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A review on integral transforms: Aboodh and finite Mellin transforms

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ABSTRACT

In the field of Mathematics and Engineering, there are several integral transforms and many researchers worked on it. The Mellin transform is a most important integral transform to solve differential problem in applied mathematics and physics. A new transform Aboodh transform arises as an alternative to solve differential equation in recent days.

In this paper, we review an integral transform specifically focusing on the Aboodh transform and the finite Mellin transform definition, application in various fields, comparative analysis, limitation, research gaps and conclusions.

Keywords: Integral Transform, Aboodh Transform, Finite Mellin Transform, Applied Mathematics, Mellin Transform

1. Introduction

Integral transforms are fundamental and basic mathematical tools used to convert differential equations into algebraic equations (Liaqat *et al.*, 2022) [1]. This transformation simplifies the search for analytical or approximate solutions in various domains of science and engineering. Among the wide lists of transforms, the Aboodh transform and the Mellin transform represent critical frameworks for solving complex problems in applied mathematics, and theoretical physics.

The Aboodh transform, first introduced by Khalid Aboodh in 2013, has interest due to its simplicity in solving initial value problems (Qureshi *et al.*, 2019) [2], (Aboodh *et al.*, 2022) [3]. On the other hand, the Mellin transform particularly its finite version, it has wide application in signal sampling theories, and complex ordinary and partial differential equation in higher physics (Butzer *et al.*, 1999) [4], (Atale, 2024) [5]. This paper review on these transforms, examining their theoretical foundations and expanding practical applications, limitation and future scope.

2. The Aboodh transform

2.1 Mathematical definition

The Aboodh Transform with parameter s of the function $f(t)$ is denoted by $A\{f(t)\} = F(s)$ and is given by

$$A\{f(t)\} = F(s) = \frac{1}{s} \int_0^{\infty} e^{-st} f(t) dt, \text{ for parameter } s > 0$$

The integral kernel is e^{-st} (Alshikh *et al.*, 2016) [6], (Benattia *et al.*, 2020) [7], (Iqbal *et al.*, 2024) [8].

2.2 Variations and extensions

The Aboodh transform has developed to tackle more complex mathematical structures, some versions of it:

- **Local Fractional Aboodh Transform (LFAT):** This transform is use to convert linear local fractional differential equations into algebraic equation, using the Mittag-Leffler function in its kernel for non-smooth functions (Ziane *et al.*, 2022) [9].
- **Double Aboodh Transform:** Used to solve two variables' problems in fractional order (Alfaqeih *et al.*, 2019) [10].
- **Triple Laplace-Aboodh-Sumudu Transform:** A hybrid transform used for solving three variable 3D Helmholtz equation and partial differential equations (Hunaiber, 2025) [11], (Al-aati *et al.*, 2022) [12].

- **Aboodh Transform Series Decomposition Method (ATSDM):** the Taylor series combine with transform for solving higher-order nonlinear boundary value problems by decomposition methods (Akinola *et al.*, 2023) [13].

3. Applications of the Aboodh transform

3.1 Ordinary and partial differential equations

- The Aboodh transform is effective for solving linear and nonlinear ordinary differential equations (ODEs) with variable coefficients (Benattia *et al.*, 2019) [14].
- The Variational Iteration Method (VIM) or the Homotopy Perturbation Method (HPM), solved Klein-Gordon equations and partial differential equations (PDEs) (Aboodh *et al.*, 2022) [3], (Sedeeg *et al.*, 2016) [15], (Cherif *et al.*, 2018) [16].

3.2 Fractional calculus and nonlinear dynamics

- **Korteweg-de Vries (KdV) and Kawahara Equations:** It play fundamental role for understanding of nonlinear wave dynamics provide and analytical solutions for the modified KdV and fifth-order Kawahara equations (Almusawa *et al.*, 2024) [17], (Iqbal *et al.*, 2024) [8], (Shah *et al.*, 2020) [18].
- **Zakharov-Kuznetsov Equation:** multi-dimensional fractional ZK equations solve using the Adomian Decomposition Method (Rashid *et al.*, 2021) [19].
- **Fokker-Planck Equations:** The ZZ transform is related to the Aboodh transform for analysis of fractional-view Fokker-Planck equations (Alshehry *et al.*, 2022) [20].
- **Telegraph Equations:** In transmission lines, transform solve fractional-order telegraph equations to model signals (Abdlrasol *et al.*, 2022) [21], (Ali *et al.*, 2020) [22].
- **Keller-Segel Equation:** KS equation applied to fractional modelling in mathematical biology [23].

