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Stochastic modeling and priority-based maintenance policy for enhanced availability in repairable systems

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ABSTRACT

This study presents a comprehensive stochastic modeling framework for the evaluation and enhancement of availability in repairable systems, emphasizing the role of maintenance prioritization. Using a continuous-time Markov chain (CTMC) approach, the system's various operational, degraded, and failed states are captured, with transitions governed by well-defined failure and repair rates. Differential equations and matrix methods are employed to derive steady-state probabilities and assess key performance metrics such as Mean Time between Failures (MTBF), Mean Time to Repair (MTTR), and steady-state availability. Sensitivity analyses reveal that system availability is highly responsive to changes in failure and repair rates, highlighting the practical importance of reliable components and rapid, efficient maintenance. The study further introduces a maintenance priority assessment framework based on computed availability, guiding decision-makers in resource allocation and preventive maintenance planning. The findings provide actionable insights for maintenance managers and system designers aiming to maximize uptime, minimize costs, and ensure sustained operational effectiveness in modern manufacturing environments.

Keywords: Reliability, Availability, Continuous-Time Markov Chain (CTMC), Steady-State Analysis, Maintenance Policy, Repairable Systems

Introduction

In the age of advanced manufacturing and pervasive automation, the uninterrupted operation of industrial systems has become a linchpin for global competitiveness and efficient production. Modern manufacturing plants, which often operate round the clock, rely heavily on the reliability and availability of complex machinery and interconnected subsystems. Reliability, in this context, denotes the likelihood that a system will function without failure over a prescribed period and under stated conditions. Availability, meanwhile, encompasses not only the frequency of failures but also the speed and effectiveness of repairs, reflecting the proportion of time a plant or machine is ready and capable of performing its intended function.

The repercussions of equipment failures and prolonged downtime are profound leading to production losses, missed delivery deadlines, compromised product quality, and increased operational costs. Industries such as automobile manufacturing, pharmaceuticals, food processing, and electronics are particularly sensitive to such disruptions. Consequently, there is an increasing trend towards predictive and preventive maintenance strategies, which utilize real-time data, advanced analytics, and optimization algorithms to anticipate failures and schedule timely interventions. Despite these advancements, the fundamental analytical backbone for evaluating system performance remains rooted in stochastic modeling, probability theory, and operations research. Among the various mathematical tools available, continuous-time Markov chains (CTMCs) are widely regarded for their ability to capture both the steady-state and transient behavior of repairable systems. By representing systems as a set of discrete operational and failed states, and modeling transitions between these states with well-defined failure and repair rates, CTMCs provide a robust framework for quantifying key performance metrics. These include Mean Time between Failures (MTBF), Mean Time to Repair (MTTR), steady-state availability, and expected downtime. Through this lens, engineers and decision-makers can design maintenance policies and allocate resources more effectively ultimately maximizing system uptime and reducing costs. This study leverages stochastic modeling and matrix-based analysis to devise a priority-based maintenance framework that enhances the availability and reliability of repairable manufacturing systems. Using additional methods, [Shakuntla et al. \(2011\)](#) [1] performed a behavioral analysis of a polytube system. The operational behavior of a bread manufacturing facility was investigated by [Kumar et al. \(2018\)](#) [2]. [Kumar et al. \(2019\)](#) [3] used the RPGT method to prioritize preventive maintenance and account for server failure in a sensitivity analysis of a cold standby system made up of two identical units. A comparative study of subsystems undergoing simultaneous failures was also conducted by [Shakuntla et al. \(2011\)](#) [4]. The behavior and mathematical formulation of a washing unit in a paper mill were investigated by [Kumar et](#)

