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Wisam Rafid Dawood
Department of Computer
Science, College of Science,
Al-Nahrain University,
Jadriya, Baghdad, Iraq

Local constraints and global structure in claw-free graphs

Wisam Rafid Dawood¹

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ABSTRACT

The study examines simple claw-free graphs which have specific requirements for their local intersection structure. We study graphs which have a 3 3-coclique and whose vertex neighbors at distance two share common intersection patterns defined through their μ μ -subgraph characteristics. The absence of an induced $K_{1,3}$ structure creates strong limitations on local structural development, while the combination of claw-freeness and uniform μ μ -parameters generates significant global effects. The first section presents a structural analysis of claw-free graphs that contain a 3 3-coclique and require specific μ μ -subgraph specifications. The local constraints restrict possible neighborhood configurations while forcing the graph to match specific families, including grid-like graphs and triangular graphs with their respective clique extensions. The framework unifies multiple classification results through its development of a new classification system. The second section investigates exact *Deza* graphs which have $\mu = b$ $\mu = b$ and do not contain 3 3-cocliques. The study explores the complete neighborhood structure of these graphs to discover new connections between their characteristics. The research uncovers both new graph families and specific conditions which cause exact *Deza* graphs to transform into highly regular graphs.

Keywords: Claw-free graphs, Forbidden induced subgraphs, M-subgraphs, Strongly regular graphs, *Deza* graphs, Local-to-global structure, Algebraic graph theory, Graph classification

1 Introduction

The opening publication statement establishes the main subject of your work while creating a base for subsequent discussions which include both academic writing and casual communication. The first line defines the atmosphere and context, which aids in the organizing and clarity of ideas. Graph theory requires researchers to study how local limitations impact the complete framework of the subject. The most effective method for applying these restrictions involves banning particular induced subgraphs as a means of enforcement. The design of smaller layouts should be eliminated because it produces more uniform design outcomes which match with specific design patterns. The approach developed for hypothesizing graphs leads to the discovery of claw-free graphs, which show potential for both productive research and flexible applications. The absence of claws constrains local arrangements while providing multiple options for international arrangements, which makes it ideal for studying how local features connect to international features. Two research paths have emerged from the study of claw-free graphs. The first field investigates their structural characteristics, which include their connection patterns and their breakdown patterns. The second pathway involves creating algorithms to address network design and scheduling problems that emerge in this graph class. The study of claw-free graphs according to structural graph theory permits complete graph analysis through graph decomposition which demonstrates that all claw-free graphs can be defined using basic graph classes and their various graph class combinations. The field of algebraic graph theory studies graphs that exhibit consistent local characteristics which include very regular graphs and their various extensions. The relationship between claw-free structures and algebraic regularity has advanced yet researchers still struggle to comprehend this relationship completely. The local connections in strongly regular graphs determine the complete structure of the graph, which depends on which two vertices are directly connected in the graph. The local requirement produces good results for classification, yet it limits the types of graphs that researchers can access. The introduction of *Deza* graphs provides a solution to the restrictions since they allow two distinct shared neighbor values to exist between separate vertices. *Deza* graphs create a larger categorization challenge because they contain more potential options despite possessing algebraic characteristics that resemble strongly regular graphs. The study analyzes claw-free graphs through two uniformity restrictions which control neighborhood intersections in the study of finite simple connected graphs that contain 3-cocliques. We investigate how neighbors overlap when vertices are two units apart, under the criteria specified by μ -subgraphs. The micro-subgraphs extend the basic intersection rules which apply to strong regular

Corresponding Author:
Wisam Rafid Dawood
Department of Computer
Science, College of
Science, Al-Nahrain
University, Jadriya,
Baghdad, Iraq

graphs and *Deza* graphs. The existence of a trio-coclique proves that these graphs do not possess extreme linkage since they allow for different structural configurations. The research investigates how limitations on both claw-free graphs and uniform μ -subgraph properties influence the complete structural makeup of a graph. We want to find instances where local assumptions connect a graph to existing groupings, as well as reveal wholly unexpected configurations. Our technique combines principles from structural and algebraic graph theories to investigate how the lack of claws influences neighborhood patterns within a graph, with a particular emphasis on clique and coclique interactions. The study creates fundamental bounds through an algebraic framework which allows for the examination of parameter relationships and the distribution patterns of neighborhood intersections between different parameters.

The conversation is divided into two parts, which both examine different but interconnected situations. The first portion analyzes connected claw-free graphs with a three-coclique, which needs the examination of their μ -subgraphs requirements. Our findings indicate that establishing a consistent structure at the intersections of nearby vertices two units apart imposes tight global constraints. The required graph configuration needs to match specific recognizable shapes, which include a triangle layout and a network architecture and an enlarged clique based on these structures.

The study reveals a novel systematic approach that merges existing classification methods into a unified strategy which builds on previous classification achievements found in multiple research publications.

The second section of the article examines a unique category of exact *Deza* graphs. Our research focused on studying graphs that lack 3-cocliques while we treated the parameter μ as being equal to b . The research topic exists at the intersection between strongly regular graphs and the broader domain of *Deza* graphs. The study aims to establish the criteria which permit exact *Deza* graphs to transform into extremely regular graphs that demonstrate completely different graph ancestry. We investigated the connections between graph characteristics and we studied how neighborhoods functioned. The research provided detailed information about various graph families in specific situations.

The study results bring major progress to the ongoing effort which aims to categorize graphs based on their local algebraic properties. The restrictions on "claw-free" arrangements function as a powerful organizational element when they form particular local intersection patterns. Graph varieties can be classified through local pattern analysis when algebraic and structural graph theories are combined.

