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Double even star decomposition of complete bipartite graphs

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ABSTRACT

In this paper we investigate (DESD) Double Even Star Decomposition of Complete Bipartite Graphs. We find some results in the concept Double Even Star Decomposition (DESD) of graphs. In (DED) $q = 2n(n+1)$. That is, the number of edges of G is double the sum of first n even numbers $2, 4, 6, \dots, 2n$. Since the number of edges of each subgraphs of G is even, we denote the DESD as $(S_2, S_4, \dots, S_{2n})$.

Keywords: Decomposition of graphs, Continuous Monotonic Decomposition, Double Even Decomposition (DED), Double Even Star Decomposition (DESD)

1. Introduction

All basic terminologies from Graph Theory are used in this paper in the sense of Frank (Harary) [1]. Gnanadhas. N and Paulraj Joseph. J discussed on Continuous Monotonic Decomposition (CMD) of graphs in (Gnanadhas *et al.*, 2000) [2]. Ebin Raja Merly. E and Gnanadhas. N introduced Arithmetic Odd decomposition (AOD) in (Ebin Raja Merly *et al.*, 2011) [3]. Joseph Varghese and A. Antony Samy introduced Double Continuous Monotonic decomposition in (Joseph Varghese *et al.*, 2010) [4]. Ebin Raja Merly. E and Suthiesh Goldy. J introduced Even Decomposition (ESD) in (Ebin Raja Merly *et al.*, 2016) [5] and Double Even Decomposition (DED) in (Suthiesh Goldy *et al.*, 2018) [6]. In this paper we investigate Double Even Star Decomposition of Complete Bipartite Graphs. The definitions which are useful for the present investigation are given below. In section 2, we recall the necessary and sufficient condition for a graph admits Even Decomposition and Double Even Decomposition. In section 3, we investigate the concept Double Even Star Decomposition (DESD).

Keywords: Decomposition of graphs, Continuous Monotonic Decomposition, Double Even Decomposition (DED), Double Even Star Decomposition (DESD).

2. Double even decomposition

Definition 2.1: An Even Decomposition is said to be Double Even Decomposition DED if the graph G can be decomposed into two copies of $(G_2, G_4, \dots, G_{2n})$ for all $n \in \mathbb{N}$ of a graph G .

Theorem 2.2: Any graph G admits Even Decomposition $(G_2, G_4, G_6, \dots, G_{2n})$, where $G_{2i} = (V_{2i}, E_{2i})$ and $|E(G_{2i})| = 2i$, $(i = 1, 2, 3, 4, \dots, n)$, if and only if $q = n(n+1)$ for each $n \in \mathbb{Z}^+$.

Remark 2.3: If any graph G admits DED $(G_2, G_4, G_6, \dots, G_{2n})$ where $|E(G_{2i})| = 2i$, $(i = 1, 2, 3, 4, \dots, n)$, if and only if $q = 2n(n+1)$ for each $n \in \mathbb{Z}^+$.

3. Double even star decomposition (DESD)

Definition 3.1 A Double Even Decomposition DED $(G_2, G_4, G_6, \dots, G_{2n})$ of G is called a Double Even Star Decomposition (DESD) if each G_{2i} is a star of size $2i$.

Definition 3.3: A **bigraph** or **bipartite** graph G is graph whose vertex set V can be partitioned into two subsets V_1 and V_2 such that every edge of G joins V_1 and V_2 . If G contains every edge joining V_1 and V_2 then G is a complete bigraph. If V_1 and V_2 have m and n vertices, we write $G = K_{m,n} = K(m,n)$. A star is a complete bipartite graph of the form $K_{1,n}$ and is denoted by S_n . Clearly $K_{m,n}$ has mn edges.

Theorem 3.4: A complete bipartite graph $K_{2^r}^{r+}$ (Harary, 1969) [1], $2 \times (2^{r+} \text{ (Harary, 1969) } [1]_{x-1})$ admits Double Even Star Decomposition $(S_2, S_4, S_6, \dots, S_{2n})$, if and only if $n = 2^{r+} \text{ (Harary, 1969) } [1]_{x-1}$, $x, r \in \mathbb{N}$.