3.3 Physics, engineering, and mechanics

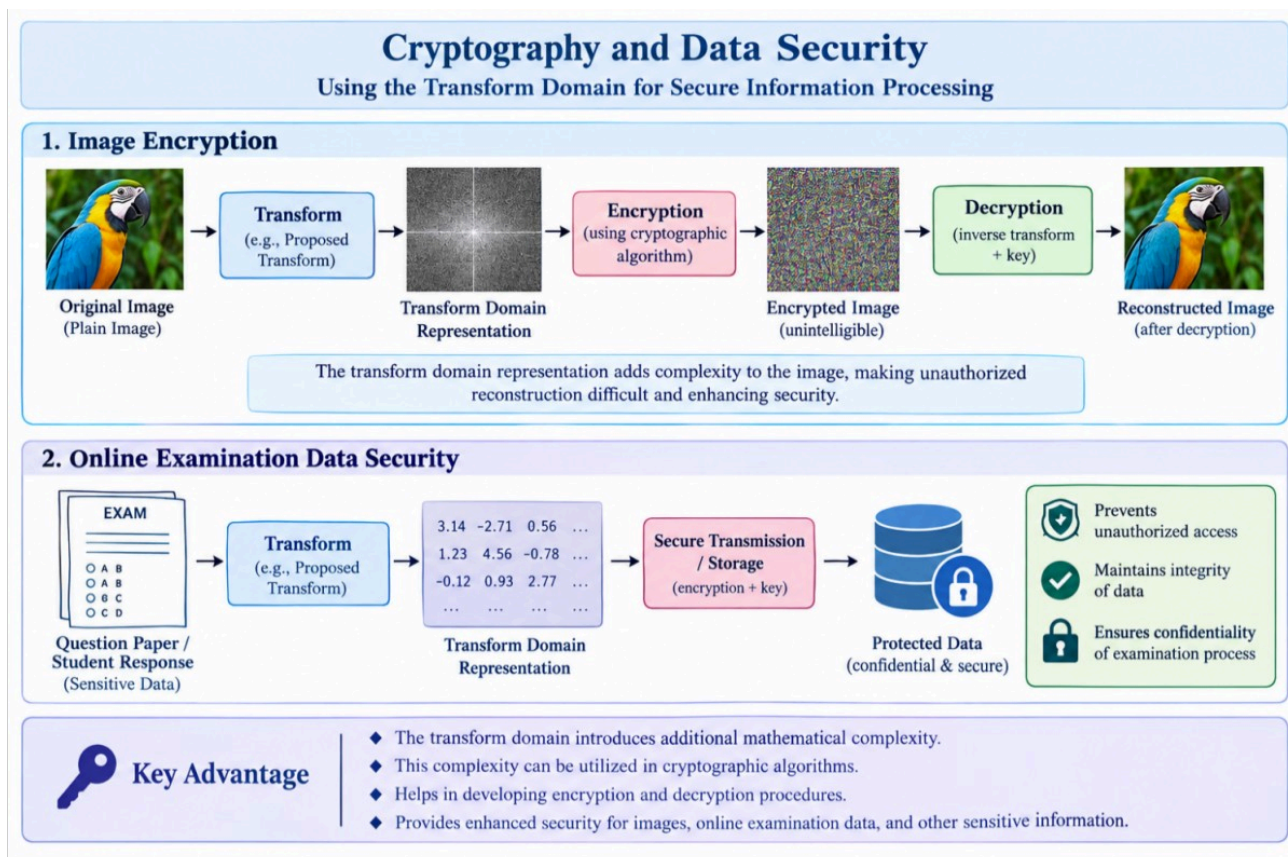
- **Quantum Mechanics:** Solving time-fractional quantum mechanics problems (Liaquat *et al.*, 2022) [1].
- **MEMS Systems:** Application of the homotopy perturbation method (Amir *et al.*, 2024) [24].
- **Fluid Dynamics and Plasma:** Modeling nonlinear waves in plasma and fluids (Shah *et al.*, 2022) [25], (Mukhtar, 2023) [26].
- **Vibrating Systems:** Analyzing vibrating beams and rotating systems (Aggarwal *et al.*, 2019) [27], (Mackerle, 1999) [28].

3.4 Cryptography and data security

- It has been proposed for use in image encryption and as a mathematical framework for online examination data security (Sedeeg *et al.*, 2016) [29].
- Its ability to map functions into a transform domain provides a layer of complexity useful for cryptographic algorithms.