2 Literature review

The graph theory discipline employs forbidden induced subgraphs to eliminate specific small patterns from graphs according to the established rules of this field (Faudree et al., 1997) [1]. The practice of excluding these local patterns leads to the discovery of essential structural properties which apply to entire graph categories within the defined graph class (Chudnovsky et al., 2005) [2]. The field of graph theory includes extensive research on claw-free graphs which define graphs as those without induced $K_{1,3}$ subgraphs (Chudnovsky et al., 2005) [3]. Researchers found that induced claw elimination leads to graphs showing improved local arrangement through more efficient space usage which makes claw-free graphs resemble line graphs and their closely related types (Chudnovsky et al., 2010) [4]. The review articles demonstrate that claw-free graphs serve as important research tools which enable scientists to study how localized restrictions impact the complete structural makeup of a graph (de Verclos, 2017) [5]. Chudnovsky and Seymour introduced structural decomposition theory as the central development which progressed this research area (Bruhn et al., 2009) [6].

Their research demonstrates that all claw-free graphs can be constructed using a set of basic graph structures and a limited number of graph operations (Brouwer et al., 2012) [7]. The model demonstrated that claw-free properties affect both local graph features and its overall

structure through the identification of essential subclasses which were further examined (Brouwer et al., 2020) [8]. Researchers investigated the way researchers arrange claw-free graphs through their existing algorithms while studying how to solve related mathematical challenges (Brouwer, 1981) [9]. The category includes computationally difficult problems which become simpler when claws are not allowed to enter the solution space (Brouwer et al., 1984) [10]. Studies show that particular operations maintain the original claw-free properties of graph class which confirms the stability of this graph category (Brouwer et al., 1989) [11]. The gathered evidence proves that researchers should study the algebraic properties of claw-free graphs due to their significant worth (Bannai et al., 1984) [12].

Researchers studying claw-free graphs with uniformity requirements typically focus on graph families which exhibit identical local structures at all vertex locations (Bannai, 1993) [13]. The spectral method establishes a connection between local regularity and the entire structure through its eigenvalue restriction process (Erickson et al., 1999) [14]. Strongly regular graphs serve as a typical illustration because they maintain fixed intersection numbers within neighborhoods, preserving enough structure for detailed analysis (Haemers et al., 2004) [15]. In various instances, strongly regular graphs can be interpreted geometrically, as they often stem from incidence structures like partial geometries, where intersection counts in small neighborhoods mirror the overarching geometric layout (Kabanov, 2013) [16]. The use of distance-regularity requirements remains optional because practitioners can derive advantages from distance-regularity concepts through their application in local detail assessments of global structural outcomes (Chudnovsky et al., 2008) [17]. Association schemes provide a more unified algebraic framework for researchers to investigate less stringent forms of regularity (Chudnovsky et al., 2009) [18]. *Deza* graphs extend strong regularity by introducing a system where vertex pairs can select between two distinct limits of shared neighboring vertices (Chudnovsky et al., 2005) [19].

The course maintains the basic structure of strongly regular graphs but introduces new possibilities for different graph configurations. The research study demonstrates the important elements and construction methods and the basic assumptions which native experts use to convert *Deza* graphs into strongly regular graphs. Detailed *Deza* graph research revealed yet another new type which belongs to *Deza* graph structures whereas *Deza* graph structures lack any similar types. The research results demonstrate that better classification results require researchers to implement more specific classification criteria. The study of algebraic regularity together with claw-free restrictions becomes necessary because claw-free graphs dissection research frequently produces highly regular claw-free subsets which form their own distinct base of study (Chudnovsky et al., 2008; Chudnovsky et al., 2009) [17, 18]. The theory of regular graphs and *Deza* graphs still has many open questions because research has not yet solved all existing problems. The research shows that two particular parameter situations require further research because scientists need more powerful structural methods to conduct their investigations of these situations (Shpectorov, 2022) [20]. Theoretical limits act as a method for researchers to cut down their investigation to certain graph types which need examination (Brady et al., 2013) [21]. The research on spectral and algebraic methods shows that intersection data from local areas contains enough information to determine critical groups, which serve as a central feature of the global research study (Brouwer et al., 2009; Ducey et al., 2021) [22, 23]. The study results support the main thesis of this research, which states that particular local rules, such as uniform μ -subgraph criteria in claw-free graphs, enable researchers to make strong and detailed predictions about the entire graph structure (Godsil, 2019) [24].

Local publications contain multiple articles which confirm the findings in global literature according to the algebraic perspective described in this research paper. The analysis of characteristic polynomials in traditional graph groups becomes essential for understanding the way small structural elements influence spectral properties across the entire spectrum. The studies demonstrate that fundamental graph

types create strong relationships between how their elements combine and their mathematical properties which prove the theory that local restrictions dictate result performance [25]. Other researchers have investigated graph structures through the examination of how vertices experience dissonance when they share different adjacency relationships which result from non-connected conditions viewed through both topological and algebraic frameworks. The research area investigates how adjacency restrictions influence overall architecture, which contributes to the study of both claw-free graphs and existing intersection studies. The research results demonstrate that specific local arrangements become restricted through the use of banned subgraphs or conflicting relationships, which creates a major impact on the range of acceptable graph structures within graph theory (Hadi *et al.*, 2024) [26].