Proof: Assume $K_{2^r}^{r+} \text{ (Harary, 1969) } [1]_{2 \times (2^{r+} \text{ (Harary, 1969) } [1]_{x-1})}$ admits DESD $(S_2, S_4, S_6, \dots, S_{2n})$. We know that $q(K_{2^r}^{r+} \text{ (Harary, 1969) } [1]_{2 \times (2^{r+} \text{ (Harary, 1969) } [1]_{x-1})}) = 2^{r+} \text{ (Harary, 1969) } [1]_{2 \times (2^{r+} \text{ (Harary, 1969) } [1]_{x-1})}$.

Therefore $2^{r+} \text{ (Harary, 1969) } [1]_{2 \times (2^{r+} \text{ (Harary, 1969) } [1]_{x-1})} = 2n(n+1)$, Implies $n = 2^{r+} \text{ (Harary, 1969) } [1]_{x-1}$.

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Conversely, To prove $K_{2^r}^{r+}$ (Harary, 1969) ^[1], $2x(2^r +$ (Harary, 1969) ^[1] _{$x-1$}) admits DESD ($S_2, S_4, S_6, \dots, S_{2n}$) Applying induction on r . We proved the result is true when $r = 1$

Suppose the result is true when $r = m$. That is $K_{2^m}^{m+}$ (Harary, 1969) ^[1], $2x(2^m +$ (Harary, 1969) ^[1] _{$x-1$}) admits DESD ($S_2, S_4, \dots, S_{2(x2m+1-1)}$).

We prove that the result is true for $r = m + 1$, that is to prove $K_{2^{m+1}}^{m+1}$ (Gnanadhas et al., 2000) ^[2], $2x(2^{m+1} +$ (Gnanadhas et al., 2000) ^[2] _{$x-1$}) admits DESD.

We have $q(K2m + 2, 2x(2m + 2x - 1)) = 2m + 2[2x(2m + 2x - 1)]$ Is equal to $2x$ (Gnanadhas et al., 2000) ^[2] $2^{2m+4} - 2x2^m +$ (Gnanadhas et al., 2000) ^[2]

Also, $q(K2m + 1, 2x(2m + 1x - 1)) = 2m + 1[2x(2m + 1x - 1)] = 2x22m + 2 - 2x2m + 1$

Therefore, $q(K2m + 2, 2x(2m + 2x - 1)) - q(K2m + 1, 2x(2m + 1x - 1))$ That is equal to $2x$ (Gnanadhas et al., 2000) ^[2] $2^{2m} +$ (Joseph Varghese et al., 2010) ^[4] $- 2x2^m +$ (Gnanadhas et al., 2000) ^[2] $- 2x$ (Gnanadhas et al., 2000) ^[2] $2^{2m} +$ (Gnanadhas et al., 2000) ^[2] $+ 2x2^m +$ (Harary, 1969) ^[1]

$= 3x$ (Gnanadhas et al., 2000) ^[2] $2^{2m} +$ (Ebin Raja Merly et al., 2011) ^[3] $- x.2^m +$ (Gnanadhas et al., 2000; Harary, 1969) ^[2, 1]

Now, $2[q(S_{2n+2}) + q(S_{2n+4}) + \dots + q(S_{4n+2})]$

Equal to $2[2n+2 + 2n+4 + \dots + 4n+4] = 3x$ (Gnanadhas et al., 2000) ^[2] $2^{2m} +$ (Ebin Raja Merly et al., 2011) ^[3] $- x2^m +$ (Gnanadhas et al., 2000) ^[2] $-----$ (Gnanadhas et al., 2000) ^[2]

where $n = 2^m +$ (Harary, 1969) ^[1] _{$x-1$} From (Harary, 1969) ^[1] and (Gnanadhas et al., 2000) ^[2] we have proved that

$q(K2m + 2, 2x(2m + 2x - 1)) - q(K2m + 1, 2x(2m + 1x - 1)) = 2[q(S_{2n+2}) + q(S_{2n+4}) + \dots + q(S_{4n+2})]$.

Therefore, $q(K2m + 2, 2x(2m + 2x - 1)) = q(K2m + 1, 2x(2m + 1x - 1)) + 2[q(S_{2n+2}) + q(S_{2n+4}) + \dots + q(S_{4n+2})]$.

