonzero because M_6 is compactly supported. The matrices in Eq. (10) are formed from the wrapped basis, and periodicity applies to the solution and its spatial derivatives. This construction specifies a consistent periodic formulation; its correspondence with the implementation used for the reported calculations still needs to be documented.

2.5.2 Finite interval shock experiment

The shock calculation prescribes $u(a,t) = u_{ex}(a,t)$ and $u(b,t) = u_{ex}(b,t)$. These endpoint values alone do not specify the complete treatment of the third spatial derivative. The endpoint spline modifications, any additional boundary trace or equivalent closure relation, and the residual rows or coefficient elimination used at the boundaries are not recorded in the available implementation description. Consequently, the finite interval shock problem cannot yet be reconstructed uniquely from the stated endpoint values.

2.6 Fully nonlinear time integration

After spatial discretization, the advective form of Eq. (1) gives $\frac{du}{dt} = f(u)$, where

$$f(u) = -A[u \odot (D_1 u)] - BD_3 u + CD_2 u \quad (12)$$

The symbol $\odot$ denotes componentwise multiplication. Let u^n denote the nodal solution at t_n and let $\Delta t = t_{n+1} - t_n$. The Crank–Nicolson approximation defines u^{n+1} as a root v of

$$R(v) = v - u_n - (\Delta t/2) [f(v) + f(u_n)] = 0 \quad (13)$$

The nonlinear convective term is retained at both time levels. For periodic nodes, or interior equations before boundary row replacement,

$$J(v) = -A[\text{diag}(D_1 v) + \text{diag}(v)D_1] - BD_3 + CD_2 \quad (14)$$

Here, $\text{diag}(q)$ is the diagonal matrix whose diagonal entries are the components of q . At Newton iteration k , the correction and updated iterate satisfy

$$[I - (\Delta t/2)J(v^{(k)})]\delta v^{(k)} = -R(v^{(k)}), \quad v^{(k+1)} = v^{(k)} + \alpha_k \delta v^{(k)}. \quad (15)$$

The matrix I is the identity. An undamped step uses $\alpha_k = 1$; a damped step uses $0 < \alpha_k < 1$. The Jacobian must be updated as the iterate changes, and imposed boundary equations replace the corresponding residual and Jacobian rows. The calculations are described as using double precision, without artificial viscosity, filtering or a limiter. The initial predictor, damping rule, stopping tolerances, iteration limit and failed-step policy are not recorded. Software and hardware details are also needed for reproducibility and any timing comparison.

2.7 Error norms and integral diagnostics

For a numerical solution and the exact shock evaluated at the same time, define the nodal error e_i and the discrete norms by

$$e_i = u_i - u_{ex}(x_i, t), \quad L_2 = \sqrt{(h \sum_i w_i |e_i|^2)}, \quad L_\infty = \max_i |e_i|. \quad (16)$$

The sums and maximum extend over all stored nodes. Composite trapezoidal quadrature on $[a, b]$ assigns weight $1/2$ to each endpoint and weight 1 to every interior node. On the periodic grid, all weights are 1 . The normalization used in the reported error values must be confirmed against Eq. (16). Given errors e_1 and e_2 on two refinement levels, observed spatial and temporal orders are calculated as $\log(e_1/e_2)/\log(h_1/h_2)$ and $\log(e_1/e_2)/\log(\Delta t_1/\Delta t_2)$, respectively. Spatial order requires temporal error to be negligible, and temporal order requires adequate spatial resolution.

Mass, energy and gradient energy are defined by

$$M(t) = \int u \, dx, \quad E(t) = \frac{1}{2} \int u^2 \, dx, \quad G(t) = \int u_x^2 \, dx. \quad (17)$$

All integrals in Eq. (17) are taken over Ω . For sufficiently smooth periodic solutions, integration by parts in Eq. (1) gives

$$dM/dt = 0, \quad dE/dt = -CG \leq 0 \quad (18)$$

Table 1. Reported computational configurations.

Experiment	Domain	(A, B, C)	N	(Δt , T)
Exact shock	$[-15, 15]$	$(1, 0.02, \sqrt{(25B/12)})$	41, 61, 81, 121	$(h/8, 4)$
Front sweep	$[-40, 40]$	$(1, 0.08, 0.05/0.20/0.50)$	256	$(0.025, 8)$
Two pulses	$[-25, 25]$	$(1, 0.15, 0.015)$	160	$(0.040, 24)$

The initial plateau for the dissipation sweep is

$$u(x, 0) = \frac{1}{2}[\tanh((x + 15)/1.25) - \tanh((x - 15)/1.25)]. \quad (19)$$

This profile contains an increasing transition near $x = -15$ and a decreasing transition near $x = 15$. Its initial total variation over the full periodic interval is therefore approximately 2. Diagnostics for the leading decreasing front must be distinguished from quantities measured over the entire plateau.