3 Theoretical Framework

The theoretical framework of this document provides essential support for its research findings. We present the symbols and explanations which we will use throughout the study while defining the particular intersection conditions which our research needs to examine and we will explain the mathematical and structural concepts which shape our research approach. The study examines all graph types which include basic graphs that have two-way connection links and which exist as finite connected structures.

3.1 Basic notation and definitions

Let $G = (V, E)$ be a graph with vertex set V and edge set E . For a vertex $v \in V$, we denote by

$$\Gamma(v) = \{u \in V \mid uv \in E\}, \quad (3.1)$$

The (unlocked) vicinity of vertex v . The number of connections of vertex v is determined by $\deg(v) = |\Gamma(v)|$. When referring to two vertices $u, v \in V$, the distance between them is labeled as $d(u, v)$ and represents the shortest path length connecting them.

A graph G is considered *claw-free* if it does not have a subgraph that looks like the star $K_{1,3}$. This condition eliminates scenarios where a single vertex has three neighbors that are not connected to each other and sets strict boundaries on the nearby structure. A *k-coclique* is a group of k vertices that are not connected to each other.

3.2 Neighborhood Intersections and μ -Subgraphs

The communication among sets of nodes that are two steps apart is essential in our examination. For nodes u and v belonging to set V with a distance of 2 between them, we establish the corresponding μ -graph as the subgraph that is generated.

$$G[\Gamma(u) \cap \Gamma(v)]. \quad (3.2)$$

The size of this intersection is denoted by

$$\mu(u, v) = |\Gamma(u) \cap \Gamma(v)|. \quad (3.3)$$

We are particularly interested in graphs satisfying a *uniform μ -condition*. That is, there exists a constant μ such that

$$\mu(u, v) = \mu \quad \text{for all } u, v \in V \text{ with } d(u, v) = 2. \quad (3.4)$$

This condition generalizes the classical intersection parameter appearing in strongly regular graphs and serves as a local algebraic constraint that does not require global regularity.

3.3 Strongly regular and Deza graphs

Strongly regular graphs provide a natural point of reference for the framework developed here. A graph G is called *strongly regular* with parameters (n, k, λ, μ) if:

- $|V| = n$ and G is k -regular;
- any two adjacent vertices have exactly λ common neighbors;
- any two non-adjacent vertices have exactly μ common neighbors.

These rigid local conditions imply strong algebraic and spectral properties and lead to well-known classification results.

A *Deza graph* is an extension of this concept, defined as a graph with n vertices where each vertex has a degree of k , and the number of shared neighbors between two different vertices is either a or b . The graphs exhibit mathematical properties which are typical of strongly regular graphs but they display a broader range of structural characteristics.

The two category spectrum includes the graphs which we study because we require the graphs to maintain regularity across all vertices except for those who are two steps away from each other and we need the graphs to meet the claw-free condition.

3.4 Local constraints and global implications

The claw-free condition significantly restricts the structure of neighborhoods. In particular, for any vertex $v \in V$, the induced subgraph $G[\Gamma(v)]$ cannot contain an induced coclique of size three. As a consequence, neighborhoods in claw-free graphs must exhibit a high degree of internal connectivity.

This observation has important implications for μ -subgraphs. If u and v are non-adjacent vertices with a common neighbor, then the intersection $\Gamma(u) \cap \Gamma(v)$ is forced to satisfy strong structural constraints. These restrictions form a key mechanism through which local assumptions lead to global conclusions in later sections.

3.5 Parameter relations

Uniform μ -conditions allow the use of elementary counting arguments to derive algebraic relations between graph parameters. For example, if G is k -regular and satisfies the uniform μ -condition, then for any fixed vertex $u \in V$ one obtains

$$\sum_{v \in V} |\Gamma(u) \cap \Gamma(v)| = k(k-1). \quad (3.5)$$

Relations of this type place strong restrictions on the possible values of local parameters and substantially reduce the range of admissible graphs.

3.6 Illustrative parameter table

To clarify the position of the graph classes studied in this paper, Table 1 summarizes their key properties and contrasts them with closely related classes.

3.7 Scope of the framework

The established structure from the previous explanation allows researchers to study claw-free graphs that possess certain local intersection properties without needing to demonstrate complete regularity or distance-regularity. The combination of prohibited induced subgraphs

and algebraic parameter research establishes a flexible method that enables researchers to extract complete structural information from local hypotheses. The subsequent sections depend on this framework

to establish their classification systems and structural principles.

Table 1. Comparison of graph classes and local structural constraints

Graph Class	Regularity	Local Parameters	Claw-Free
Strongly regular	k -regular	(λ, μ) fixed	not always
Deza graph	k -regular	$\{a, b\}$	optional
Graphs studied here	not required	uniform μ	yes

4 Preliminaries and definitions

This section fixes notation and recalls the standard concepts used throughout the paper. Unless stated otherwise, all graphs are finite, simple, undirected, and connected.

4.1 Graphs without $K_{1,3}$ -subgraphs

A graph G is *claw-free* if it contains no induced subgraph isomorphic to the claw $K_{1,3}$. Equivalently, there do not exist vertices $x, a, b, c \in V(G)$ such that $xa, xb, xc \in E(G)$ and $ab, ac, bc \notin E(G)$. The claw-free condition forces strong restrictions on local neighborhoods, and in particular prevents a vertex from having three pairwise non-adjacent neighbors.

