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Comparing the outdate quantities for quantity based and useful lifetime based fixed lifetime inventory models

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ABSTRACT

The ordering policy tells when to order and how much to order. There are many fixed lifetime ordering policies in the literature, some are based on the quantity of items on hand while others are based on the number of useful lifetimes remaining on the quantity on hand. In this work, we compare three ordering policies, two quantity base policies and one useful lifetime based policy. The outdate and shortage quantities are our primary concern. Numerical simulation was used to generate results.

Keywords: Order, Policy, Outdates, Shortages, Inventory

Introduction

The ordering policy of a fixed lifetime inventory system is an expression that tells the inventory manager when to order and how much to order. Some ordering policies are based on the quantity of items on hand while others are based on the number of useful lifetimes remaining on the quantity on hand. The goal of a good ordering policy is to minimize the outdate and shortage quantities. The egg inventory (chicken eggs) is our focus in this work.

The useful, safe and high quality lifetime of chicken eggs is four (4) weeks. Eggs can be stored through any of the following ways; refrigerated, hard-boiled or unrefrigerated. Refrigerated eggs remain consumable 4 weeks from the time of purchase. Hard-boiled eggs are best consumed 1 week, if placed in the refrigerator, whether peeled or unpeeled while unrefrigerated eggs are best consumed two weeks from the time of purchase. After 4 weeks, the eggs go bad and are no longer safe for consumption. There are three simple test that can be conducted to determine if the eggs are good or bad.

The

float test: place the egg in a bowl of water. A fresh egg will sink to the bottom on its side. An older egg, still safe for consumption will stand on one end while a bad egg will float to the top.

The

smell test: on breaking a bad or rotten egg, there will be a distinct sulfur smell that is poisonous to humans.

Appearance

broken eggs with a pink or iridescent white should be discarded

In this work, we shall compare three ordering policies on the egg inventory, our aim is to determine which ordering policy minimizes the outdate quantity. The ordering policies to be considered are the (Q, r) ordering policy by (Chiu, 1994) [1], the (s, S) ordering policy by (Fries, 1975) [2] and the $(y, m - 1)$ ordering policy by (Izevbizua et al., 2021) [3]. The (Q, r) and (s, S) ordering policies, base the decision to reorder on the quantity of items on hand while the $(y, m - 1)$ ordering policy, base the decision to reorder on the number of useful lifetime remaining on the items on hand in inventory.

Two of the biggest problems of the fixed lifetimes inventory system (like the egg inventory) is the quantity of outdates and the shortages associated with the inventory system. Outdates refer to items in inventory whose useful lifetime is over (that is expired items) and shortage refer to situation where inventory managers cannot meet the customers demand. Expired items are dangerous for consumption and leads to loss of revenue while shortages can lead to lost of customers goodwill. The goal of every inventory manager is to minimize these two occurrences. Next we briefly discuss the three ordering policies and run a simulation on them.

(Q, r) ordering policy: the policy was developed by (Chiu, 1994) [1] and has been used by authors like (Izevbizua et al., 2021; Izevbizua et al., 2019; Liu et al., 1999) [3, 4, 5] in the literature. The policy states that

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Q is ordered whenever on hand inventory drops to r . The decision to order is based on the on hand inventory dropping to r . The reorder frequency is determined by demand, if demand is low, the reorder frequency is low and if demand is high the reorder frequency is

high. When demand is low, it takes longer time for items to drop to the reorder point and this can lead to outdates. The policy is easy to implement and can be applied to small, medium and larger scarce businesses.

The total cost function obtained by (Chiu, 1994) [1] is given in equation (1)

$$EAC(Q, r) = \left[(K + cQ + pES + wER) / ET \right] + hOH \quad (1)$$

Substituting ES , ER and OH in equation (1) gives

$$EAC(Q, r) = \left[\frac{K + cQ + p \int_r^\infty (d_L - r) f_L(d_L) d_L + w \int_0^{r+Q} (r + Q - d_{m+L}) f_{m+L}(d_{m+L}) - \int (r - d_{m+L}) f_{m+L}(d_{m+L})}{ET + h \left(r - DL + \frac{Q}{2} \right)} \right] \quad (2)$$

