

International Journal of Statistics and Applied Mathematics

ISSN Online: 2456-1452
NAAS Rating (2026): 4.49
IJSAM 2026; 11(9): 172-186
www.mathsjournal.com
Received: 19-07-2026
Accepted: 20-08-2026
Published: 25-09-2026

Bishnupriya Behera

Assistant Professor,
Department of Statistics,
Rajendra University, Bolangir-
767002, Odisha, India Email:

Priyaranjan Dash

Professor, Department of
Statistics, Utkal University,
Bhubaneswar-751004, Odisha,
India Email:

Chakrالا Sreelatha

Assistant Professor,
Department of Statistics,
Rajendra University, Bolangir-
767002, Odisha, India Email:

Corresponding Author:

Chakrالا Sreelatha

Assistant Professor,
Department of Statistics,
Rajendra University,
Bolangir-767002, Odisha,
India Email:

Exploring auxiliary information for estimating finite population mean by an improved class of estimators

Bishnupriya Behera¹, Priyaranjan Dash², Chakrالا Sreelatha³

DOI: <https://www.doi.org/10.22271/math.2026.v11.i9b.2700>

ABSTRACT

In the present paper, an improved class of estimator of population mean has been proposed and utilizes the auxiliary information for developing the same estimators by combining the sample mean of the study variable. The bias and mean squared error of the proposed estimator has been obtained up to first order of approximation for the large sample. The optimum value of characterizing scalar which minimizes the mean squared error has been obtained and the minimum value of the mean squared error of the proposed estimator has been obtained by using the optimum value. A comparison of the proposed estimator has been made with mean per unit estimators and some other existing estimators of population mean. The performance of the proposed classes was analysed both theoretically and numerically by considering some practical examples.

Keywords: Auxiliary variable, Efficient estimators, Bias and Mean square error, Efficiency

1. Introduction

Within the field of statistics, the estimation of population parameters has proven to be a difficult task. Sampling methods have played an important role in developing various estimators suitable for different situations. The fact is that sampling is the modern approach towards all kinds of data driven inferences. It is the art and science of obtaining any needed information at the lowest possible cost and achieving an objective index of accuracy. Drawing a sample using simple random sampling from a homogeneous population with regard to the study's features has shown to be an effective technique for drawing inferences about population parameters. In most of circumstances, the statistician focus on estimation of the population mean or total. For instance, we might be interested in estimating the average or total annual income of a household in a locality. The estimation can be improved by utilizing some auxiliary variables that are highly correlated with the variable for which the estimate is needed. The use of supplementary information in sampling procedures is as old as survey sampling itself. It is a common practice to employ knowledge about auxiliary variables to create better designs and more effective estimators. This information may be used in the planning phase of the survey, in the estimation process, or at both stages simultaneously. The literature on survey sampling outlines a wide range of methods for utilizing auxiliary information (see, e.g., Cochran, 1977; Murthy, 1967) [1, 2]. Cochran (1977) [1] used auxiliary information to develop the ratio estimator for estimating the population mean. In the ratio estimator, the study variable and the auxiliary variable had a strong positive correlation, and the regression line should be intersected at the origin. When there is a strong negative correlation between the research variable and the auxiliary variable, Murthy (1964) rediscovers the Robson (1957) product estimator, which is used quite successful in these situations. Further, if the regression line of the study variable on auxiliary variable is a straight line passing through origin and the variance of study variable about this line is proportional to the magnitude of auxiliary variable, the ratio estimator is just as accurate as the regression estimator. However, in many situations the regression line does not pass through the origin. The ratio estimator does not perform as well in these cases as the regression estimator does. Consequently, authors such as Srivastava (1967) [3], Reddy (1973, 1974), Walsh (1970) [4], Chakrabarty (1979) [5], Bandyopadhyay (1980) [6], Srivenkataramana (1980) [7], Upadhyaya *et al.* (1999) [8], Singh *et al.* (2003) [9] to suggested modified ratio estimators in order to provide better alternatives. In recent years, new estimators have also emerged, in addition to many other contributions seen in sampling literature. Here we mention some estimators suggested by Choudhury *et al.* (2012) [10], Swain (2013), Jerajuddin *et al.* (2016) [11], Gupta *et al.* (2018) [12], Muili and Audu (2019), Dar *et al.* (2022), Lall *et al.* (2024) [13], etc. The product method of estimation has also been used by some authors when the study variable and auxiliary variable are negatively correlated. The ratio and product estimators have been combined by some authors to propose some more efficient estimators. The estimator proposed by Singh *et al.* (2003) [9], Perri (2007), Sifawa *et al.* (2021) [14] combines ratio and product estimators and has a smaller mean square error than the classical ratio

estimator. In recent years, some work has been done to propose efficient families of estimators and one can go through Kadilar and Cingi (2004), Subzar *et al.* (2018), Singh *et al.* (2020) [15], etc.

2. Summary of some existing estimators

Consider a finite population $U = \{U_1, U_2, \dots, U_N\}$ of N units. Let (y, x) denote the study variable and the auxiliary variable taking values (y_i, x_i) , $(i = 1, 2, \dots, N)$ respectively on the i th unit of U . The

population means of these two variables be $\bar{Y} = N^{-1} \sum_i y_i$ and $\bar{X} = N^{-1} \sum_i x_i$ for y and x respectively. Let us select a sample of size n from U by using simple random sampling without replacement (SRSWOR) scheme and observe (y, x) . Let us suppose that $\bar{y}$ and $\bar{x}$ are sample means of y and x respectively based on n sample units. We assume that the population mean $\bar{X}$ of the auxiliary variable x is known to us in advance. To study the large sample behaviour of different estimators, let us define

$$\theta = \frac{1}{n} - \frac{1}{N}, \quad C_y^2 = \frac{S_y^2}{\bar{Y}^2}, \quad C_x^2 = \frac{S_x^2}{\bar{X}^2}, \quad C_{yx} = \frac{S_{yx}}{\bar{Y}\bar{X}}$$

$$S_x^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{X})^2, \quad S_y^2 = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{Y})^2, \quad S_{yx} = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{X})(y_i - \bar{Y})$$

$$f = \frac{n}{N}, \quad G = \rho \frac{C_y}{C_x}, \quad \delta = \frac{n}{N-n}, \quad \delta_p = \frac{\bar{X}}{\bar{X} + n}$$