Therefore, $K_{2^m}^{m+}$ (Gnanadhas et al., 2000) ^[2], $2x(2^m +$ (Gnanadhas et al., 2000) ^[2] _{$x-1$}) admits DESD ($S_2, S_4, \dots, S_{2(x2m+1-1)}$).

Therefore the result is true for $r = m + 1$.

Hence, $K_{2^r}^{r+}$ (Harary, 1969) ^[1], $2x(2^r +$ (Harary, 1969) ^[1] _{$x-1$}) admits Double Even Star Decomposition ($S_2, S_4, S_6, \dots, S_{2n}$).

Theorem 3.5: A complete bipartite graph $K_{2^r}^{r+}$ (Harary, 1969) ^[1], $2x(2^r +$ (Harary, 1969) ^[1] _{$x+1$}) admits Double Even Star Decomposition ($S_2, S_4, S_6, \dots, S_{2n}$), if and only if $n = 2^r +$ (Harary, 1969) ^[1] _{$x, x, r \in \mathbb{N}$} .

Proof: Assume $K_{2^r}^{r+}$ (Harary, 1969) ^[1], $2x(2^r +$ (Harary, 1969) ^[1] _{$x+1$}) admits DESD ($S_2, S_4, S_6, \dots, S_{2n}$) We know that $q(K_{2^r}^{r+}$ (Harary, 1969) ^[1], $2x(2^r +$ (Harary, 1969) ^[1] _{$x+1$}) = $2^r +$ (Harary, 1969) ^[1]. $2x(2^r +$ (Harary, 1969) ^[1] _{$x+1$}).

Therefore $2^r +$ (Harary, 1969) ^[1], $2x(2^r +$ (Harary, 1969) ^[1] _{$x+1$}) = $2n(n+1)$, Implies $n = 2^r +$ (Harary, 1969) ^[1] _{x}

Conversely, To prove $K_{2^r}^{r+}$ (Harary, 1969) ^[1], $2x(2^r +$ (Harary, 1969) ^[1] _{$x+1$}) admits DESD ($S_2, S_4, S_6, \dots, S_{2n}$) Applying induction on ' r '.

We proved the result is true when $r = 1$

Suppose the result is true when $r = m$. That is $K_{2^m}^{m+}$ (Harary, 1969) ^[1], $2x(2^m +$ (Harary, 1969) ^[1] _{$x+1$}) admits DESD

($S_2, S_4, \dots, S_{2(x2m+1)}$).

We prove that the result is true for $r = m + 1$, that is to prove $K_{2^{m+1}}^{m+1}$ (Gnanadhas et al., 2000) ^[2], $2x(2^{m+1} +$ (Gnanadhas et al., 2000) ^[2] _{$x+1$}) admits DESD.

We have $q(K2m + 2, 2x(2m + 2x + 1)) = 2m + 2[2x(2m + 2x + 1)]$ Is equal to $2x$ (Gnanadhas et al., 2000) ^[2] $2^{2m} +$ (Joseph Varghese et al., 2010) ^[4] $+ 2x2^m +$ (Gnanadhas et al., 2000) ^[2]

Also, $q(K2m + 1, 2x(2m + 1x + 1)) = 2m + 1[2x(2m + 1x + 1)]$.

Is equal to $2x$ (Gnanadhas et al., 2000) ^[2] $2^{2m} +$ (Gnanadhas et al., 2000) ^[2] $+ 2x2^m +$ (Harary, 1969) ^[1]

Therefore, $q(K2m + 2, 2x(2m + 2x + 1)) - q(K2m + 1, 2x(2m + 1x + 1))$ That is equal to $2x$ (Gnanadhas et al., 2000) ^[2] $2^{2m} +$ (Joseph Varghese et al., 2010) ^[4] $+ 2x2^m +$ (Gnanadhas et al., 2000) ^[2] $- 2x$ (Gnanadhas et al., 2000) ^[2] $2^{2m} +$ (Gnanadhas et al., 2000) ^[2] $- 2x2^m +$ (Harary, 1969) ^[1]

$= 4.2x$ (Gnanadhas et al., 2000) ^[2] $2^{2m} +$ (Gnanadhas et al., 2000) ^[2] $- 2x$ (Gnanadhas et al., 2000) ^[2] $2^{2m} +$ (Gnanadhas et al., 2000) ^[2] $- 2.2x.2^m +$ (Harary, 1969) ^[1] $+ 2x.2^m +$ (Harary, 1969) ^[1]