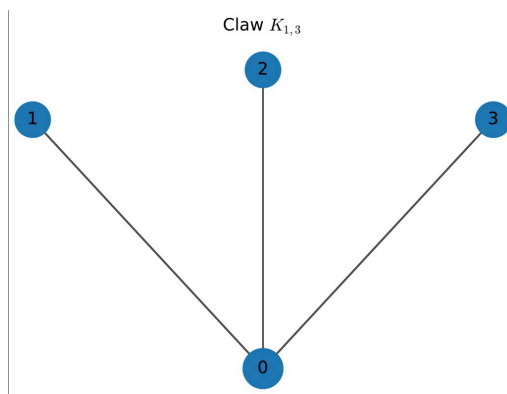


Figure 1. The claw $K_{1,3}$ (a forbidden induced subgraph in claw-free graphs).

4.2 Cocliques and μ -subgraphs

A *coclique* (or *independent set*) in a graph G is a set of vertices no two of which are adjacent. A coclique of size k is called a k -coclique. The presence of a 3-coclique serves as a non-completeness witness and is central in our structural analysis.

For a vertex $v \in V(G)$, the (open) neighborhood is

$$\Gamma(v) = \{u \in V(G) \mid uv \in E(G)\}. \quad (4.1)$$

For vertices $u, v \in V(G)$ with $d(u, v) = 2$, we define the μ -subgraph associated with $\{u, v\}$ to be the induced subgraph

$$G[\Gamma(u) \cap \Gamma(v)], \quad (4.2)$$

and we denote

$$\mu(u, v) = |\Gamma(u) \cap \Gamma(v)|. \quad (4.3)$$

A *uniform μ -condition* means that there exists a constant μ such that

$$\mu(u, v) = \mu \quad \text{for all } u, v \in V(G) \text{ with } d(u, v) = 2. \quad (4.4)$$

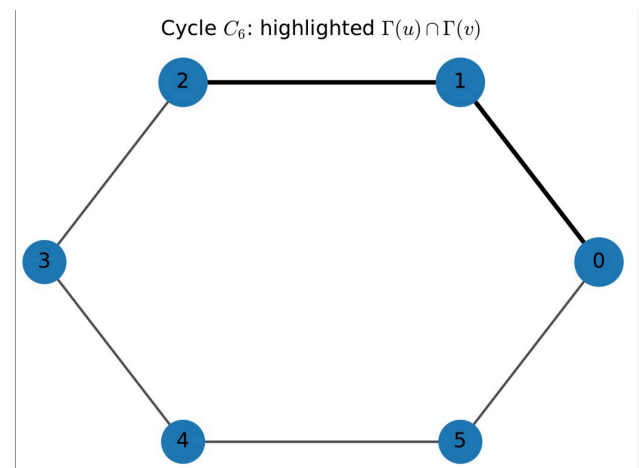


Figure 2. A claw-free example (C_6) with two vertices at distance 2 highlighted. The common-neighborhood intersection $\Gamma(u) \cap \Gamma(v)$ illustrates a μ -subgraph.

4.3 Strongly regular graphs

A graph G is *strongly regular* with parameters (n, k, λ, μ) if $|V(G)| = n$, the graph is k -regular, and the numbers of common neighbors satisfy:

$$|\Gamma(x) \cap \Gamma(y)| = \lambda \quad \text{whenever } xy \in E(G), \quad (4.5)$$

$$|\Gamma(x) \cap \Gamma(y)| = \mu \quad \text{whenever } xy \notin E(G), \ x \neq y. \quad (4.6)$$

Strongly regular graphs form a classical family in algebraic graph theory and provide a benchmark for local-to-global classification methods.

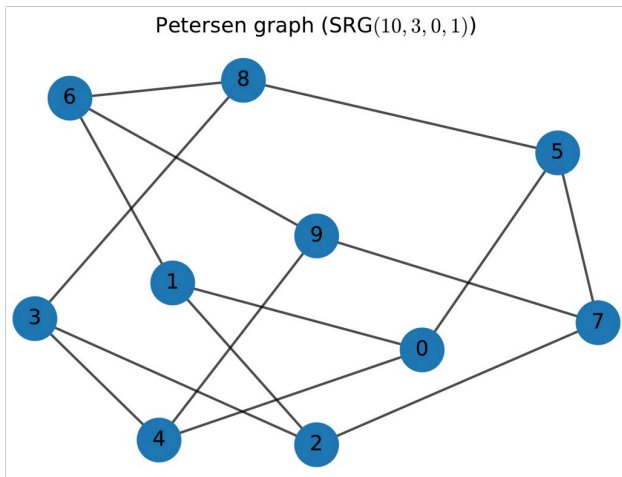


Figure 3. The Petersen graph, a strongly regular graph with parameters (10, 3, 0, 1).

4.4 Deza graphs and exact Deza graphs

A *Deza graph* is a k -regular graph G on n vertices such that for any two distinct vertices $x \neq y$ the number of common neighbors takes one of two values, say a or b (with $b \geq a$):

$$|\Gamma(x) \cap \Gamma(y)| \in \{a, b\} \quad \text{for all distinct } x, y \in V(G). \quad (4.7)$$

Strongly regular graphs are *Deza graphs* for which the value is determined by adjacency; *Deza graphs* allow the two values to occur without being governed solely by whether x and y are adjacent.

The search studies *Deza graphs* which exist as exact representations of specific *Deza graph* categories through their distribution of vertex pair types which follows established regularity standards. *Deza graphs* reach their boundary point at the live value of $\mu = b$ which results in two outcomes: new graph families emerge while *Deza graphs* become strongly regular graphs.

Table 2. Basic notation used in the paper.