Several estimators / class of estimators (t) for estimating population mean $\bar{Y}$ proposed by different researchers. Their bias and mean square

error (MSE) respectively to the first order of approximation can be presented in Table-1 as:

Table 1. Various estimators, their biases and mean squared errors with their optimal conditions

Estimators	Bias	MSE	Optimal condition(α)
Mean per Unit Estimator (no auxiliary information case)			
$t_0 = \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$		$\theta \bar{Y}^2 C_y^2$	...
Cochran(1940) - Ratio Estimator			
$t_1 = \frac{\bar{y}}{\bar{x}} \bar{X}$	$\theta \bar{Y} [C_x^2 - C_{yx}]$	$\theta \bar{Y}^2 [C_y^2 + C_x^2 - 2C_{yx}]$	...
Robson(1957) and Murthy(1964) - Product Estimator			
$t_2 = \frac{\bar{y}}{\bar{x}} \bar{x}$	$\theta \bar{Y} C_{yx}$	$\theta \bar{Y}^2 [C_y^2 + C_x^2 + 2C_{yx}]$	...
Srivastava(1967) Estimator			
$t_3 = \bar{y} \left(\frac{\bar{x}}{\bar{X}} \right)^{\alpha_1}$	$\theta \bar{Y} \left[\frac{\alpha_1 - 1}{2} C_x^2 + C_{yx} \right]$	$\theta \bar{Y}^2 [C_y^2 + \alpha_1^2 C_x^2 + 2\alpha_1 C_{yx}]$	$\alpha_{1(opt.)} = -G$
Chakrabarty(1979) Estimator			
$t_4 = (1 - \alpha_2) \bar{y} + \alpha_2 \bar{X} \left(\frac{\bar{y}}{\bar{x}} \right)$	$\frac{\bar{Y}}{n} \alpha_2 [C_x^2 - C_{yx}]$	$\frac{\bar{Y}^2}{n} [C_y^2 + \alpha_2^2 C_x^2 - 2\alpha_2 C_{yx}]$	$\alpha_{2(opt.)} = G$
Srivenkataramana (1980) Estimator			
$t_5 = \bar{y} \left(\frac{N\bar{X} - n\bar{x}}{(N-n)\bar{X}} \right)$	$-\theta \bar{Y} \delta C_{yx}$	$\theta \bar{Y}^2 [C_y^2 + \delta^2 C_x^2 - 2\delta C_{yx}]$	...

(continued on next page)

Estimators	Bias	MSE	Optimal condition(α)
Singh and Espejo(2003) Estimator			
$t_6 = \bar{y} \left(\alpha_3 \left(\frac{\bar{X}}{\bar{x}} \right) + (1 - \alpha_3) \left(\frac{\bar{x}}{\bar{X}} \right) \right)$	$\theta \bar{Y} \left[\alpha_3 C_x^2 + (1 - \alpha_3) C_{yx} \right]$	$\theta \bar{Y}^2 \left[C_y^2 + (1 - 2\alpha_3)^2 C_x^2 + 2(1 - 2\alpha_3) C_{yx} \right]$	$\alpha_{3(opt.)} = \frac{1 + G}{2}$
Swain (2013) Estimator			
$t_7 = \bar{y} \left(\frac{\bar{X}}{\alpha_4 \bar{x} + (1 - \alpha_4) \bar{X}} \right)$	$\theta \bar{Y} \left[\alpha_4^2 C_x^2 - \alpha_4 C_{yx} \right]$	$\theta \bar{Y}^2 \left[C_y^2 + \alpha_4^2 C_x^2 - 2\alpha_4 C_{yx} \right]$	$\alpha_{4(opt.)} = G$
Jerajuddin and Kishun (2016) Estimator			
$t_8 = \bar{y} \left(\frac{\bar{X} + n}{\bar{x} + n} \right)$	$\theta \bar{Y} \left[\delta_p^2 C_x^2 - 2\delta_p C_{yx} \right]$	$\theta \bar{Y}^2 \left[C_y^2 + \delta_p^2 C_x^2 - 2\delta_p C_{yx} \right]$	...
Gupta and Yadav (2018) Estimator			
$t_9 = \bar{y} \left(\alpha_5 + (1 - \alpha_5) \left(\frac{\bar{X} + n}{\bar{x} + n} \right) \right)$	$\theta \bar{Y} (1 - \alpha_5) \left[\delta_p^2 C_x^2 - \delta_p C_{yx} \right]$	$\theta \bar{Y}^2 \left[C_y^2 + (\alpha_5 - 1)^2 \delta_p^2 C_x^2 + 2(\alpha_5 - 1) \delta_p C_{yx} \right]$	$\alpha_{5(opt.)} = 1 - \frac{G}{\delta_p}$
Muili and Audu(2019)			
$t_{10} = \bar{y} \left(\frac{\bar{X} + n}{\bar{x} + n} \right)^{\alpha_6}$	$\theta \bar{Y} \left[\frac{\alpha_6(\alpha_6 + 1)}{2} \delta_p^2 C_x^2 - \alpha_6 \delta_p C_{yx} \right]$	$\theta \bar{Y}^2 \left[C_y^2 + \alpha_6^2 \delta_p^2 C_x^2 - 2\alpha_6 \delta_p C_{yx} \right]$	$\alpha_{6(opt.)} = \frac{G}{\delta_p}$
Singh and Nigam(2020) Estimator			
$t_{11} = \bar{y} \left(\frac{\bar{X} - \alpha_7 \bar{x}}{(1 - \alpha_7) \bar{X}} \right)$	$-\theta \bar{Y} \left(\frac{\alpha_7}{1 - \alpha_7} \right) C_{yx}$	$\theta \bar{Y}^2 \left[C_y^2 + \left(\frac{\alpha_7}{1 - \alpha_7} \right)^2 C_x^2 - 2 \left(\frac{\alpha_7}{1 - \alpha_7} \right) C_{yx} \right]$	$\alpha_{7(opt.)} = \frac{G}{1 + G}$
Lall and Dash (2024) Estimator			
$t_{12} = \bar{y} \left(\frac{N\bar{X} - n\bar{x}}{(N - n)\bar{X}} \right)^{\alpha_8}$	$\theta \bar{Y} \left[\frac{\alpha_8(\alpha_8 - 1)}{2} \delta^2 C_x^2 - \alpha_8 \delta C_{yx} \right]$	$\theta \bar{Y}^2 \left[C_y^2 + \alpha_8^2 \delta^2 C_x^2 - 2\alpha_8 \delta C_{yx} \right]$	$\alpha_{8(opt.)} = \frac{G}{\delta}$