Implies $3x$ (Gnanadhas et al., 2000) ^[2] $2^{2m} +$ (Ebin Raja Merly et al., 2011) ^[3] $+ x.2^m +$ (Gnanadhas et al., 2000; Harary, 1969) ^[2, 1] Now, $2[q(S_{2n+2}) + q(S_{2n+4}) + \dots + q(S_{4n})]$

Equal to $2[2n+2 + 2n+4 + \dots + 4n] = 2(3n$ (Gnanadhas et al., 2000) ^[2] $+ n)$

$= 3x$ (Gnanadhas et al., 2000) ^[2] $2^{2m} +$ (Ebin Raja Merly et al., 2011) ^[3] $+ x2^m +$ (Gnanadhas et al., 2000) ^[2] $-----$ (Gnanadhas et al., 2000) ^[2] where $n = 2^m +$ (Harary, 1969) ^[1] _{x}

From (Harary, 1969) ^[1] and (Gnanadhas et al., 2000) ^[2] we have proved that

$q(K2m + 2, 2x(2m + 2x + 1)) - q(K2m + 1, 2x(2m + 1x + 1)) = 2[q(S_{2n+2}) + q(S_{2n+4}) + \dots + q(S_{4n})]$.

Therefore, $q(K2m + 2, 2x(2m + 2x + 1)) = q(K2m + 1, 2x(2m + 1x + 1)) + 2[q(S_{2n+2}) + q(S_{2n+4}) + \dots + q(S_{4n})]$.

Therefore, $K_{2^m}^{m+}$ (Gnanadhas et al., 2000) ^[2], $2x(2^m +$ (Gnanadhas et al., 2000) ^[2] _{$x+1$}) admits DESD ($S_2, S_4, \dots, S_{2(x2m+1)}$).

Therefore the result is true for $r = m + 1$.

Hence, $K_{2^r}^{r+}$ (Harary, 1969) ^[1], $2x(2^r +$ (Harary, 1969) ^[1] _{$x+1$}) admits Double Even Star Decomposition ($S_2, S_4, S_6, \dots, S_{2n}$)

Theorem 3.6: A complete bipartite graph $K_{2,x(x+1)}$ admits Double Even Star Decomposition ($S_2, S_4, S_6, \dots, S_{2n}$), $x \in \mathbb{N}$ if and only if $n = x$.

Proof: Assume $K_{2,x(x+1)}$ admits DESD ($S_2, S_4, S_6, \dots, S_{2n}$)

We know that $q(K_{2,x(x+1)}) = 2x(x+1)$. Therefore $2x(x+1) = 2n(n+1)$, Implies $n = x$.

Conversely, To prove $K_{2,x(x+1)}$ admits DESD ($S_2, S_4, S_6, \dots, S_{2n}$), $x \in \mathbb{N}$.

Let $V_1 = \{u_1, u_2\}$ and $V_2 = \{v_1, v_2, v_3, \dots, v_{x(x+1)}\}$ be the bipartition of $V(K_{2,x(x+1)})$.

Let T_1 denotes the star $S_{x(x+1)}$ of $K_{2,x(x+1)}$ rooted at u_1 . Clearly $q(T_1) = x(x+1)$.

Then $T_1 = \cup S_{2i1}$, where $2i_1 = 2, 4, 6, \dots, 2n$ and $\sum 2i_1 = x(x+1)$. Thus the decomposition of T_1 is ($S_2, S_4, S_6, \dots, S_{2n}$).

Let T_2 denotes the star $S_{x(x+1)}$ of $K_{2,x(x+1)}$ rooted at u_2 . Clearly $q(T_2) = x(x+1)$,

then $T_2 = \cup S_{2i2}$, where $2i_2 = 2, 4, 6, \dots, 2n$ and $\sum 2i_2 = x(x+1)$.

Thus the decomposition of T_2 is ($S_2, S_4, S_6, \dots, S_{2n}$).

The decomposition of $T_1 \cup T_2$ is DESD ($S_2, S_4, S_6, \dots, S_{2n}$), Which is the required Double Even Star Decomposition of $K_{2,x(x+1)}$.

Illustration 3.7. $K_{2,12}$ admits DESD.