where

$$\text{shortage} = ES = \int_r^\infty (d_L - r) f_L(d_L) d_L$$

$$\text{outdates} = ER = \int_0^{r+Q} (r + Q - d_{m+L}) f_{m+L}(d_{m+L}) d_{m+L}$$

$$\text{holding cost} = OH = r - DL + \frac{Q}{2}$$

where

K = fixed ordering cost

c = replenishment cost

p = shortage cost

w = outdate cost

h = holding cost

Q = order quantity

r = reorder point

ET = cycle time

The (s, S) ordering policy was used by (Fries, 1975) [2] to obtain the optimal ordering policy for perishable commodities with fixed life-time. The quantity of items ordered is not fixed and new orders arrive inventory every period. The total cost function obtained by (Fries, 1975) [2] is given as

$$f_n(W) = -cW_1 + V(W_{m-1}) + G_{n-1}(W, y), \quad W \geq 0, \quad n = 1, 2, 3, \dots \quad (3)$$

where

$$G_{n-1}(W, y) = cy + L(y) + \alpha \int_0^\infty f_{n-1}(z) \phi(t) dt$$

$$L(y) = h \int_0^y (y - t) f(t) dt + p \int_y^\infty (t - y) f(t) dt$$

$$V(x) = v \int_0^x (x - t) f(t) dt$$

The $(y, m - 1)$ ordering policy was designed by (Izevbizua et al., 2021) [3]. The policy is based on the number of useful lifetime remaining on the items on hand and the quantity ordered is not fixed. The cost function as obtained by (Izevbizua et al., 2021) [3] is Using the same set of conditions for three models, a 30 weeks simulation was carried out and the results for each model is as shown.

Table 1. Chiu's model: $Q = 50$, $r = 10$, $m = 4$ weeks

c = ordering cost

h = holding cost

p = shortage cost

v = outdate cost

m = lifetime

W_j = on hand inventory

α = discounting factor

week	Quantity ordered	Demand	outdates	shortages
1	50	25	-	-
2	25	15	-	-
3	10/50	10	-	-
4	50	10	-	-
5	40	10	-	-

(continued on next page)

week	Quantity ordered	Demand	outdates	shortages
6	30/50	20	10	-
7	50	15	-	-
8	35	15	-	-
9	20/50	18	2	-
10	50	18	-	-
11	32	12	-	-
12	20	18	-	-
13	2/50	16	-	-
14	36	20	-	-
15	20	10	-	-
16	10/50	17	-	-
17	43	15	-	-
18	28	16	-	-
19	12/50	10	2	-
20	50	18	-	-
21	32	18	-	-
22	14/50	13	1	-
23	50	10	-	-
24	40	10	-	-
25	30	22	8	-
26	50	15	-	-
27	35	15	-	-
28	20	10	-	-
29	10/50	20	-	-
30	40	20	-	-

Table 1 shows a 30 weeks (Q, r) ordering policy for egg, where the quantity order $Q = 50$ and the reorder point $r = 10$. New orders are placed only when on hand inventory drops to 10. From the table,

50 creates of eggs arrived, week 1 and 25 was used to meet demand. The demand for the second week was 15, reducing on hand inventory to 10 and a new order placed. With a useful lifetime of 4 weeks, any create not used to meet demand by the end of the fourth week expires and must be discarded.

$$C(x, y) = ky + h \int_0^{x+y} (x + y - t)f(t)dt + v \int_t^{x+y} (x + y)f(t)dt + \theta \int_0^y (y - t)f(t)dt \quad (4)$$

y = ordered quantity

x = items with one useful life left

θ = outdate cost

v = shortage cost

k = ordering cost

h = holding cost

Table 2. Fries model: order every week and quantity ordered not fixed. $m = 4$ weeks

Week	Quantity Ordered	Demand	Outdates	Shortages
1	50	15	-	-
2	35/40	15	-	-
3	20/40/30	8	-	-
4	12/40/30/20	10	2	-
5	40/30/20/10	35	5	-
6	30/20/10/20	32	-	-
7	18/10/20/30	15	3	-
8	10/20/30/20	30	-	-

(continued on next page)