The transformation of functions from the original domain into a suitable transform domain introduces additional mathematical complexity, which can be exploited in cryptographic algorithms. This property may be utilized to develop encryption and decryption procedures, enhance data confidentiality, and provide additional security for sensitive information transmitted or stored in digital systems. In particular, transform-domain representations can be incorporated into image encryption schemes to alter the structure and statistical characteristics of the original image, thereby making unauthorized reconstruction more difficult.

Furthermore, the transform-based approach may be investigated for securing online examination systems, where confidentiality and integrity of question papers, student responses, and examination-related data are important. Thus, the proposed transform provides a mathematically structured framework that can potentially contribute to the development of secure information-processing techniques.



4. The finite Mellin transform

4.1 Mathematical definition

The Finite-Mellin Transform with parameter p of the function $f(x)$ is denoted by $M_f\{f(x)\} = F(p)$ and is given by

$$M_f\{f(x)\} = F(p) = \int_0^a \left(\frac{a^{2p}}{x^{p+1}} - x^{p-1} \right) f(x) dx, \text{ for parameter } p > 0 \quad (4.1)$$

- It is classically used for functions on an infinite region (Khairnar *et al.*, 2012) [30].
- The transform is also used in heat transfer and diffusion problem.
- The finite Mellin transform is a most recent adaptation for functions (Atale, 2024) [5], (Butzer *et al.*, 2000) [31].
- The finite Mellin transform for discrete signal analysis and finite-field applications (Katz, 2012) [32], (Katz, 2012) [33].

4.2 Connections to Ramanujan's master theorem

- The finite Mellin transform provides an integral representation for meromorphic functions based on their power series coefficients with connected to Ramanujan's Master Theorem (Atale, 2024) [5], (Bradshaw *et al.*, 2024) [34].
- This allows for the evaluation of integrals that appear in number theory and mathematical physics.

5. Applications of the Mellin transform

5.1 Signal analysis and sampling theory

- **Mellin-bandlimited Signals:** Analysis of signals with specific integral transform (Butzer *et al.*, 1999) [4].
- **Signal Processing:** Implementing digital algorithms for signal processing in automated control systems (Makarov *et al.*, 2019) [35].

5.2 Physics and statistical mechanics

- **Statistical Mechanics:** Used to analyze the ideal quantum gases and other problems in thermodynamics with statistical mechanics (Grandy *et al.*, 1981) [36], (Iwata, 1960) [37].
- **Particle Physics:** This transform is evaluating loop amplitudes in Feynman diagrams and simplify the calculation of complex scattering processes (Atale, 2024) [5].
- **Yang-Mills Theory:** Applied to perturbative aspects of deformed Yang-Mills theory (Devika, 2023) [38].
- **Electromagnetics:** Used in electromagnetic field modeling (Mystilidis *et al.*, 2022) [39].

5.3 Financial mathematics and option pricing

- **American Options:** transformation of the Black-Scholes equation for pricing American options (Frontczak *et al.*, 2008) [40], (Alobaidi *et al.*, 2014) [41].
- **Vanilla Power Options:** Valuation of power options with non-dividend yields (Nwozo *et al.*, 2014) [42].

6. Comparative analysis

Current research focuses on the relationships between different integral transforms.

- **Aboodh verses Laplace and Elzaki:** The Aboodh transform Comparative studies show performance in solving first and second-order ODEs (Alshikh *et al.*, 2016) [6].
- **Aboodh and Mohand:** Comparative studies show both are effective for engineering problems like heat conduction (Aggarwal *et al.*, 2019) [27].
- **ZZ Transform:** The Aboodh and Laplace transforms is related to the ZZ transform used for solving fractional-order Fokker-Planck equations (Mahgoub *et al.*, 2016) [43], (Alshehry *et al.*, 2022) [20], (Jena *et al.*, 2020) [44].
- **Ouideen Transform:** This new transform offering high accuracy for linear differential equations by introduced as a generalization of the Aboodh and Sumudu transforms [45].