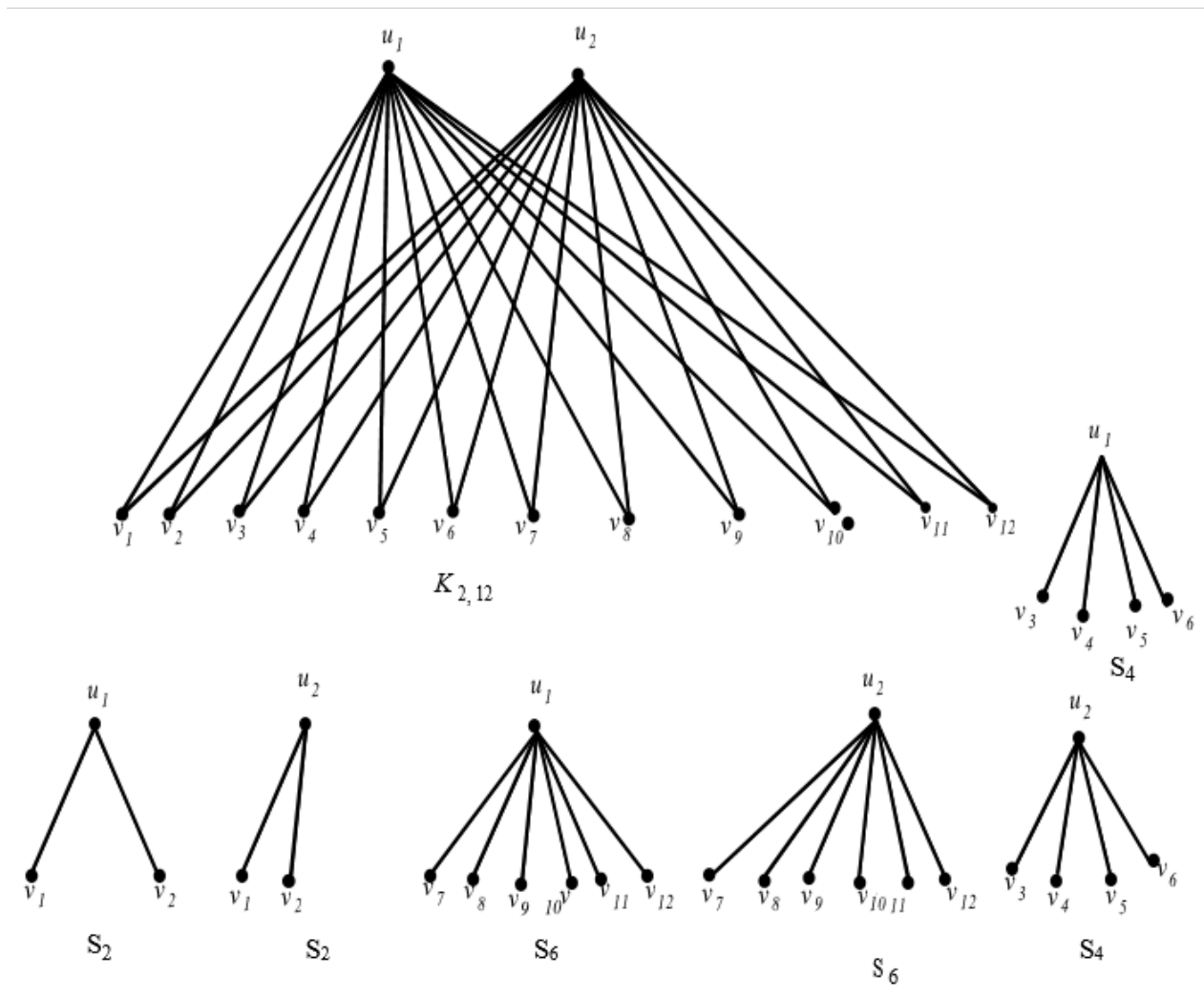


Figure 3.