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al. (2019) [5]. Kumari *et al.* (2021) [6] investigated constrained scenarios using Particle Swarm Optimization (PSO). Rajbala *et al.* (2022) [7] used a heuristic approach to solve the redundancy allocation problem in a cylinder manufacturing plant. Review of Literature: Singh, R., & Mehta, P. (2024) [8] This study focuses on improving preventive maintenance strategies in automated manufacturing systems through the application of advanced Markov modeling techniques. The research demonstrates that using Markov-based optimization models can lead to significant reductions in unexpected downtime and enhance overall production efficiency in automated environments. Sharma, V., Gupta, N., & Patel, S. (2025) [9] investigate the reliability assessment of repairable systems by integrating artificial intelligence-driven monitoring with traditional reliability modeling. The study underscores the value of leveraging emerging technologies to strengthen maintenance management practices and ensure higher system uptime in industrial contexts.

2. Scope of the study

This study is focused on the development and application of stochastic modeling and analytical tools to evaluate and optimize the availability of repairable systems in manufacturing environments. The research is specifically concerned with systems where components or subsystems may fail and subsequently be repaired, as commonly found in automated plants and industrial machinery networks. By employing continuous-time Markov chain modeling, the study captures the full range of system states operational, degraded, and failed and the transitions governed by statistical failure and repair rates. The scope extends to the formulation of differential equations, construction of transition rate matrices, and computation of steady-state probabilities and availability metrics. Additionally, the study introduces a structured approach to maintenance prioritization, enabling resource allocation decisions based on the criticality of components and their impact on overall system performance. Practical scenarios are modeled, including sensitivity analyses where failure and repair rates are varied. The findings and methodologies are intended to inform maintenance policy, reliability-centered asset management, and the design of future industrial systems. While the core modeling is generalizable, the case studies and results are tailored to typical manufacturing plant configurations.

3. Need of the study

The increasing complexity and automation of industrial systems have elevated the importance of reliability and availability as cornerstones of operational excellence. Even minor equipment failures can cascade into significant production delays, financial losses, and reputational damage. In this context, the ability to predict, analyze, and optimize the availability of repairable systems is vital for sustaining competi-

tive advantage and meeting stringent customer demands. Traditional reactive maintenance approaches are no longer sufficient in an environment where efficiency, productivity, and cost control are paramount. Despite the proliferation of data-driven maintenance technologies, many plants still lack a rigorous, quantitative foundation for prioritizing maintenance actions and optimizing resource allocation. This study addresses this gap by providing a stochastic modeling approach, grounded in Markov analysis, which integrates component failure/repair data, system structure, and operational priorities. By systematically evaluating how failure and repair rates influence availability, the research empowers maintenance managers to devise more effective, proactive strategies minimizing downtime, maximizing uptime, and ensuring long-term asset health. The need for such a study is underscored by the trend toward predictive maintenance, the growing reliance on automation, and the critical role of availability in modern manufacturing economics.

4. Assumption and notations

- Repair restores the system to its previous operational state, not necessarily to the fully operational condition unless indicated by the model.
- Uppercase letters (P, Q, R, S , and T) show **original components** of the state.
- Lowercase letters (p, q, r , and s) show **modified components**.
- α_i is failure rate from state i to next failed state.
- β_i is repair rate from failed state back to previous state.

5. State transition diagram

A transition diagram, also known as a state diagram, visually illustrates how a system shifts from one state to another in response to specific events or conditions. Each arrow in the diagram indicates a possible transition, labeled with the corresponding input or trigger that causes the change. By examining the transition diagram, one can trace the sequence of state changes for the system, gaining insight into its behavior and logic. In this context, the diagram models a system with primary states such as PQRS¹ and PQRS², as well as multiple intermediate states. The movement between these states is governed by failure rates (α_i), representing the likelihood of transitioning to a degraded or failed condition, and repair rates (β_i), which denote the return to an operational state. Transition diagrams provide a clear and intuitive way to represent complex system dynamics, making them invaluable tools in computational modeling and reliability analysis.