7. Limitations of the evidence

The following limitations are identified in the current literature:

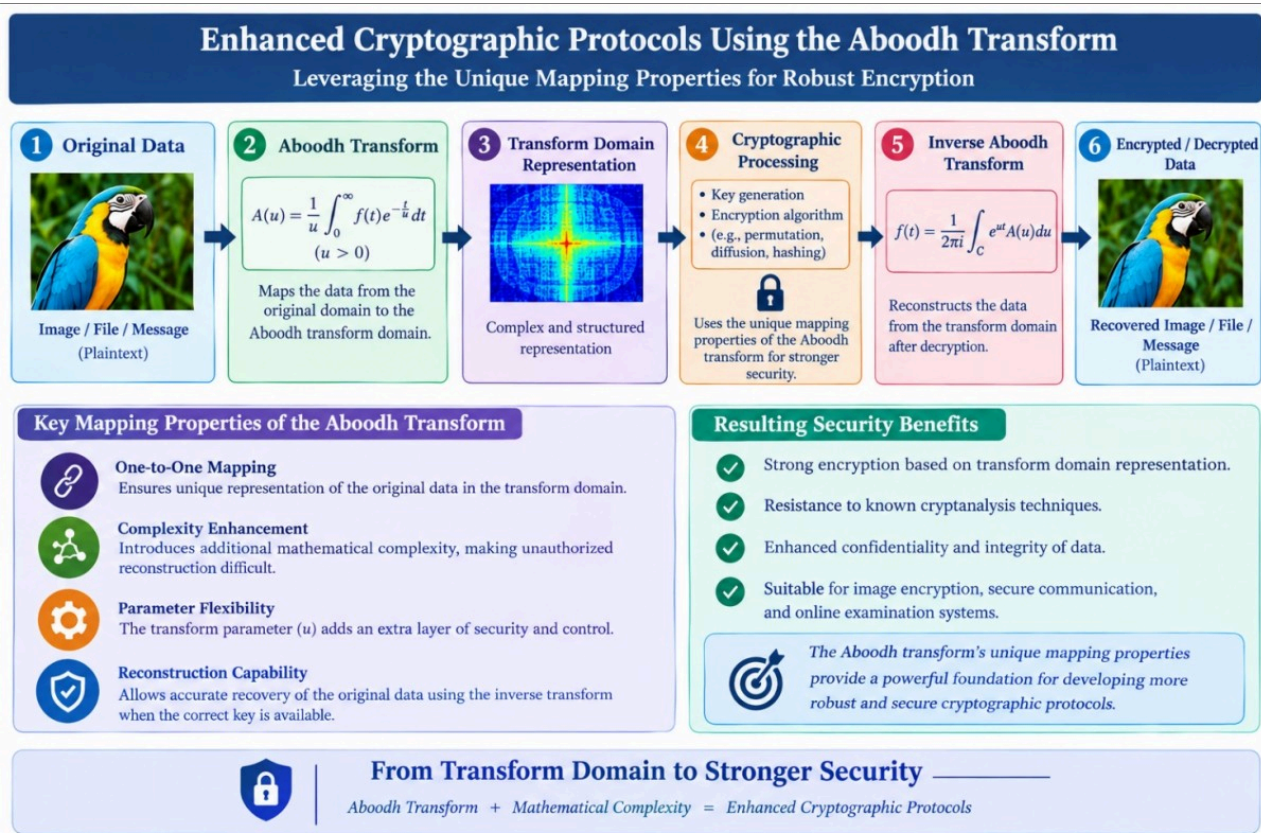
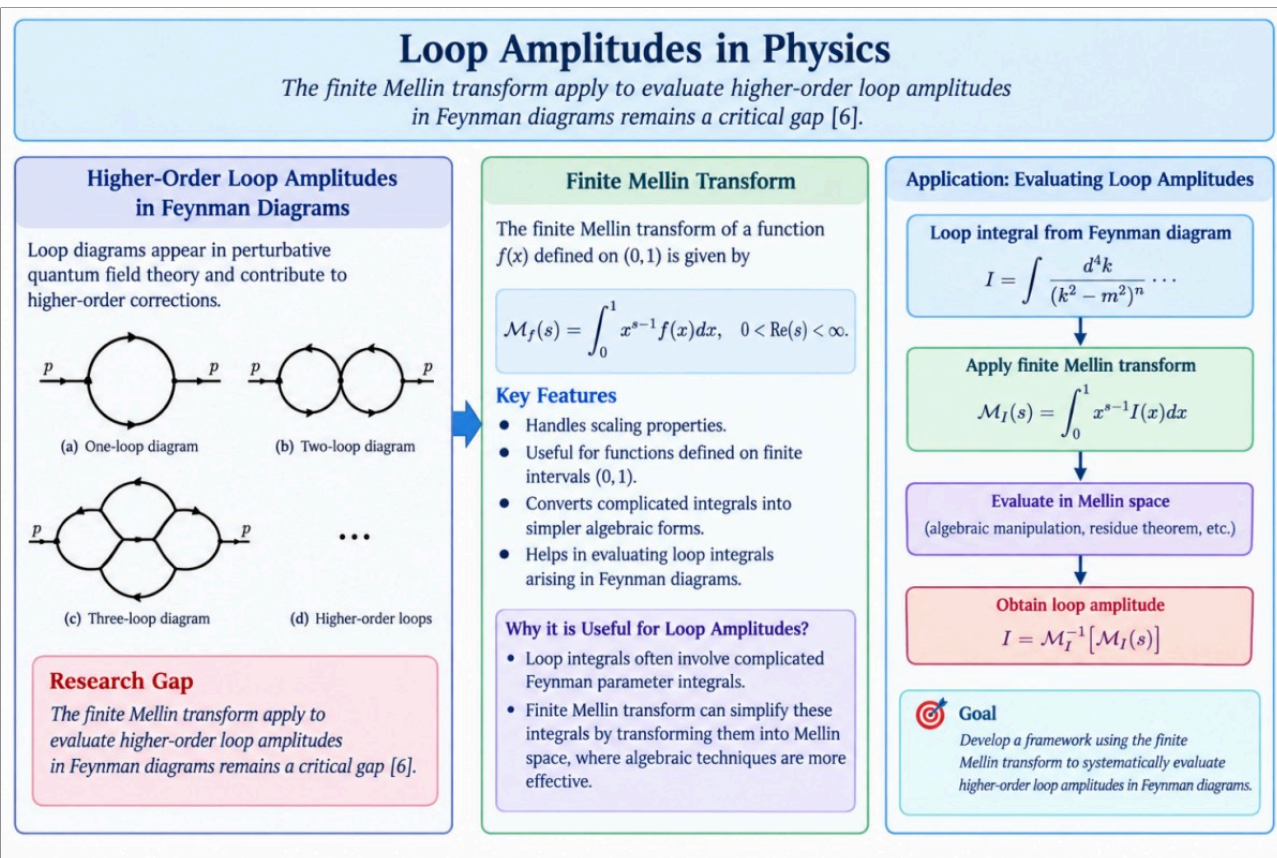
- **Function Class Restrictions:** The finite Mellin transform is limited to functions with series expansions (Atale, 2024) [5].
- **Inversion Stability:** the Mellin transform inversion can be unstable and requiring functional equivalence analysis (Hawkins, 2016) [46].
- **Convergence Speed:** they may still face challenges with highly complex nonlinear boundary conditions (Aboodh *et al.*, 2022) [3], (Akinola *et al.*, 2023) [13].
- **Limited Long-term Projections:** For broad theoretical generalization across all possible operators required more time so, the research is focused on demonstrating efficacy for specific equations (Benattia *et al.*, 2020) [7], (Benattia *et al.*, 2019) [14].