Table 3.1. List of first 7 DESD of $K_{2, x(x+1)}$

n	$x(x+1)$	DESD
1	2	(S_2)
2	6	(S_2, S_4)
3	12	(S_2, S_4, S_6)
4	20	(S_2, S_4, S_6, S_8)
5	30	$(S_2, S_4, \dots, S_{10})$
6	42	$(S_2, S_4, \dots, S_{12})$
7	56	$(S_2, S_4, \dots, S_{14})$

Theorem 3.8: A Complete bipartite graph $K_{2, 8x}$ (Gnanadhas *et al.*, 2000) [2] admits Double Even Star Decomposition $(S_2, S_4, \dots, S_{2n})$ if and only if $n = 4x - 1, x \in \mathbb{N}$.

Proof:	Assume K^2	admits DESD $(S, S, ..., S),$	2	$) = 4(8x^2$
	K	Then $q(K$		$- 2x).$
				That is
				$4(8x^2 -$
				$2x) = 2n(n$
				$+ 1)$
				Implies
				$2(4x - 1)$
				$(4x) =$
				$2n(n + 1) .$
				Implies n
				$= 4x - 1.$
$4, 8x - 2x$			2	4
				$2n$

(continued on next page)

4, $8x - 2x$

Conversely, to prove $K_{4, 8x}$ (Gnanadhas *et al.*, 2000) ^[2] - $2x$ admits DESD ($S_2, S_4, \dots, S_{2n}$) Let $V_1 = \{u_1, u_2, u_3, u_4\}$

$V_2 = \{v_1, v_2, v_3, \dots, v_{8x} \text{ (Gnanadhas et al., 2000) }^{[2]} - 2x\}$ be the partition of $V(K \text{ (Gnanadhas et al., 2000) }^{[2]})$.

4, $8x - 2x$

Let T_1 denotes the star S (Gnanadhas *et al.*, 2000) ^[2] of K (Gnanadhas *et al.*, 2000) ^[2] rooted at u Clearly $q(T) = 8x$ (Gnanadhas *et al.*, 2000) ^[2] - $2x$,

$8x - 2x$ 4, $8x - 2x$ 1, 1

then $T_1 = \cup SR_{2i_1}$, where, $2i_1 = 2, 4, 6, \dots, \dots, \dots, 2n$ and $\sum 2i_1 = 8x - 2x$

$n-7$ $n-3$ $3n+3$ $3n+7$ 2

2 2 2 2

T_1 can be decomposed into ($S_2, S_4, \dots, S_{n-7}, S_{n-3}, \dots, S_{3n+3}, S_{3n+7}, \dots, S_{2n}$).

2 2 2 2

Let T_2 denotes the star S (Gnanadhas *et al.*, 2000) ^[2] of K (Gnanadhas *et al.*, 2000) ^[2] rooted at u Clearly $q(T) = 8x$ (Gnanadhas *et al.*, 2000) ^[2] - $2x$,

$8x - 2x$ 4, $8x - 2x$ 2, 2

then $T_2 = \cup SR_{2i_2}$, Where $2i_2 = 2, 4, 6, \dots, n-7, n-3, \dots, 3n+3, 3n+7, \dots, 2n$ and $\sum 2i_2 = 8x$ (Gnanadhas *et al.*, 2000) ^[2] - $2x$

2 2 2 2

T_2 can be decomposed into ($S_2, S_4, \dots, S_{n-7}, S_{n-3}, \dots, S_{3n+3}, S_{3n+7}, \dots, S_{2n}$).

2 2 2 2

Let T_3 denotes the star S (Gnanadhas *et al.*, 2000) ^[2] of K (Gnanadhas *et al.*, 2000) ^[2] rooted at u , Clearly $q(T) = 8x$ (Gnanadhas *et al.*, 2000) ^[2] - $2x$,

$8x - 2x$ 4, $8x - 2x$ 3 3

then $T_3 = \cup SR_{2i_3}$, where $2i_3 = n+1, n+5, \dots, 3n-1$, and $\sum 2i_3 = 8x$ (Gnanadhas *et al.*, 2000) ^[2] - $2x$.

2 2 2 2

Thus the decomposition of T_3 is $\left(\frac{S_{n+1}}{2}, \frac{S_{n+5}}{2}, \dots, \frac{S_{3n-1}}{2} \right)$

Let T_4 denotes the star S_{8x} (Gnanadhas *et al.*, 2000) ^[2] - $2x$ of $K_{4, 8x}$ (Gnanadhas *et al.*, 2000) ^[2] - $2x$ rooted at u_4 , clearly $q(T_4) = 8x$ (Gnanadhas *et al.*, 2000) ^[2] - $2x$,

then $T_4 = \cup S_{2i_4}$ where, $2i_4 = \frac{n+1}{2}, \frac{n+5}{2}, \dots, \frac{3n-1}{2}$, and $\sum 2i_4 = 8x$ (Gnanadhas *et al.*, 2000) ^[2] - $2x$.