Week	Quantity Ordered	Demand	Outdates	Shortages
9	30/20/40	15	-	-
10	15/20/40/20	10	5	-
11	20/40/20/30	38	-	-
12	22/20/30/40	18	4	-
13	20/30/40/20	20	-	-
14	30/40/20/30	25	5	-
15	40/20/30/10	38	2	-
16	20/30/10/20	50	-	-
17	10/20/50	30	-	-
18	50/50	30	-	-
19	25/50/20	10	-	-
20	15/50/20/10	14	1	-
21	50/20/10/20	40	10	-
22	20/10/20/25	18	2	-
23	10/20/25/20	15	-	-
24	15/25/20/20	40	-	-
25	20/20/30	18	2	-
26	20/30/20	20	-	-
27	30/20/30	18	-	-
28	12/20/30/20	10	2	-
29	20/30/20/20	25	-	-
30	25/20/20/30	20	5	-

Table 2 shows a 30 weeks, weekly order, using Fries model. 50 creates of eggs were received by the first week and 15 was used to meet demand. At the start of the second week, 35 creates of eggs from the first order were brought forward and 40 creates arrived. The demand in the second week is 15 reducing the first order to 20 while 40 and 30 represents the second and third orders.

The demand in the third week is 8, leaving 12 creates from the first order together with 40, 30 and 20 from the second, third and fourth

orders.

The demand in the fourth week is 10, leaving 2 from the first order with 40, 30 and 20 from the second, third and fourth orders. Since the useful lifetime is 4 weeks, the remaining two from the first order at the beginning of the fifth week outdates and are discarded. The process continues with the other orders.

Table 3. Izevbizua and Batubo model:

$$(y, m - 1), \quad m = 4 \text{ weeks}$$

Weeks	Quantity Ordered	Demand	Outdates	Shortages
1	60	35	-	-
2	25	10	-	-
3	15/40	25	-	-
4	30	15	-	-
5	15/30	10	-	-
6	5/30	15	-	-
7	20/40	15	-	-
8	5/40	15	-	-
9	30/45	20	-	-
10	15/45	14	1	-
11	45/35	20	-	-
12	25/35	23	2	-
13	35/40	30	-	-
14	5/40	25	-	-
15	20/40	21	-	-
16	39	15	-	-
17	14/40	25	-	-
18	29	20	-	-
19	9/40	15	-	-
20	34	14	-	-
21	20/35	15	-	-
22	5/35	15	-	-
23	25/40	16	-	-

(continued on next page)

Weeks	Quantity Ordered	Demand	Outdates	Shortages
24	9/40	10	-	-
25	39/20	8	-	-
26	31/20	20	-	-
27	11/20/20	8	3	-
28	20/20/30	25	-	-
29	15/30	14	-	-
30	1/30	20	-	-

Table 3 shows the $(y, m - 1)$ model, where a new order is placed when the useful lifetime remaining on the items on hand is one week. 60 crates of eggs were received in week one and the demand for the week was 35. The number of crates brought forward to the second

week was 25 and the demand was 10, reducing the first order to 15. By the third week with one useful lifetime remaining on the items on hand a new order of 40 crates was ordered and received. The process continues and items not used to demand within four weeks outdates.

Table 4. gives a summary of the number of outdates associated with each model

week	Chiu model	Fries model	Izevbizua and Batubo
1	-	-	-
2	-	-	-
3	-	-	-
4	-	2	-
5	-	5	-
6	10	-	-
7	-	3	-
8	-	-	-
9	2	-	-
10	-	5	1
11	-	-	-
12	-	4	2
13	-	-	-
14	-	5	-
15	-	2	-
16	-	-	-
17	-	-	-
18	-	-	-
19	2	-	-
20	-	1	-
21	-	10	-
22	1	2	-
23	-	-	-
24	-	-	-
25	8	2	-
26	-	-	3
27	-	-	-
28	-	2	-
29	-	-	-
30	-	5	-
Total	23	48	06

The three models deal with fixed lifetime products. The ordering policy of the first two is quantity base and that of the third model is useful lifetime based. The quantity ordered by the first model is fixed while that of the second and third model is not fixed. The first and third model have a fixed time for reordering while the second model orders every period. After a 30 weeks simulation of all three models, Table 4 shows that the first model have 23 outdates, the second model have 48 outdates and third model have 6 outdates. Ordering policies based on the number of useful lifetime can help reduce the number of outdates associated with the fixed life time inventories.

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