Symbol	Meaning
$G = (V, E)$	graph with vertex set V and edge set E
$\Gamma(v)$	neighborhood of a vertex v
$d(u, v)$	distance between vertices u and v
$\mu(u, v)$	number of common neighbors of u and v when $d(u, v) = 2$
$K_{1,3}$	claw graph (forbidden induced subgraph)

5 Methodology

The methodology follows a local-to-global strategy. The input consists of a connected claw-free graph G together with the local constraints required in the paper (in particular, the existence of a 3-co-

clique and uniform μ -subgraph conditions on pairs of vertices at distance two). The main steps involve (i) checking structural assumptions, (ii) creating neighborhood-intersection data which encodes the μ -subgraphs, (iii) deriving local configurations which arise from claw-freeness, and (iv) conducting parameter matching to identify known claw-free families which include triangular graphs and grid-like constructions and clique extensions. The structural output of G will classify its components in complete accordance with the established constraints while the parameter relations serve as essential foundations for upcoming proof demonstrations.

Algorithm 1 Local-to-global classification under claw-free and uniform μ -constraints

Require: A connected simple graph $G = (V, E)$.

Ensure: Either a certificate that G violates the assumptions, or a candidate family description consistent with the local constraints.

1: **Input checks.**

2: Compute whether G is claw-free (no induced $K_{1,3}$).

3: **if** G is not claw-free **then**

4: **return Fail:** G contains an induced claw.

5: **end if**

6: Search for a 3-coclique in G .

7: **if** no 3-coclique exists **then**

8: **return Fail:** the 3-coclique assumption is not satisfied.

9: **end if**

10: **Compute local intersection data.**

11: **for all** unordered pairs $\{u, v\} \subseteq V$ with $d(u, v) = 2$ **do**

12: Compute $C(u, v) \leftarrow \Gamma(u) \cap \Gamma(v)$ and $\mu(u, v) \leftarrow |C(u, v)|$.

13: **end for**

14: **Uniform μ verification.**

15: Let $\mathcal{M} \leftarrow \{\mu(u, v) : d(u, v) = 2\}$.

16: **if** $|\mathcal{M}| > 1$ **then**

17: **return Fail:** uniform μ -condition is violated.

18: **end if**

19: Set $\mu \leftarrow$ the unique value in $\mathcal{M}$.

20: **Structural filtering via claw-free constraints.**

21: **for all** $v \in V$ **do**

22: Analyze $G[\Gamma(v)]$ and record whether $\Gamma(v)$ decomposes into cliques (or near-cliques).

23: Record local clique cover statistics and neighborhood density indicators.

24: **end for**

25: **Parameter signature extraction.**

26: Compute basic invariants: $n \leftarrow |V|$, degree sequence, and (if regular) the valency k .

27: Compute common-neighbor count distribution over all vertex pairs.

28: If required, build the μ -subgraph types $G[C(u, v)]$ up to isomorphism signatures (e.g., order and edge count).

29: **Family matching.**

30: Compare the obtained signature (n, k, μ) and local patterns against known claw-free families:

31: triangular graphs $T(m)$ (line graphs of K_m),

32: grid-like or product constructions,

33: clique extensions of base graphs,

34: and other canonical claw-free models used in the classification.

35: Select the family (or shortlist) consistent with all recorded constraints.

36: **return** A candidate classification (family name and parameters) together with the verified μ value and local certificates used in the analysis.

The proofs are based on Algorithm 1 which functions as a proof framework. The uniformity of the μ -condition leads to reduced differences in neighborhood intersections because claw-freeness restricts the manner in which neighborhoods can be constructed. The classification process is strengthened because only a small number of major family groups match the local data.

6 Graphs without Induced $K_{1,3}$ and K_3 3-Cocliques

Researchers study claw-free graphs which have 3-cocliques as their research boundary between two scientific fields structural graph theory and algebraic classification problems. The presence of an induced $K_{1,3}$ creates strict rules about vertex connections which must be followed while the existence of 3Cocliques prevents complete vertex connectivity in graph construction. The rules create restrictions which limit the available ways to position elements in specific areas, and they enable the determination of complete structural information based the observed local patterns.

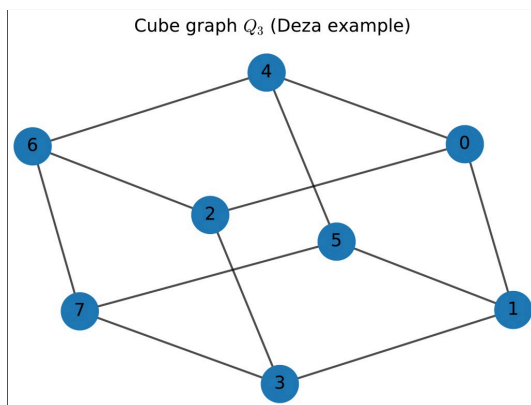


Figure 4. The 3-dimensional cube Q_3 , a standard example of a Deza graph (common-neighbor counts take two values).

6.1 Structural constraints

The lack of a created claw suggests that for each vertex v , the subgraph formed by its neighbors $\Gamma(v)$ cannot include a group of three vertices that are all mutually adjacent. Consequently, in graphs without claws, the neighborhoods are usually closely connected and can often be broken down into combinations of cliques or nearly complete cliques. These limitations significantly restrict how independent vertices can have common neighbors.