8. Future directions and research gaps

The literature points toward several promising research directions:

- **Semi-Derivative Interpretation:** The Aboodh transform using semi-derivatives interpretation expected to future breakthroughs. (Qureshi *et al.*, 2019) [2].
- **Loop Amplitudes in Physics:** The finite Mellin transform apply to evaluate higher-order loop amplitudes in Feynman diagrams remains a critical gap (Atale, 2024) [5].
- **Extension of Ramanujan's Theorem:** aims to explore finite-limit analogs of Frullani's theorem (Atale, 2024) [5] and extend research for the class of functions applicable to Ramanujan's Master Theorem.
- **Enhanced Cryptographic Protocols:** The Aboodh transform developing more robust encryption methods based on the unique mapping properties (Sedeeg *et al.*, 2016) [29].

- **Cell Growth Modeling:** application of Mellin transforms to functional differential equations arising in cell growth models (Hall *et al.*, 1989) [47].



9. Conclusion

In modern mathematics the Aboodh and finite Mellin transforms are essential components for analysis. The Aboodh transform's ability to handle fractional and nonlinear dynamics Meanwhile, the finite

Mellin transform provides application in signal processing, financial mathematics, and theoretical physics. As researchers continue to explore properties, generalizations of these transforms, inversion theorem, uniqueness, hybrid model, the dualities and their impact on scientific and engineering problem-solving. It is expected to grow, offering new insights into the fundamental dynamics of complex systems.

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