3. The proposed estimator

Motivated by Lall *et al.* (2024) [13] estimator of population mean we propose a generalized estimator of the population mean by using the auxiliary information as,

(3.1)

$$t = \frac{n}{N} \bar{y} + \left(\frac{N - n}{N} \right) \bar{y} \left(\frac{N\bar{X} - n\bar{x}}{(N - n)\bar{X}} \right)^\alpha,$$

where α is any real constant to be defined such that the mean squared error of the proposed is minimum.

$$E(\varepsilon_0) = E(\varepsilon_1) = 0, \quad E(\varepsilon_0)^2 = \theta C_y^2, \quad E(\varepsilon_1)^2 = \theta C_x^2, \quad E(\varepsilon_0 \varepsilon_1) = \theta \rho C_y C_x$$

In order to find the Bias and mean square error of these estimators, let us define,

$$\bar{y} = \bar{Y}(1 + \varepsilon_0), \quad \bar{x} = \bar{X}(1 + \varepsilon_1),$$

Such that,

Now expressing t in terms of ε 's we have,

$$\begin{aligned} t &= \frac{n}{N} \bar{Y}(1 + \varepsilon_0) + \left(\frac{N-n}{N} \right) \bar{Y}(1 + \varepsilon_0) \left(\frac{N\bar{X} - n\bar{x}(1 + \varepsilon_1)}{(N-n)\bar{X}} \right)^\alpha \\ &= \frac{n}{N} \bar{Y}(1 + \varepsilon_0) + \left(\frac{N-n}{N} \right) \bar{Y}(1 + \varepsilon_0) \left(1 - \frac{n}{N-n} \varepsilon_1 \right)^\alpha \\ &= \frac{n}{N} \bar{Y}(1 + \varepsilon_0) + \left(\frac{N-n}{N} \right) \bar{Y}(1 + \varepsilon_0) (1 - \delta \varepsilon_1)^\alpha, \quad \delta = \frac{n}{N-n} \end{aligned}$$

(3.2)

$$t = \bar{Y}(1 + \varepsilon_0) \left(\frac{n}{N} + \left(\frac{N-n}{N} \right) (1 - \delta \varepsilon_1)^\alpha \right)$$

Expanding the right-hand side of equation (3.2) retaining the terms up to the second order of ε 's, so we can have,

$$t \cong \bar{Y}(1 + \varepsilon_0) \left(\frac{n}{N} + \left(\frac{N-n}{N} \right) \left(1 - \alpha \delta \varepsilon_1 + \frac{\alpha(\alpha-1)}{2} \delta^2 \varepsilon_1^2 \right) \right), \quad \delta = \frac{n}{N-n}$$

(3.3)

Subtracting $\bar{Y}$ and taking expectation on both side we get bias of the estimator t as

$$B(t) = \theta \bar{Y} \left(\frac{N-n}{N} \right) \left(\frac{\alpha(\alpha-1)}{2} \delta^2 C_x^2 - \alpha \delta \rho C_y C_x \right)$$

(3.4)

The proposed class of estimator up to first order will be unbiased for $\bar{Y}$ if

$$\left(\frac{(\alpha-1)}{2} \delta C_x - \rho C_y \right) = 0 \Rightarrow \frac{(\alpha-1)}{2} \delta = \rho \frac{C_y}{C_x} = \frac{\beta}{R} \Rightarrow \alpha = 1 + 2 \left(\frac{N-n}{n} \right) \left(\frac{\beta}{R} \right)$$

(3.5)

So, when $\beta \approx R$, i.e., the regression line of y on x is linear passing through origin then the above unbiasedness condition reduces to $1 + 2 \left(\frac{N-n}{n} \right)$. So, when the sample size increases, then $\left(\frac{N-n}{n} \right) \rightarrow 0$, hence $\alpha \rightarrow 1$. In this case, our proposed class of estimator reduces to

$$t = \frac{n}{N} \bar{y} + \left(\frac{N-n}{N} \right) \bar{y} \left(\frac{N\bar{X} - n\bar{x}}{(N-n)\bar{X}} \right) \quad (3.6)$$

which is an unbiased estimator of $\bar{Y}$. It utilizes the information as $I(x) = \left(\frac{N\bar{X} - n\bar{x}}{(N-n)\bar{X}} \right)$, where $E[I(x)] = 1$. As we increase the sample size, $I(x) \rightarrow 1$ i.e. the difference between the population total $N\bar{X}$ and the sample total $n\bar{x}$ is minimized. So when the sample selected based on the auxiliary characteristic is representative then the proposed class of estimator provides a better estimate of $\bar{Y}$.