Thus the decomposition of T_4 is $\left(\frac{S_{n+1}}{2}, \frac{S_{n+5}}{2}, \dots, \frac{S_{3n-1}}{2} \right)$

The decomposition of $T_1 \cup T_2 \cup T_3 \cup T_4$ is

($s_2, s_4, s_6, \dots, S_{n-7}, S_{n+1}, S_{n+9}, \dots, S_{n-3}, S_{n+1}, S_{n+5}, \dots, S_{3n-5}, S_{3n+3},$

4 4 4 2 2 2 4 4

$S_{3n+11}, \dots, S_{5n-3}, S_{5n+5}, \dots, S_{3n-1}, S_{3n+3}, S_{3n+7}, \dots, S_{7n-1}, S_{7n+7}, \dots, S_{2n}$)

4 4 8 2 2 2 4 4

Which is the required Double Even Star Decomposition of K (Gnanadhas *et al.*, 2000) ^[2].

4, $8x - 2x$

Table 3.2. List of first 9 DESD of $K_{4, 8x}$ (Gnanadhas *et al.*, 2000) ^[2] - $2x$.

x	$8x^2 - 2x$	$q(K_{4, 8x^2 - 2x})$	n	Double ESD
1	6	24	3	(S_2, S_4, S_6)
2	28	112	7	($S_2, S_4, S_6, \dots, S_{14}$)
3	66	264	11	($S_2, S_4, S_6, \dots, S_{22}$)
4	120	480	15	($S_2, S_4, S_6, \dots, S_{30}$)
5	190	760	19	($S_2, S_4, S_6, \dots, S_{38}$)
6	288	1104	23	($S_2, S_4, S_6, \dots, S_{46}$)
7	378	1512	27	($S_2, S_4, S_6, \dots, S_{54}$)
8	496	1984	31	($S_2, S_4, S_6, \dots, S_{62}$)
9	630	2520	35	($S_2, S_4, S_6, \dots, S_{70}$)

Theorem 3.9: A Complete bipartite graph $K_{4, 8x^2 + 2x}$ admits Even Star Decomposition ($S_2, S_4, \dots, S_{2n}$) if and only if $n = 4x, x \in \mathbb{N}$.

Proof : Assume $K_{4, 8x^2 + 2x}$ admits DESD ($S_2, S_4, \dots, S_{2n}$). Then $q(K_{4, 8x^2 + 2x}) = 4(8x^2 + 2x)$

That is $4(8x^2 + 2x) = 4(8x^2 + 2x) = 2n(n+1)$. Implies $2(4x+1)(4x) = 2n(n+1)$ Implies $n = 4x$

Conversely, to prove $K_{4, 8x^2 + 2x}$ admits DESD ($S_2, S_4, \dots, S_{2n}$)

Let $V_1 = \{u_1, u_2, u_3, u_4\}$

$V_2 = \{v_1, v_2, v_3, \dots, v_{(Gnanadhas et al., 2000)^{[2]}}\}$ be the partition of $V(K \text{ (Gnanadhas et al., 2000) }^{[2]})$.

$8x + 2x$ 4, $8x + 2x$

Let T_1 denotes the star S

2

$8x + 2x$

of K

2

4, $8x$

$+2x$

rooted at u_1 ,

Clearly $q(T_1) = 8x$ (Gnanadhas *et al.*, 2000) ^[2] + $2x$, then T_1

$= \cup S_{2i_1}$

where, $2i_1 = 2, 4, 6, \dots, n, n+4, \dots, 3n+4, 3n+8, \dots, 2n$ and $\sum 2i_1 = 8x$ (Gnanadhas *et al.*, 2000) ^[2] + $2x$

2 2 2 2

T_1 can be decomposed into ($S_2, S_4, \dots, S_n, S_{n+4}, \dots, S_{3n+4}, S_{3n+8}, \dots, S_{2n}$).