3.2 Travelling shock errors

The available error summary contains the coarsest and finest values from the coupled spatial and temporal refinement. These values are retained in Table 2.

Table 2. Reported error values at the two available refinement levels.

Reported level	L_2	L_∞
Coarsest	3.425×10^{-3}	3.100×10^{-3}
Finest	2.941×10^{-5}	3.895×10^{-5}

Both norms decrease substantially, with reductions of approximately two orders of magnitude. The intermediate errors, exact correspon-

the analytical Jacobian $J(v) = \frac{\partial f}{\partial v}$ is

The same identities hold on the real line when the solution and the required derivatives decay sufficiently at infinity. Boundary fluxes generally remain for a finite interval shock, so Eq. (18) is used only for the periodic experiments here. Numerical estimates of M , E and G use consistent grid quadrature, with u_x approximated by $D_1 u$. The relative mass drift is $|M(t) - M(0)|/|M(0)|$ when $M(0) \neq 0$. These continuous identities provide diagnostic targets; they do not by themselves prove that the fully discrete scheme conserves mass exactly or produces monotone energy decay.

Periodic total variation is the sum of successive absolute nodal differences, including the final-to-first difference. The leading front thickness is measured between the 0.9 and 0.1 amplitude crossings of the decreasing transition. When a profile is oscillatory, the selected crossings must be specified so that the 10–90% thickness can be reproduced.

3 Results

3.1 Computational configurations

Table 1 summarizes the reported nondimensional coefficients, domains and discretization parameters. The shock starts from Eq. (5) at $t = 0$. The remaining experiments use periodic initial data for a plateau and for two localized pulses.

Here, T denotes the final simulation time and N counts stored nodes according to Section 2.4. The finest shock grid has $N = 121$, $h = 0.25$ and $\Delta t = 0.03125$. Its endpoint data are exact values, subject to the closure limitation in Section 2.5.2. The periodic grids omit the repeated right endpoint. A separate temporal refinement study is described with $N = 161$, but its time steps and error rows are not available.

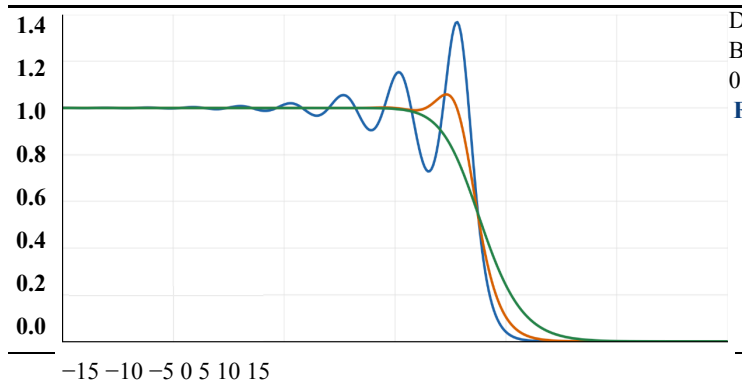
dence between grid levels and error rows, and the time at which the errors were measured are not recorded. Table 2 therefore supports an improvement under refinement but does not determine a verified convergence order. Because h and Δt are reduced together, even a complete coupled table would need further analysis to separate spatial and temporal contributions.

The separate temporal study is described as recovering second-order behaviour before spatial error becomes dominant. This is consistent with the formal temporal accuracy of Eq. (13), but the absent time-step and error table prevents an independent estimate of the observed order. No runtime measurements or direct comparisons with other discretizations are available.

3.3 Effect of dissipation on the front

For $A = 1$ and $B = 0.08$, the critical dissipation value is $\sqrt{(2AB)} = 0.4$. The three values $C = 0.05, 0.20$ and 0.50 therefore give $\chi = 0.125, 0.500$ and 1.250 , respectively. Figure 1 shows the front profiles attributed to $T = 8$. Their changing shape illustrates the attenuation of dispersive oscillations as C increases.

Amplitude u at $T = 8$



Displayed position (coordinate origin unconfirmed)

Blue: $C = 0.05$, $\chi = 0.125$ Orange: $C = 0.20$, $\chi = 0.500$ Green: $C = 0.50$, $\chi = 1.250$

Figure 1. Front profiles at $T = 8$ for the three dissipation coefficients. The displayed coordinate spans -15 to 15 . Its origin relative to the initial transition in Eq. (19) is not documented; the curves support a comparison of shape, but their absolute front positions remain unconfirmed.

Table 3. Reported diagnostics for the dissipation sweep at $T = 8$.