Again, from Equation (3.3), we have

$$t - \bar{Y} \cong \bar{Y} \left[\varepsilon_0 - \left(\frac{N-n}{N} \right) \left\{ \alpha \delta \varepsilon_1 - \frac{\alpha(\alpha-1)}{2} \delta^2 \varepsilon_1^2 + \alpha \delta \varepsilon_0 \varepsilon_1 \right\} \right] \quad (3.7)$$

Squaring both sides of (3.7) and retaining the terms up to first order approximation, we have

$$(t - \bar{Y})^2 \cong \bar{Y}^2 \left[\varepsilon_0^2 + \left(\frac{N-n}{N} \right)^2 \alpha^2 \delta^2 \varepsilon_1^2 - 2 \left(\frac{N-n}{N} \right) \alpha \delta \varepsilon_0 \varepsilon_1 \right] \quad (3.8)$$

Taking expectations of both sides of (3.8) and putting the values of different expectations we get the MSE of the estimator t , up to the first order of approximation, as

$$MSE(t) = \theta \bar{Y}^2 \left[C_y^2 + \left(\frac{N-n}{N} \right)^2 \alpha^2 \delta^2 C_x^2 - 2 \left(\frac{N-n}{N} \right) \alpha \delta \rho C_y C_x \right] \quad (3.9)$$

The optimum value of α that minimizes the MSE of proposed estimator t . So

$$\alpha_{opt.} = \frac{\rho C_y}{f C_x} \quad (3.10)$$

Now by using the value of $\alpha_{opt.}$, then (3.4) becomes

$$B'(t) = \theta \bar{Y} \delta \left(\frac{\rho C_y}{C_x} - \frac{1}{2} \right) C_{yx} \quad (3.11)$$

Substituting the optimum value of α in (3.9) then the minimum value of MSE of proposed estimator t is

$$MSE(t) = \theta \bar{Y}^2 (1 - \rho^2) C_y^2 \quad (3.12)$$

4. Theoretical comparison

We first compare the mean square error (MSE) of the proposed class of estimators t with the different competing estimators /class of estimators proposed by different authors. Again, the classes are compared using the bias criterion.

4.1 Comparison of efficiency

4.1.1 With Simple mean estimator (t_0)

The proposed estimator t is preferred to t_0 if $MSE(t) \leq Var(t_0)$ i.e.,

$$\alpha \left(\alpha - \frac{2\beta}{fR} \right) \leq 0$$

$$\Rightarrow 0 < \alpha < \frac{2\beta}{fR} \quad (4.1)$$

4.1.2 With Ratio estimator (t_1)

The proposed estimator t is preferred to t_1 if $MSE(t) \leq MSE(t_1)$ i.e.,

$$(\alpha f - 1) \left((\alpha f + 1) - \frac{2\beta}{R} \right) \leq 0$$

$$\Rightarrow \frac{1}{f} < \alpha < \frac{1}{f} \left(\frac{2\beta}{R} - 1 \right) \quad (4.2)$$

4.1.3 With Product estimator (t_2)

The proposed estimator t is preferred to t_2 if $MSE(t) \leq MSE(t_2)$ i.e.,

$$(\alpha f + 1) \left((\alpha f - 1) - \frac{2\beta}{fR} \right) \leq 0$$

$$\Rightarrow -\frac{1}{f} < \alpha < \frac{1}{f} \left(\frac{2\beta}{R} + 1 \right) \quad (4.3)$$

4.1.4 With Srivastava estimator (t_3)

The proposed estimator t is preferred to t_3 if $MSE(t) \leq MSE(t_3)$ i.e.,

$$\alpha \left[\alpha(f-1) - \frac{2\beta}{R} \right] \leq 0$$

$$\Rightarrow 0 < \alpha < \frac{2\beta}{(f-1)R} \quad (4.4)$$

4.1.5 With Chaktabarty estimator (t_4)

The proposed estimator t is preferred to t_4 if $MSE(t) \leq MSE(t_4)$ i.e.,

$$\alpha \left(\alpha - \frac{2\beta}{(f+1)R} \right) \leq 0$$

$$\Rightarrow 0 \leq \alpha \leq \frac{2\beta}{(f+1)R} \quad (4.5)$$

4.1.6 With Srivenkataramana estimator (t_5)

The proposed estimator t is preferred to t_5 if $MSE(t) \leq MSE(t_5)$ i.e.,

$$\alpha \left(\alpha - \frac{2\beta}{(f+\delta)R} \right) \leq 0$$

$$\Rightarrow 0 < \alpha < \frac{2\beta}{(f+\delta)R} \quad (4.6)$$

4.1.7 With Singh and Espejo estimator (t_6)

The proposed estimator t is preferred to t_6 if $MSE(t) \leq MSE(t_6)$ i.e.,

$$\left[\alpha(f-2) - \left(\frac{2\beta}{R} - 1 \right) \right] \leq 0$$

$$\Rightarrow 0 < \alpha < \frac{1}{(f-2)} \left(\frac{2\beta}{R} - 1 \right) \quad (4.7)$$

4.1.8 With Swain estimator (t_7)

The proposed estimator t is preferred to t_7 if $MSE(t) \leq MSE(t_7)$ i.e.,

$$\alpha \left(\alpha - \frac{2\beta}{(f+1)R} \right) \leq 0$$

$$\Rightarrow \frac{\left(f + \frac{2\beta}{R} + 1 \right) - \sqrt{\left(f + \frac{2\beta}{R} + 1 \right)^2 - 8\frac{\beta f}{R}}}{2f} < \alpha < \frac{\left(f + \frac{2\beta}{R} + 1 \right) + \sqrt{\left(f + \frac{2\beta}{R} + 1 \right)^2 - 8\frac{\beta f}{R}}}{2f} \quad (4.12)$$

4.1.13 With Lall and Dash estimator (t_{12})

The proposed estimator t is preferred to t_{12} if $MSE(t) \leq MSE(t_{12})$ i.e.,

$$\Rightarrow 0 < \alpha < \frac{2\beta}{(f+1)R} \quad (4.8)$$

4.1.9 With Jerajuddin and Kishun estimator (t_8)

The proposed estimator t is preferred to t_8 if $MSE(t) \leq MSE(t_8)$ i.e.,

$$\left[\alpha f - \left(\frac{2\beta}{R} - \delta_p \right) \right] \leq 0$$

$$\Rightarrow 0 < \alpha < \frac{1}{f} \left(\frac{2\beta}{R} - \delta_p \right) \quad (4.9)$$