6. Mathematical modeling

$$\frac{dp_1}{dt} = -(\alpha_1 + \alpha_3 + \alpha_5 + \alpha_6)p_1 + \beta_1p_2 + \beta_3p_6 + \beta_5p_8 + \beta_6p_9 \quad (1)$$

$$\frac{dp_2}{dt} = \alpha_1p_1 - (\beta_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6)p_2 + \beta_2p_3 + \beta_3p_4 + \beta_4p_5 + \beta_5p_{10} + \beta_6p_{11} \quad (2)$$

$$\frac{dp_3}{dt} = \alpha_2p_2 - \beta_2p_3 \quad (3)$$

$$\frac{dp_4}{dt} = \alpha_3p_2 - \beta_3p_4 \quad (4)$$

$$\frac{dp_5}{dt} = \alpha_4p_2 - \beta_4p_5 \quad (5)$$

$$\frac{dp_6}{dt} = \alpha_3 p_1 - \beta_3 p_6 \quad (6)$$

$$\frac{dp_7}{dt} = \alpha_4 p_1 - \beta_4 p_7 \quad (7)$$

$$\frac{dp_8}{dt} = \alpha_5 p_1 - \beta_5 p_8 \quad (8)$$

$$\frac{dp_9}{dt} = \alpha_6 p_1 - \beta_6 p_9 \quad (9)$$

$$\frac{dp_{10}}{dt} = \alpha_5 p_2 - \beta_5 p_{10} \quad (10)$$

$$\frac{dp_{11}}{dt} = \alpha_6 p_2 - \beta_6 p_{11} \quad (11)$$

Now, Matrix form

$$\frac{dp(t)}{dt} = P(t)Q, \dots (12)$$

$$P(t) = [p_1, p_2, \dots, p_{11}] \text{ and}$$

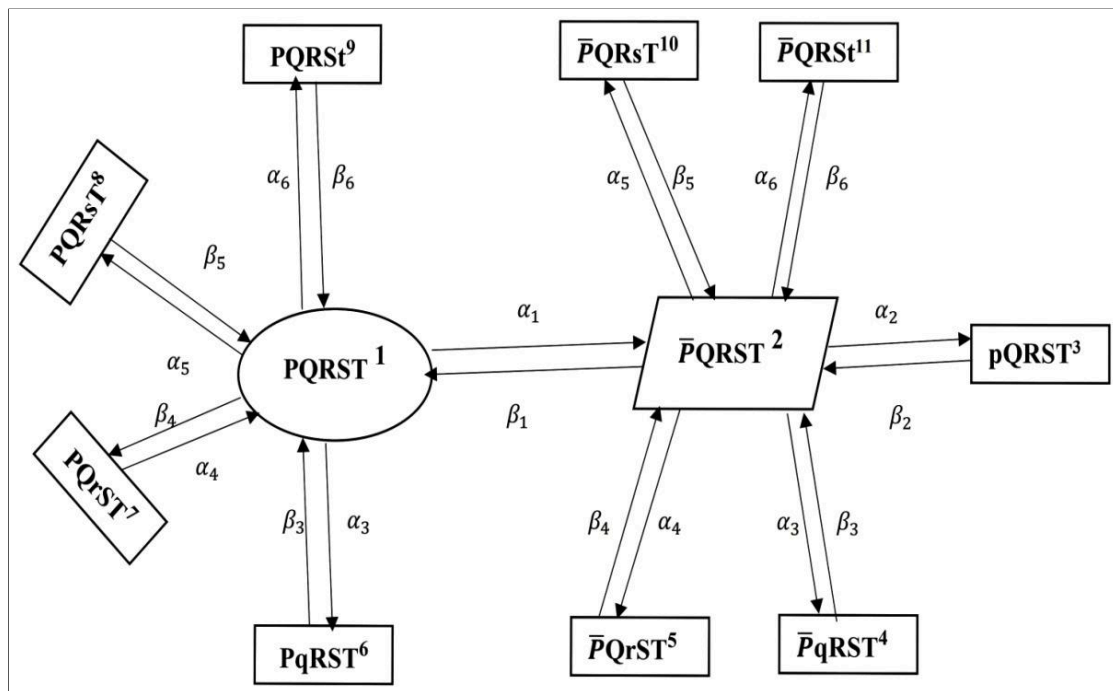


Figure 1. Transition Diagram

The steady-state probabilities are then obtained by solving $PQ = 0$ subject to the normalization condition $\sum P_i = 1$.