Let T_2 denotes the star S

2

$8x + 2x$

of K

2

2

4, $8x$

2

$+2x$

2

rooted at u_2 ,

2

Clearly $q(T_2) = 8x$ (Gnanadhas *et al.*, 2000) ^[2] + $2x$, then T_2

$= \cup S_{2i_2}$

Where, $2i_2 = 2, 4, 6, \dots, n, n+4, \dots, 3n+4, 3n+8, \dots, 2n$ and $\sum 2i_2 = 8x$ (Gnanadhas *et al.*, 2000) ^[2] + $2x$.

2 2 2 2

T_2 can be decomposed into ($S_2, S_4, \dots, S_n, S_{n+4}, \dots, S_{3n+4}, S_{3n+8}, \dots, S_{2n}$).

Let T_3 denotes the star S

2
 $8x + 2x$
 of K
 2
 2
 $4, 8x$
 2
 $+ 2x$
 2
 rooted at u_3 ,
 2
 Clearly $q(T_3) = 8x$ (Gnanadhas *et al.*, 2000) $[2] + 2x$, then T_3
 $= \cup S_{2i3}$
 where, $2i_3 = n+8, n+12, \dots, 3n-4, 3n$, and $\sum 2i_3 = 8x$ (Gnanadhas *et al.*, 2000) $[2] + 2x$
 $2 \ 2 \ 2 \ 2$
 Thus the decomposition of T_3 is $(S_{n+8}, \dots, S_{3n-4}, S_{3n})$
 $2 \ 2 \ 2$
 Let T_4 denotes the star S_{8x} (Gnanadhas *et al.*, 2000) $[2] + 2x$
 of $K_4, 8x$
 $+ 2x$ rooted at u_4 , clearly $q(T_4) = 8x$ (Gnanadhas *et al.*, 2000) $[2] + 2x$,
 then T
 $4 = \cup S_{2i4}$
 where, $2i_4 = n+8, n+12, \dots, 3n-4, 3n$ and $\sum 2i_4 = 8x$ (Gnanadhas *et al.*, 2000) $[2] + 2x$.
 $2 \ 2 \ 2 \ 2$
 Thus the decomposition of T_4 is $(S_{n+8}, \dots, S_{3n-4}, S_{3n})$
 $2 \ 2 \ 2$
 The decomposition of $T_1 \cup T_2 \cup T_3 \cup T_4$ is

$$(s_2, s_4, s_6, \dots, \frac{S_{n-7}}{4}, \frac{S_{n+1}}{4}, \frac{S_{n+9}}{4}, \dots, \frac{S_{n-3}}{2}, \frac{S_{n+1}}{2}, \frac{S_{n+5}}{2}, \dots, \frac{S_{3n-5}}{4}, \frac{S_{3n+3}}{4},$$
 $S_{3n+11}, \dots, S_{5n-3}, S_{5n+5}, \dots, S_{3n-1}, S_{3n+3}, S_{3n+7}, \dots, S_{7n-1}, S_{7n+7}, \dots, S_{2n})$
 $4 \ 4 \ 8 \ 2 \ 2 \ 2 \ 4 \ 4$
 Which is the required Double Even Star Decomposition of K (Gnanadhas *et al.*, 2000) $[2]$.
 $4, 8x + 2x$
 $4, 8x + 2x$

Table 3.3. List of first 10 Double ESD of K (Gnanadhas *et al.*, 2000) $[2]$.

x	n	Double ESD
1	4	(S_2, S_4, S_6, S_8)
2	8	$(S_2, S_4, S_6, \dots, S_{16})$
3	12	$(S_2, S_4, S_6, \dots, S_{24})$
4	16	$(S_2, S_4, S_6, \dots, S_{32})$
5	20	$(S_2, S_4, S_6, \dots, S_{40})$
6	24	$(S_2, S_4, S_6, \dots, S_{48})$
7	28	$(S_2, S_4, S_6, \dots, S_{56})$
8	32	$(S_2, S_4, S_6, \dots, S_{64})$
9	36	$(S_2, S_4, S_6, \dots, S_{72})$
10	40	$(S_2, S_4, S_6, \dots, S_{80})$

Remark 3.11: Complete bipartite graph $K_{3,s}, K_{6,s}, \dots, K_{w,s}$ does not admit DESD where w is odd or odd multiples.

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