4.1.10 With Gupta and Yadav estimator (t_9)

The proposed estimator t is preferred to t_9 if $MSE(t) \leq MSE(t_9)$ i.e.,

$$\left[\alpha(f + \delta_p) - \left(\frac{2\beta}{R} + \delta_p \right) \right] \leq 0$$

$$\Rightarrow 0 < \alpha < \frac{1}{(f + \delta_p)} \left(\frac{2\beta}{R} + \delta_p \right) \quad (4.10)$$

4.1.11 With Mulli and Audu estimator (t_{10})

The proposed estimator t is preferred to t_{10} if $MSE(t) \leq MSE(t_{10})$ i.e.,

$$\left[\alpha(f + \delta_p) - \left(\frac{2\beta}{R} \right) \right] \leq 0$$

$$\Rightarrow 0 < \alpha < \frac{1}{(f + \delta_p)} \left(\frac{2\beta}{R} \right) \quad (4.11)$$

4.1.12 With Singh and Nigam estimator (t_{11})

The proposed estimator t is preferred to t_{11} if $MSE(t) \leq MSE(t_{11})$ i.e.,

$$\left[\alpha f + \left(\frac{\alpha}{1-\alpha} \right) - \left(\frac{2\beta}{R} \right) \right] \leq 0$$

$$\alpha \left(\alpha - \frac{2\beta}{(f-2)\delta R} \right) \leq 0$$

$$\Rightarrow 0 < \alpha < \frac{2\beta}{(f-2)\delta R} \quad (4.13)$$

4.2 Comparison of bias

4.2.1 With Simple mean estimator (t_0)

The proposed estimator t is preferred to t_0 if $B(t) \leq B(t_0)$ i.e.,

$$\theta \bar{Y} \delta \left(\frac{\rho C_y}{C_x} - \frac{1}{2} \right) C_{yx} \leq 0 \Rightarrow \left(\frac{\rho C_y}{C_x} - \frac{1}{2} \right) \leq 0 \Rightarrow \frac{\rho C_y}{C_x} \leq \frac{1}{2}$$

The proposed estimator t is preferred to t_1 if $B(t) \leq B(t_1)$ i.e.,

$$\text{For } C_y \approx C_x, \text{ it reduces to } \rho < \frac{1}{2} \quad (4.14)$$

4.2.2 With Ratio estimator (t_1)

$$\begin{aligned} & \left(\frac{n}{N-n} \right) \left(\frac{\beta}{R} \right)^2 + \frac{2N-3n}{2(N-n)} \frac{\beta}{R} - 1 \leq 0 \\ \Rightarrow & \left(\frac{(3n-2N)}{4n} \right) - \left(\frac{N-n}{2n} \right) \sqrt{\left(\frac{2N-3n}{2(N-n)} \right)^2 + \frac{4n}{N-n}} < \frac{\beta}{R} < \left(\frac{(3n-2N)}{4n} \right) + \\ & \left(\frac{N-n}{2n} \right) \sqrt{\left(\frac{2N-3n}{2(N-n)} \right)^2 + \frac{4n}{N-n}} \end{aligned}$$

For $C_y \approx C_x$, it reduces to

$$\left(\frac{(3n-2N)}{4n} \right) - \left(\frac{N-n}{2n} \right) \sqrt{\left(\frac{2N-3n}{2(N-n)} \right)^2 + \frac{4n}{N-n}} < \rho < \left(\frac{(3n-2N)}{4n} \right) + \left(\frac{N-n}{2n} \right) \sqrt{\left(\frac{2N-3n}{2(N-n)} \right)^2 + \frac{4n}{N-n}} \quad (4.15)$$

4.2.3 With Product estimator (t_2)

The proposed estimator t is preferred to t_2 if $B(t) \leq B(t_2)$ i.e.,

For $C_y \approx C_x$, it reduces to

$$\left(\frac{n}{N-n} \right) \left(\frac{\beta}{R} - \frac{1}{2} \right) - 1 \leq 0 \quad 0 < \rho < \left(\frac{2N-n}{2n} \right) \quad (4.16)$$

$$\Rightarrow 0 < \frac{\beta}{R} < \left(\frac{2N-n}{2n} \right)$$

4.2.4 With Srivastava estimator (t_3)

The proposed estimator t is preferred to t_3 if $B(t) \leq B(t_3)$ i.e.,

$$\begin{aligned} & \left(\frac{n}{N-n} \right) \left(\frac{\beta}{R} \right)^2 - \frac{N}{2(N-n)} \left(\frac{\beta}{R} \right) + 1 \leq 0 \\ \Rightarrow & \frac{N - \sqrt{(N-2n)^2 + 4n^2}}{4n} < \frac{\beta}{R} < \frac{N + \sqrt{(N-2n)^2 + 4n^2}}{4n} \end{aligned}$$

For $C_y \approx C_x$, it reduces to

$$\frac{N - \sqrt{(N-2n)^2 + 4n^2}}{4n} < \rho < \frac{N + \sqrt{(N-2n)^2 + 4n^2}}{4n} \quad (4.17)$$