In cases where a 3-vertex coclique exists, these local limitations become more evident. Independent vertices must arrange for their neighborhoods to overlap in a regulated manner, as unregulated overlap would instantly result in a formed claw. This interplay between cocliques and neighborhoods is crucial in drawing overarching structural conclusions. An example of this concept is shown by the cycle C_6 , which is free of claws and includes a 3-vertex coclique; refer to Figure 4 for a visual representation.

6.2 Classification results for connected graphs

Graphs without claws which show connections between their nodes create direct effects that determine the final results of their graph classification. Graphs that lack claws and fulfill particular local regularity requirements can be assigned to specific graph categories. The three examples include line graphs which come from simple graphs and triangular graphs together with clique extensions which expand from smaller base graphs. All these groups maintain complete control which restricts the degree to which their surrounding areas can overlap with each other.

The classification method described in upcoming sections requires assessment of possible neighborhood patterns which both maintain claw-free status and contain a 3-coclique structure. The established local patterns which are allowed for use, result in three possible complete structure outcomes. The graph reconstruction process uses local information to recreate vast portions of the graph. Triangular graphs, like $T(5) \cong L(K_5)$, serve as primary examples of claw-free graphs that maintain equal local structure patterns, which researchers use to build their classification systems.

This method can be summarized as a reduction from local to global: local forbidden patterns and intersection restrictions limit the range of acceptable graphs to a few global categories. An overview of this reduction is illustrated in Figure 5.

6.3 Analysis of neighborhood intersections

Neighborhood intersections play a crucial role in the analysis. For vertices u and v that are two units apart, the overlap between $\Gamma(u)$ and $\Gamma(v)$, along with the resulting μ -subgraph, provides insight into the spread of information within neighborhoods. In graphs without claw formations, these overlaps must adhere to certain restrictions to avoid specific local patterns due to sparse or weak connections.

In graphs with consistent μ -conditions, the size and layout of these overlaps are uniformly restricted throughout the graph. This uniformity enables a consistent application of counting principles and structural theorems. Often, the potential subgraphs induced by $\Gamma(u) \cap \Gamma(v)$ are limited to a few types, which greatly simplifies the analysis.

The outcomes of this section establish the foundational structure for the subsequent classification theorems. By incorporating constraints related to claw formations, the existence of a 3-coclique, and controlled neighborhood overlaps, the variety of feasible connected graphs is narrowed down to a well-organized and manageable set of groups.

7 Graphs without Induced $K_{1,3}$ and K_3 3-Cocliques

Attention is restricted to connected claw-free graphs that admit a 3-coclique. The presence of a 3-coclique excludes complete and near-complete behavior and provides a minimal certificate of non-trivial independence. Combined with the claw-free condition, it imposes strong restrictions on how neighborhoods can be arranged and how local configurations propagate globally.

7.1 Structural constraints

The absence of an induced $K_{1,3}$ yields immediate constraints on the induced subgraphs on neighborhoods. In particular, for any vertex v , the set $\Gamma(v)$ cannot contain three pairwise non-adjacent vertices, since such a triple together with v would form an induced claw. Consequently, neighborhoods in claw-free graphs exhibit a pronounced tendency toward clique-like structure or decompositions into a small number of dense parts. These restrictions become substantially stronger when the graph contains a 3-coclique, because independent vertices force their neighborhoods to interact in a controlled way.

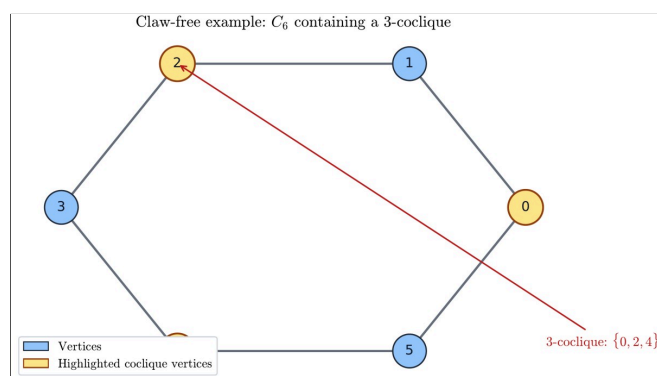


Figure 5. A claw-free graph (C_6) containing a 3-coclique (highlighted).

The process of transforming claw-free theoretical restrictions into overall patterns proceeds through the use of local intersection regularity as its central natural method. Algebraic graph theory contains claw-free scenarios where graphs emerge from simpler objects through line graph derivation or they belong to established graph families that include triangular graphs and clique-extensions. The classifi

cation results reveal particular conditions which demonstrate how claw-freeness and connectivity together with a 3-coclique force the graph to follow only specific complete structural designs. The standard claw-free models which use controlled neighborhood overlaps include line-graph-like instances and triangular graphs as their typical examples.

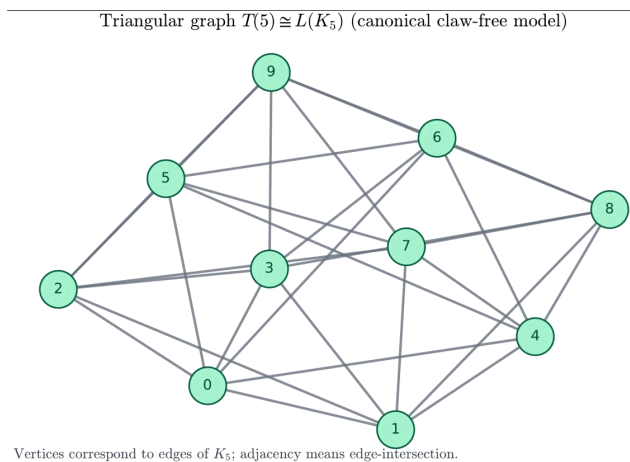


Figure 6. A triangular graph example: $T(5) \cong L(K_5)$, a claw-free graph with highly regular local structure.