C	χ	max u	min u	Thickness	Total variation
0.05	0.125	1.358112	-0.007409	1.232459	3.701492
0.20	0.500	1.048937	-0.000066	2.071154	2.115303
0.50	1.250	0.999999	0.000000	4.096420	1.999998

Table 3 records the reported leading front thickness and the total variation of the full periodic solution. The region used to calculate the maximum and minimum values is not specified. Relative to the unit plateau, the reported maximum corresponds to an overshoot of approximately 35.8% for $C = 0.05$ and 4.9% for $C = 0.20$. At $C = 0.50$, the reported extrema are approximately 0 and 1, and the total variation is approximately 2. The transition thickness is about 3.3 times its value at $C = 0.05$.

These trends agree qualitatively with the local eigenvalue criterion in Section 2.3. The two subcritical cases have oscillatory tails, whereas

the supercritical case shows a smooth transition at the displayed resolution. Independent refinement is still needed to establish convergence of the overshoot, wavelength and thickness. In addition, Eq. (19) places the initial decreasing front near $x = 15$, while Figure 1 places the displayed transition near $x = 4$ at $T = 8$. A coordinate translation may explain this discrepancy, but its value cannot be inferred reliably without the original plotting data.

3.4 Interaction of two unequal pulses

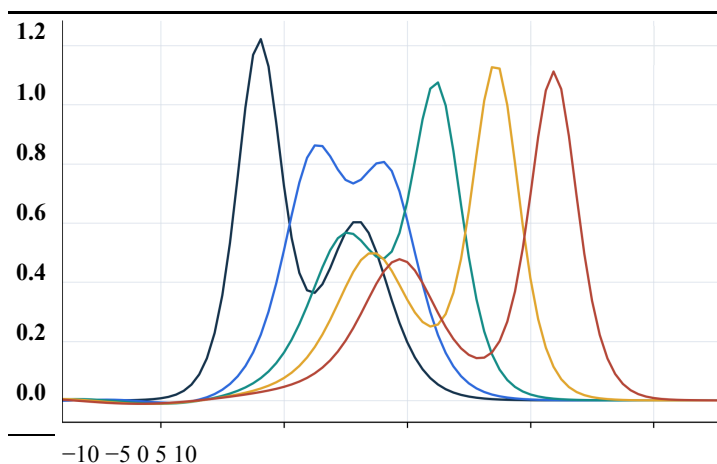
The pulse experiment begins with the sum of two localized profiles,

$$u(x, 0) = a_1 \operatorname{sech}^2[k_1(x + 6)] + a_2 \operatorname{sech}^2[k_2(x + 2)] \quad (20)$$

$$a_1 = 1.2, a_2 = 0.6, k_j = \sqrt{(Aa_j/(12B))}, j = 1, 2. \quad (21)$$

For the zero-background KdV equation with $C = 0$, an isolated pulse of amplitude a has speed $Aa/3$. The taller pulse is initially centred at $x = -6$, behind the smaller pulse centred at $x = -2$, so their amplitude-dependent speeds favour an interaction. The parameter $C = 0.015$ introduces dissipation. Equation (20) is an algebraic superposition of single-pulse profiles, rather than an exact two-soliton solution, and the nonlinear term uu_x couples their subsequent evolution. Amplitude $u(x, t)$

broader trailing component. Small negative values are visible in the curves. The reported maximum relative mass drift is approximately 3×10^{-5} , and the reported energy decrease over the simulated interval is approximately 17.2%. These aggregate values are consistent with dissipative evolution, but time-resolved diagnostic values and an energy-balance residual are not available. Figure 2 consequently supports a qualitative interpretation of the interaction without establishing unconditional numerical stability or independently verifying the integral diagnostics.



Position x

Figure 2. Evolution of two unequal pulses at $t = 0, 6, 12, 18$ and 24 . The periodic domain is $[-25, 25]$, and the displayed region is approximately $-14 \leq x \leq 13$.

The profiles show the approach and substantial overlap of the pulses, followed by the appearance of a leading localized maximum and a

4 Discussion

4.1 Interpretation of the numerical results

The formulation combines spline-based spatial differentiation with a time discretization that retains the nonlinear convective product at both endpoints of each step. The analytical Jacobian gives the corresponding Newton system explicitly. The exact travelling wave fixes the benchmark speed and far-field states, while the integral identities provide physical reference quantities for the periodic calculations. These components establish a consistent mathematical framework for evaluating the numerical outputs.

The reported shock errors decrease as the coupled grid is refined, suggesting improved resolution of the travelling profile. Their interpretation remains limited by the incomplete boundary closure and refinement records. The dissipation sweep provides a complementary test: increasing C suppresses the upstream wavetrain, reduces overshoot and broadens the transition. This behaviour is consistent with the change from complex to real upstream eigenvalues. Since the periodic plateau contains two transitions and evolves in time, the local travelling-wave criterion should be applied to the leading front rather than treated as a global result for the complete field.