7. Transition rate matrix in table form

The transition rate matrix, often referred to as the generator matrix, is a fundamental component in continuous-time Markov models. Each element q_{ij} represents the rate at which the system moves from state S_i to state S_j , for $i \neq j$. The structure of the matrix is derived from the system's state transition diagram, capturing every possible transition between states. Off-diagonal elements reflect the rates of these transi-

tions, while each diagonal element is set so that the sum of the entries in every row equals zero, maintaining the probabilistic balance required for Markov processes:

Now from 11×11 matrix Q , the steady-state probability satisfies:

$$\pi Q = 0, \dots (13)$$

$$\sum_{i=1}^{11} \pi_i \quad (14)$$

We get

$-(\alpha_1 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6)$	α_1	0	0	0	α_3	α_4	α_5	α_6	0	0
β_1	$-(\beta_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6)$	α_2	α_3	α_4	0	0	0	0	α_5	α_6
0	β_2	$-\beta_2$	0	0	0	0	0	0	0	0
0	β_3	0	$-\beta_3$	0	0	0	0	0	0	0
0	β_4	0	0	$-\beta_4$	0	0	0	0	0	0
β_3	0	0	0	0	$-\beta_3$	0	0	0	0	0
β_4	0	0	0	0	0	$-\beta_4$	0	0	0	0
β_5	0	0	0	0	0	0	$-\beta_5$	0	0	0
β_6	0	0	0	0	0	0	0	$-\beta_6$	0	0
0	β_5	0	0	0	0	0	0	0	$-\beta_5$	0
0	β_6	0	0	0	0	0	0	0	0	$-\beta_6$

then

$$\pi_1 = \frac{1}{D}, \quad (15)$$

$$A_v = \pi_1 + \pi_2 \quad (17)$$

Where

$$D = 1 + \frac{\alpha_1}{\beta_1} + \frac{\alpha_1 \alpha_2}{\beta_1 \beta_2} + \frac{\alpha_1 \alpha_3}{\beta_1 \beta_3} + \frac{\alpha_1 \alpha_4}{\beta_1 \beta_4} + \frac{\alpha_3}{\beta_3} + \frac{\alpha_4}{\beta_4} + \frac{\alpha_5}{\beta_5} + \frac{\alpha_6}{\beta_6} \dots (16)$$

Therefore,

$$A_v = \frac{1 + \frac{\alpha_1}{\beta_1}}{1 + \frac{\alpha_1}{\beta_1} + \frac{\alpha_1 \alpha_2}{\beta_1 \beta_2} + \frac{\alpha_1 \alpha_3}{\beta_1 \beta_3} + \frac{\alpha_1 \alpha_4}{\beta_1 \beta_4} + \frac{\alpha_3}{\beta_3} + \frac{\alpha_4}{\beta_4} + \frac{\alpha_5}{\beta_5} + \frac{\alpha_6}{\beta_6}} \quad (18)$$

This is the steady-state availability corresponding to the matrix.

8. Data analysis and results

The table and graph illustrate the effect of failure and repair rates on the steady-state availability of the system.