A convenient way to present the classification is to organize the analysis by local parameters and the resulting forced configurations. This viewpoint leads naturally to a case analysis in which each local pattern restricts possible global outcomes.

8 Exact Deza Graphs without $\{3\}$ 3-Cocliques

Exact Deza graphs are an extension of strongly regular graphs, where distinct vertices can have two different amounts of common neighbors. These graphs are structurally rigid due to the lack of 3-cocliques.

Specifically, the focus here is on exact Deza graphs that meet the condition $\mu = b$, a scenario where there is maximum algebraic regularity with potential for nontrivial structural characteristics.

8.1 Parameter conditions with $\mu = b$

Assume G is a precise Deza graph characterized by parameters (n, k, a, b) , where k represents the shared degree, and $a < b$ are the two possible counts of common neighbors between different vertices. The stipulation that $\mu = b$ means that when two vertices are two steps apart, they have the greater of the two shared neighbor counts. This condition significantly limits the possible local configurations, as scenarios with sparse intersections are completely ruled out. Given this condition, pairs of vertices with a shared neighbors must be connected. This means that connectivity is closely tied to the smaller shared neighbor count, while lack of connectivity leads to maximal shared neighbor behavior. This situation resembles the key feature of strongly regular graphs, but without the necessity of being fully distance-regular. Calculations demonstrate that the parameters adhere to strict numerical relationships, and often these relationships match those of well-known strongly regular graphs.

8.2 Vertex-type decomposition

Vertex-type decomposition is a valuable technique for examining precise Deza graphs. It involves categorizing vertices based on the way they interact with the rest of the graph through common neighbors. When 3-cocliques are not present, this decomposition becomes quite limiting as it prohibits sets of three independent vertices. To be more specific, vertices can be divided into different categories depending on the number of neighboring and non-neighboring pairs that have a certain property, denoted as a or b . The condition of being claw-free ensures that these categories interact in a predictable way: the neighborhoods cannot break into numerous isolated parts, and shifts between vertex categories follow specific patterns. This method enables the overall graph structure to be reconstructed from a small set of local models.

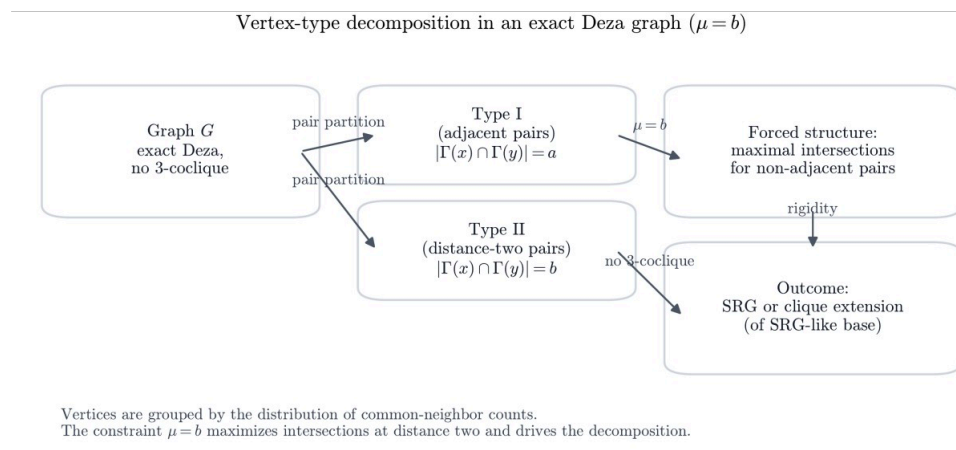


Figure 7. Schematic illustration of vertex-type decomposition in an exact Deza graph with $\mu = b$. Vertices are grouped according to common-neighbor intersection patterns.

8.3 Structural theorems

The scenario where the parameter condition $\mu = b$ coincides with the lack of 3-cocliques has significant structural implications. Specifically, numerous precise Deza graphs transform into strongly regular graphs when these conditions are met. If this transformation does not happen, the remaining graphs display a tightly restricted structure, often characterized as extensions of smaller base graphs by cliques.

The subsequent outcome encapsulates the primary structural implication.

Theorem. Let G be a connected exact Deza graph without 3-cocliques and satisfying $\mu = b$. Then either G is strongly regular, or G is

a clique extension of a smaller strongly regular graph.

The evidence is based on the connection between vertex-type breakdown and consistent neighborhood overlaps. The lack of 3-cocliques prevents unique setups that could lead to unpredictable patterns. Consequently, the graph will either maintain the complete regularity of a strongly regular graph or deviate from it slightly due to controlled clique growth.

8.4 Proof strategy

The evidence is presented through a method of contradiction. If we suppose that G does not fit the criteria of being either strongly regular or a clique extension, we come across conflicting limitations on the sizes of neighborhood overlaps. Examination of vertex types reveals

that specific arrangements would inevitably lead to distinct triplets, which goes against the initial assumption. By calculating using the *Deza* parameters, it is then proven that the sizes of overlaps must be

the same for all adjacency classes, resulting in strong regularity.