The pulse calculation shows nonlinear interaction and changes in the relative shapes of the leading and trailing structures. An energy de

crease is physically expected when $C > 0$. Quantitative collision inelasticity, however, cannot be isolated from viscous attenuation using these profiles alone. Such a measurement would require a corresponding $C = 0$ control calculation, separate pulse amplitudes before and after interaction, and converged integral diagnostics.

4.2 Limitations and reproducibility

A reproducible assessment requires the complete finite interval boundary rows and spline closure, confirmation of the periodic wrapping convention, and the nonlinear solver settings. The refinement records must identify every grid, time step, error norm and measurement time. The front study additionally requires the plotting coordinate origin, the region used for the extrema, and the crossing rule used for thickness. For the pulse study, diagnostic histories are needed to test the energy balance rather than relying only on endpoint or aggregate values.

The available evidence does not establish an accuracy or efficiency advantage over compact finite differences or cubic Hermite collocation. A fair comparison would use the same equations, domains, initial data, boundary conditions and measurement times, together with comparable nonlinear solver tolerances. Reporting errors alongside runtimes and problem sizes would then permit the accuracy and computational cost of the competing methods to be assessed.

5 Conclusion

A fully nonlinear quintic B-spline differential quadrature formulation with Crank–Nicolson integration has been presented for the Korteweg–de Vries–Burgers equation. The exact shock benchmark, spline derivative stencils and Newton Jacobian are stated consistently. The reported results indicate reduced shock errors under refinement, damping and broadening of dispersive fronts as dissipation increases, and nonlinear interaction of unequal pulses. The finest reported shock errors are $L_2 = 2.941 \times 10^{-5}$ and $L_\infty = 3.895 \times 10^{-5}$; the pulse calculation reports a maximum relative mass drift of approximately 3×10^{-5} and an energy loss of approximately 17.2%.

These findings support a preliminary computational assessment of the formulation. Complete boundary and solver records, refinement tables, a consistent front coordinate and time-resolved diagnostics are needed to confirm the reported numerical performance. Direct comparisons under matched conditions should precede any claim of superior accuracy or efficiency.

References

1. Korteweg DJ, de Vries G. On the change of form of long waves advancing in a rectangular canal, and on a new type of long stationary waves. *Philosophical Magazine*. 1895;39(240):422–443.
2. Burgers JM. A mathematical model illustrating the theory of turbulence. *Advances in Applied Mechanics*. 1948;1:171–199.
3. Su CH, Gardner CS. Korteweg–de Vries equation and generalizations. III. Derivation of the Korteweg–de Vries equation and Burgers equation. *Journal of Mathematical Physics*. 1969;10(3):536–539.
4. Johnson RS. A non-linear equation incorporating damping and dispersion. *Journal of Fluid Mechanics*. 1970;42(1):49–60.
5. Canosa J, Gazdag J. The Korteweg–de Vries–Burgers equation. *Journal of Computational Physics*. 1977;23(4):393–403.
6. Bellman R, Kashef BG, Casti J. Differential quadrature: A technique for the rapid solution of nonlinear partial differential equations. *Journal of Computational Physics*. 1972;10(1):40–52.
7. Bellman R, Kashef BG, Lee ES, Vasudevan R. Differential quadrature and splines. *Computers & Mathematics with Applications*. 1975;1(3–4):371–376.
8. de Boor C. A Practical Guide to Splines. Revised edition. New York: Springer; 2001.
9. Başhan A, Karakoç SGB, Geyikli T. Approximation of the KdVB equation by the quintic B-spline differential quadrature method. *Kuwait Journal of Science*. 2015;42(2):67–92.
10. Koroche KA, Chemeda HM. Sixth-order compact finite difference method for solving KDV-Burger equation in the application of wave propagations. *Iranian Journal of Numerical Analysis and Optimization*. 2022;12(2):277–300.
11. Parumasur N, Adetona RA, Singh P. Efficient solution of Burgers', modified Burgers' and KdV–Burgers' equations using B-spline approximation functions. *Mathematics*. 2023;11(8):1847.
12. Ari M. A cubic Hermite collocation approach for solving the Korteweg–de Vries–Burgers equation. *Eskişehir Technical University Journal of Science and Technology A – Applied Sciences and Engineering*. 2025;26(4):445–459.
13. Crank J, Nicolson P. A practical method for numerical evaluation of solutions of partial differential equations of the heat-conduction type. *Proceedings of the Cambridge Philosophical Society*. 1947;43(1):50–67.
14. Kudryashov NA. On “new travelling wave solutions” of the KdV and the KdV–Burgers equations. *Communications in Nonlinear Science and Numerical Simulation*. 2009;14(5):1891–1900.

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