4.2.5 With Chaktabarty estimator (t_4)

The proposed estimator t is preferred to t_4 if $B(t) \leq B(t_4)$ i.e.,

$$\left[\left(\frac{n}{N-n} \right) \left(\rho - \frac{1}{2} \right) - 1 \right] C_x + \rho C_y \leq 0$$

$$\Rightarrow 0 < \frac{\beta}{R} < \left(1 - \frac{f}{2} \right)$$

For $C_y \approx C_x$, it reduces to

$$0 < \rho < \left(1 - \frac{f}{2} \right) \quad (4.18)$$

The proposed estimator t is preferred to t_6 if $B(t) \leq B(t_6)$ i.e.,

$$\left[\left(\frac{n}{N-n} \right) \left(\frac{\beta}{R} - \frac{1}{2} \right) \right] \rho C_y C_x \leq 0$$

$$\Rightarrow \frac{\beta}{R} < \frac{1}{2}$$

For $C_y \approx C_x$, it reduces to

$$\rho < \frac{1}{2} \quad (4.20)$$

4.2.6 With Srivenkataramana estimator (t_5)

The proposed estimator t is preferred to t_5 if $B(t) \leq B(t_5)$ i.e.,

$$\left[\left(\frac{n}{N-n} \right) \left(\frac{\beta}{R} + \frac{1}{2} \right) - 1 \right] \rho C_y C_x \leq 0$$

$$\Rightarrow \frac{\beta}{R} < -\frac{1}{2}$$

For $C_y \approx C_x$, it reduces to

$$\rho < -\frac{1}{2} \quad (4.19)$$

4.2.8 With Swain estimator (t_7)

The proposed estimator t is preferred to t_7 if $B(t) \leq B(t_7)$ i.e.,

$$\left[\left(\frac{n}{N-n} \right) \left(\frac{\beta}{R} - \frac{1}{2} \right) \right] \rho C_y C_x \leq 0$$

$$\Rightarrow \frac{\beta}{R} < \frac{1}{2}$$

For $C_y \approx C_x$, it reduces to

$$\rho < \frac{1}{2} \quad (4.21)$$

4.2.7 With Singh and Espejo estimator (t_6)

$$\left[\left(\frac{n}{N-n} \right) \left(\frac{\rho C_y C_x}{C_x^2} - \frac{1}{2} \right) \right] \rho C_y C_x + 2\delta_p \rho C_y C_x - \delta_p^2 C_x^2 \leq 0$$

$$\Rightarrow \frac{C_y}{C_x} \cdot \frac{(N-n)}{2n} \left[\left(\frac{n}{2(N-n)} - 2\delta_p \right) - \sqrt{\left(2\delta_p - \frac{n}{2(N-n)} \right)^2 + \frac{4n}{N-n} \delta_p^2} \right] < \frac{\beta}{R} < \frac{C_y}{C_x} \cdot \frac{(N-n)}{2n} \left[\left(\frac{n}{2(N-n)} - 2\delta_p \right) + \sqrt{\left(2\delta_p - \frac{n}{2(N-n)} \right)^2 + \frac{4n}{N-n} \delta_p^2} \right]$$

For $C_y \approx C_x$, it reduces to

4.2.9 With Jerajuddin and Kishun estimator (t_8)

The proposed estimator t is preferred to t_8 if $B(t) \leq B(t_8)$ i.e.,

(4.22)

$$\frac{(N-n)}{2n} \left[\left(\frac{n}{2(N-n)} - 2\delta_p \right) - \sqrt{\left(2\delta_p - \frac{n}{2(N-n)} \right)^2 + \frac{4n}{N-n} \delta_p^2} \right] < \rho < \frac{(N-n)}{2n} \left[\left(\frac{n}{2(N-n)} - 2\delta_p \right) + \sqrt{\left(2\delta_p - \frac{n}{2(N-n)} \right)^2 + \frac{4n}{N-n} \delta_p^2} \right]$$

4.2.10 With Gupta and Yadav estimator (t_9)

The proposed estimator t is preferred to t_9 if $B(t) \leq B(t_9)$ i.e.,

$$\left[\left(\frac{1}{N-n} \right) \left(N \frac{\beta}{R} - \frac{n}{2} \right) - \delta_p \right] \leq 0$$

$$\Rightarrow 0 < \frac{\beta}{R} < \left[\frac{f}{2} + \left(\frac{N-n}{N} \right) \delta_p \right]$$

For $C_y \approx C_x$, it reduces to

$$0 < \rho < \left[\frac{f}{2} + \left(\frac{N-n}{N} \right) \delta_p \right] \quad (4.23)$$

4.2.11 With Mulli and Audu estimator (t_{10})

The proposed estimator t is preferred to t_{10} if $B(t) \leq B(t_{10})$ i.e.,

$$\left[\left(\frac{n}{N-n} + \frac{1}{4} \right) \rho C_y - \left(\frac{n}{2(N-n)} - \frac{\delta_p}{4} \right) C_x \right] \leq 0$$

$$\Rightarrow 0 < \rho < \left[\frac{C_x}{C_y} \cdot \frac{\left(\frac{n}{2(N-n)} - \frac{\delta_p}{4} \right)}{\left(\frac{n}{N-n} + \frac{1}{4} \right)} \right] \Rightarrow 0 < \rho < \left[\frac{C_x}{C_y} \cdot \frac{(4n-2)(N-n)\delta_p}{2(N+3n)} \right]$$

For $C_y \approx C_x$, it reduces to

$$0 < \rho < \left[\frac{(4n-2)(N-n)\delta_p}{2(N+3n)} \right] \quad (4.24)$$

$$0 < \rho < \left[\frac{n}{n(C_x^2 + 1) + N} \right] C_x^2 \quad (4.25)$$

4.2.12 With Singh and Nigam estimator (t_{11})

The proposed estimator t is preferred to t_{11} if $B(t) \leq B(t_{11})$ i.e.,

$$\left[\left(\frac{n(C_x^2 + 1) + N}{(N-n)C_x^2} \right) \rho - \left(\frac{n}{2(N-n)} \right) \frac{C_x}{C_y} \right] \leq 0$$

$$\Rightarrow 0 < \rho < \left[\frac{n}{n(C_x^2 + 1) + N} \right] \frac{C_x^3}{C_y}$$