Table 1. Vary failure rates (α), fixed repair rates $\beta=0.1$

	0.001	0.9615
0.002		0.9257
0.003		0.8923
0.004		0.8610
0.005		0.8317
0.006		0.8041
0.007		0.7782
0.008		0.7538
0.009		0.7308
0.010		0.7091

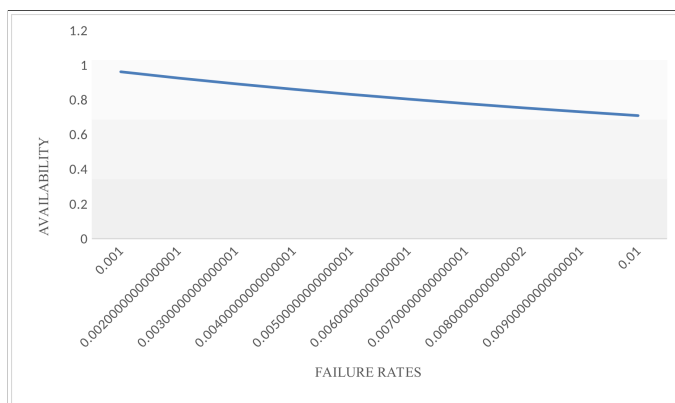


Figure 2. Availability versus failure rates

The data in Table 1 clearly show that as failure rates rise, system availability steadily declines. This pattern is also illustrated in Figure 2. The graph highlights that the highest levels of availability occur when failure rates are kept very low and repair rates are maximized. In such scenarios, the system predominantly remains in its functional states, leading to optimal or near-optimal operational efficiency.

The figures in Table 2 reveal that system availability improves as failure rates decrease. This upward trend is also confirmed by Figure 3, which shows that the highest availability is reached when component failures are rare and repair actions are swift.

Table 2. Vary Repair Rate (β), Fix Failure Rate $\alpha=0.001$

	0.02	0.8266
0.04		0.9042
0.06		0.9357
0.08		0.9518
0.10		0.9615
0.12		0.9680
0.14		0.9726
0.16		0.9761
0.18		0.9788
0.20		0.9810

Table 3. Availability Analysis under Low Failure and High Repair Rates

α	B	availability
0.001	0.10	96.15%
0.001	0.20	98.10%
0.001	0.50	99.22%
0.001	1.0	99.61%

The data presented in Table 3 indicate that maintaining low failure rates alongside high repair rates significantly enhances system performance. As a result, the system achieves peak availability, reflecting an environment that is both highly reliable and operationally efficient.

9. Conclusion

In summary, this study demonstrates the effectiveness of stochastic modeling and priority-driven maintenance in improving the availability of repairable manufacturing systems. By leveraging continuous-time Markov chain analysis, the research accurately captures the dynamic transitions between operational, degraded, and failed states. The results highlight that system availability is most influenced by the rates of failure and repair emphasizing the necessity for both reliable components and efficient maintenance practices. Implementing a maintenance prioritization framework allows for better resource allocation and proactive management of critical subsystems, reducing downtime and operational risks. The findings also underscore the value of steady-state probability analysis in maintenance planning and decision-making. Ultimately, the approach outlined here provides a robust mathematical basis for optimizing plant performance and can be adapted to various industrial contexts. This work serves as a guide for maintenance engineers and managers seeking to enhance system reliability, maximize uptime, and minimize costs through informed, data-driven strategies.

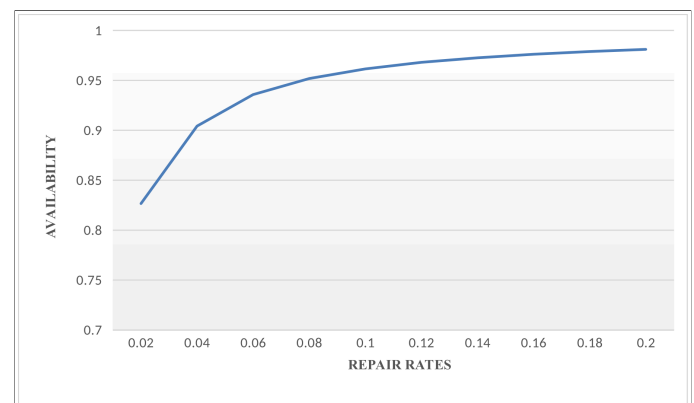


Figure 3. Availability versus repair rates

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