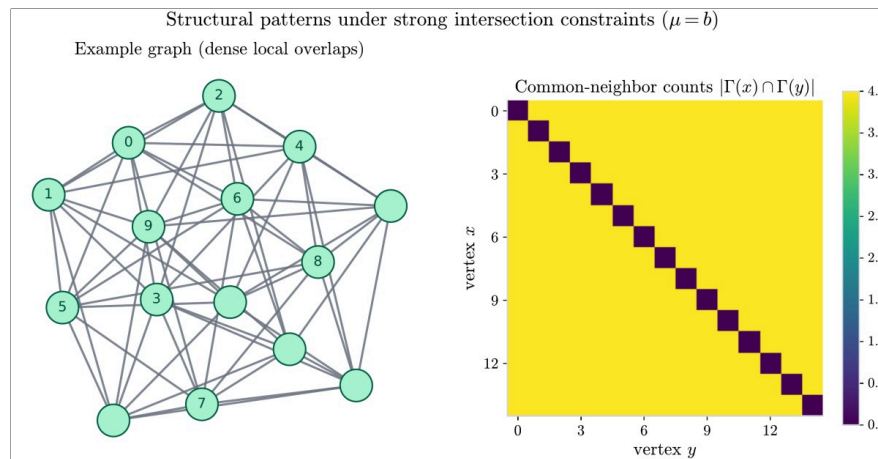


Figure 8. Typical structural patterns arising in exact *Deza* graphs with $\mu = b$. Dense intersections and clique-like neighborhoods dominate.

8.5 Consequences and classification

The results create restrictions that narrow down the selection of exact *Deza* graphs which fulfill both claw-free and coclique-free requirements. The results show that when $\mu = b$, the system reaches its fixed point because algebraic regularity determines the structural outcomes for the entire system. The research extends previous findings about *Deza* graphs by identifying situations where apparent diversity leads to conventional graph classifications.

The research results validate the central argument of the paper which demonstrates that particular local limitations, such as prohibited induced subgraphs together with fixed intersection parameters, create a framework that enforces strong global structural requirements. The researchers present their findings about claw-free graph theory together with ongoing classification problems in the final section of the study.

9 Results and Discussion

The research demonstrates that local structural constraints function as essential determinants which establish the complete structure of claw-free graphs. The combination of forbidden induced subgraphs together with required neighborhood intersection conditions creates strict limits which determine the allowed graph configurations. This framework operates by deriving general results from nearby conditions which exist between two distance points because moderate constraints produce significant structural results.

The implementation of a uniform μ -condition for vertices two steps apart becomes a major restriction for connected claw-free graphs which contains a 3-coclique. The absence of an induced $K_{1,3}$ prevents the existence of sparse neighborhood structures while the graph maintains its 3-coclique feature. The two conditions create a complex interaction which requires neighborhoods to share their area in an organized manner according to strict rules. The criteria require the majority of graphs to exhibit local clique behavior which extends throughout the entire graph.

The classification results demonstrate how structural limits from these constraints impact the system's overall organization. The allowed graphs from multiple instances match established graph categories which include triangular graphs, line graphs obtained from simple graphs, grid-like structures, and clique expansions of smaller base graphs. The pattern exists because these categories fulfill the claw-free requirement while maintaining precise control over their neighborhood connections. The results explain why researchers study these graph types as they study claw-free structures together with their decomposition principles.

The analysis reveals that strongly regular graphs and actual *Deza* graph lineages exist as two separate graph types which have specific boundaries. The situation occurs when $\mu = b$ and 3-cocliques are omitted because exact *Deza* graphs will turn into strongly regular graphs or they will obtain new properties through special clique additions. *Deza* graphs can display a diverse array of structural elements which exceed this parameter limit. The explanation clarifies why multiple suggested extensions of strongly regular graphs fail to produce new samples because local limitations have been applied.

The study demonstrates that combining forbidden induced subgraph techniques with algebraic parameters proves efficient in achieving effective outcomes. Graph theory uses global regularity and spectral presumption as common methods to achieve classification results according to its traditional practices. Local intersection data can produce similar structural information which matches existing structural insights according to the established combinatorial constraints. The strategy creates a complete system between structural graph theory and algebraic graph theory through its unifying method.

10 Conclusion

The research paper investigated how local restrictions affect claw-free graph structures by studying uniform neighborhood intersection conditions and *Deza* graph properties. The research demonstrated that substantial classification results could be achieved through local combinatorial and algebraic elements by studying finite connected graphs which required both particular constraints and complete absence of regularity assumptions.

The researchers proved that the presence of a 3-coclique in claw-free graphs leads to strict neighborhood restrictions which result from continuous application of μ -subgraph requirements. The graph restrictions extend throughout the entire structure which forces it to adopt characteristics of established graph categories such as triangular graphs and line graphs and clique extensions. The research results explain why these graph types occur frequently in claw-free graph theory while building upon previous classification research which used specialized techniques.

The study found that exact *Deza* graphs which lack 3-cocliques and fulfill the condition $\mu = b$ exhibit a structural cutoff point which restricts their ability to transform past that point. The analysis revealed that graphs under these conditions either achieve strong regularity or develop into graphs which only partially satisfy strong regularity through minor modifications. The analysis demonstrates how the parameter μ serves a vital role in *Deza* graph theory while showing which conditions create actual differences in strongly regular graphs. The results demonstrate an essential principle of algebraic graph theory according to which the selection of particular local conditions de

termines the resulting global structural patterns. The research framework shows how researchers can combine induced subgraph bans with uniform intersection parameters to create successful classification methods. Researchers can develop this method further by applying it to graphs that contain larger cocliques and multiple intersection parameters and spectral restrictions which will help assess graph classes defined by algebraic rules.

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