4.2.13 With Lall and Dash estimator (t_{12})

The proposed estimator t is preferred to t_{12} if $B(t) \leq B(t_{12})$ i.e.,

$$\left[\left(\frac{n}{N-n} + \frac{1}{2} \right) \left(\frac{\beta}{R} \right) \right] \leq 0$$

$$\Rightarrow \frac{\beta}{R} < 0$$

For $C_y \approx C_x$, it reduces to

$$\text{For } C_y \approx C_x, \text{ it reduces to } \rho < 0 \quad (4.26)$$

Table 2. Description of Sources of population

Pop.No.	Source	x	y
2.	Murthy (1967), P-228	Fixed capital in ('000) rupees	Output for factories in ('000) rupees
2.	Murthy (1967), P-398	No. of workers	No. of absentees
3.	Sukhatme&Sukhatme (1970), P-51	Total cultivated area of the village (acres)	Area under Rice (acres)
4.	Cochran (1977), P-325	Population of U.S. in 1950(in millions)	Population of U.S. in 1960(in millions)
5.	Murthy (1967), P-228	No. of workers	Output for factories in ('000) rupees
6.	Murthy (1967), P-228	No. of workers	No. of absentees

(continued on next page)

Table 3. Description of Some Important Parameters of The Population

P.No.		N	$\bar{X}$	$\bar{Y}$	ρ	R	C_x	C_y	C_{yx}	α_{opt}
		15	630	4108.4	0.82	6.52	0.11	0.09	0.01	2.05
2.	2.	16	77.31	6.88	0.77	0.09	0.54	0.61	0.25	2.75
3.	3.	15	1011.13	782.93	0.96	0.77	0.58	0.7	0.38	3.47
4.	4.	10	58.8	101.1	0.65	1.72	0.14	0.16	0.01	1.47
5.	5.	15	108.13	4108.4	0.87	37.99	0.14	0.09	0.01	1.71
6.	6.	14	72.5	10.29	0.54	0.14	0.33	0.58	0.1	2.61

Table 4. Absolute Percent Relative Bias of different estimators

Pop.No.		t_0	t_1	t_2	t_3	t_4	t_5	t_6
$n = 5$								
		0.00	0.05	0.10	0.02	0.03	0.05	0.07
2.	2.	0.00	0.60	3.28	0.27	0.52	1.49	1.58
3.	3.	0.00	0.61	4.77	0.32	0.70	2.38	1.37
4.	4.	0.00	0.05	0.13	0.02	0.04	0.13	0.09
5.	5.	0.00	0.10	0.13	0.03	0.06	0.07	0.12
6.	6.	0.00	0.09	1.25	0.04	0.08	0.69	0.54
Pop.No.		t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t
$n = 5$								
		0.00	0.04	0.03	0.02	5.81	0.06	0.04
2.	2.	0.01	0.32	0.30	0.16	9.52	2.14	1.50
3.	3.	0.09	0.62	0.72	0.34	16.63	3.84	2.45
4.	4.	0.00	0.03	0.02	0.01	4.89	0.12	0.07
5.	5.	0.00	0.09	0.05	0.03	4.06	0.07	0.05
6.	6.	0.01	0.00	0.00	0.00	10.38	0.93	0.61

Table 5. Percent Relative Efficiency of different estimators

Pop.No.		t_0	t_1	t_2	t_3	t_4	t_5	t_6
$n = 5$								
		100	224.77	24.39	322.86	322.54	282.40	108.23
2.	2.	100	231.81	32.01	248.45	246.41	200.10	150.46
3.	3.	100	1039.15	30.01	1174.75	1199.06	309.35	160.04
4.	4.	100	174.89	37.70	191.78	191.70	175.23	112.89
5.	5.	100	157.76	17.94	440.20	438.92	420.13	109.38
6.	6.	100	147.76	54.42	148.47	148.38	141.85	120.98

(continued on next page)

Pop.No.		t_0	t_1	t_2	t_3	t_4	t_5	t_6
Pop.No.		t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t
$n = 5$								
		323.10	228.19	322.56	322.87	0.02	323.38	324.39
2.	2.	250.45	242.65	248.37	249.38	24.36	253.08	254.73
3.	3.	1130.22	1029.99	1200.52	1175.90	20.58	1095.09	1220.41
4.	4.	191.84	183.04	191.75	191.80	0.11	191.91	192.95
5.	5.	440.94	180.57	439.23	440.30	0.05	440.52	441.93
6.	6.	148.56	148.54	148.56	148.56	5.37	149.79	159.50

Table 6. Skewness of different estimators

Pop.No.		t_0	t_1	t_2	t_3	t_4	t_5	t_6
$n = 5$								
		0.16	-0.03	0.15	0.05	0.04	0.15	0.24
2.	2.	0.15	0.22	0.62	0.04	0.04	-0.12	0.18
3.	3.	0.40	-0.21	0.69	-0.09	-0.08	0.00	0.56
4.	4.	0.00	0.05	0.09	0.00	-0.01	0.06	0.03
5.	5.	0.16	0.25	0.11	0.60	0.60	0.63	0.31
6.	6.	0.13	0.16	0.38	0.14	0.14	0.09	0.11
Pop.No.		t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t
$n = 5$								
		0.05	-0.03	0.04	0.05	-0.15	0.07	0.07
2.	2.	0.03	0.13	0.04	0.04	-0.84	0.07	0.04
3.	3.	-0.07	-0.21	-0.08	-0.09	-1.17	-0.03	-0.02
4.	4.	0.00	0.03	-0.01	0.00	-0.14	0.02	0.01
5.	5.	0.61	0.24	0.60	0.61	-0.12	0.63	0.62
6.	6.	0.14	0.15	0.14	0.14	-0.45	0.17	0.16

Table 7. Kurtosis of different estimators

Pop.No.		t_0	t_1	t_2	t_3	t_4	t_5	t_6
$n = 5$								
		2.59	2.48	2.71	2.60	2.60	2.51	2.63
2.	2.	2.64	2.87	3.17	2.73	2.74	2.71	2.79
3.	3.	2.15	2.45	2.50	2.63	2.61	2.11	2.31

(continued on next page)

Pop.No.		t_0	t_1	t_2	t_3	t_4	t_5	t_6
4.	4.	2.48	2.57	2.46	2.61	2.61	2.56	2.49
5.	5.	2.59	2.82	2.76	1.83	1.82	1.67	2.64
6.	6.	2.58	2.72	2.81	2.70	2.70	2.64	2.59
Pop.No.		t_7	t_8	t_9	t_{10}	t_{11}	t_{12}	t
$n = 5$								
		2.60	2.48	2.60	2.60	2.67	2.60	2.60
2.	2.	2.73	2.78	2.73	2.73	4.23	2.74	2.75
3.	3.	2.68	2.44	2.60	2.63	3.78	2.54	2.54
4.	4.	2.61	2.58	2.61	2.61	2.54	2.62	2.62
5.	5.	1.83	2.81	1.82	1.83	2.75	1.85	1.84
6.	6.	2.70	2.71	2.70	2.70	3.04	2.73	2.72

Figure 1. Bar diagram showing the Performance of PRB of Proposed Estimator with Different Competing Estimators with Sample Size $n = 5$

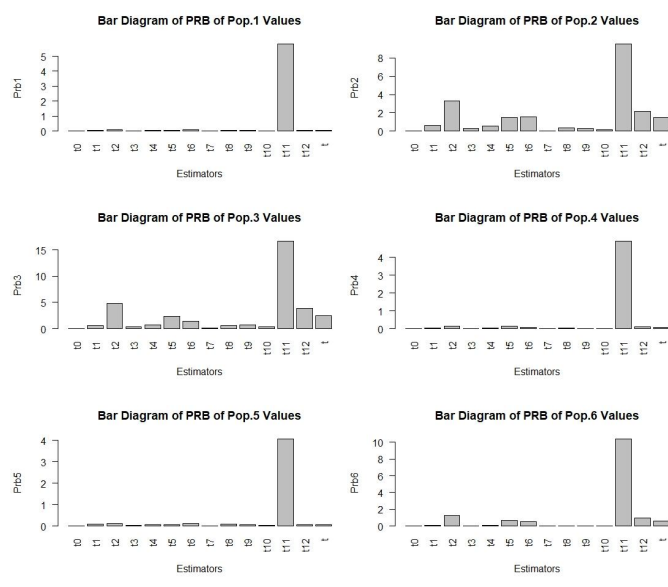


Figure 2. Line diagram showing the Performance of PRE of Proposed Estimator with Different Competing Estimators with Sample Size $n = 5$

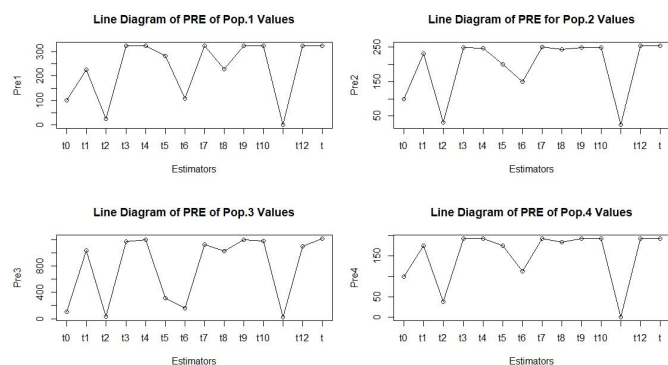


Figure 3. Histogram showing the Sampling Distribution of the Proposed Estimator with Sample Size $n = 5$

5. Numerical illustration

In this present paper, the performance of the proposed class of estimators with the different existing estimators has discussed, we have used 6 (six) natural population data sets. The sources of the population and the description of the variables y and x are listed in Table-2. These six populations are studied for sample size $n = 5$. Table-3 gives the values of different parameters such as N , $\bar{X}$, $\bar{Y}$, ρ , R , C_x , C_y , C_{yx} and the optimum value denoted as α_{opt} of constant α used in proposed class of estimators for which it gives the minimum variance for different sets of population. Now, we study the performance of the proposed class of estimators with some competing estimators namely simple mean estimator (t_0), ratio estimator (t_1), product estimator (t_2) and the dif

ferent classes of estimators due to Srivastava (t_3) Chakrabarty (t_4), Srivenkataramana (t_5), Singh and Espejo (t_6) Swain (t_7), Jerajuddin and Jai Kishun (t_8), Gupta and Yadav (t_9), Muili and Audu (t_{10}), Singh and Nigam (t_{11}) and Lall and Dash (t_{12}) by considering all possible samples of sizes 5 and studied absolute percent relative bias, percent relative efficiency, skewness and kurtosis which are described in Tables-4, 5, 6 & 7 respectively. Table-4 gives the Absolute Percent Relative Bias (PRB) of the proposed class of estimators for its optimum case. Table-5 describe the Percent Relative Efficiency (PRE) of the proposed class of estimator in its optimum (MVB) case (t_{opt}). It reveals that the proposed class of estimators in optimum case has maximum PRE for all the populations. From the Table-6, we have seen that for the sample of size 5, the skewness of the proposed class of estimators varies from -0.02 to 0.62 shows that the sampling distribution is almost symmetrical. Again, Table-7 shows the kurtosis of the proposed class of estimators with other classes of estimators. For the sample size 5, the kurtosis of the proposed class of estimators varies from 1.84 to 2.75 which shows that the proposed class of estimators becomes asymptotically mesokurtic. Again, Table-8 shows the comparison of PRB of proposed class of estimators among with other competing estimators through bar diagram. Now, Table-9 shows the comparison of PRE of proposed estimator among with other competing estimators through line diagram. From this diagram it has seen that the PRE of proposed class of estimators is high as compared to other. so the proposed estimator(t) is more better than the other competing estimators. Also, Table-10 shows the nature of the distribution of the proposed estimator in its optimum case, we have constructed the histogram of the values of proposed estimator for sample sizes 5 considering all possible samples. The histograms reveal that the distribution of the estimator becomes asymptotically symmetrical and close to a standard normal distribution at a faster rate than other competing estimators.

6. Conclusion

In this paper an effort has been made to propose a generalized estimator of the population mean by using the auxiliary information Motivated by Lall *et al.* (2024) [13] estimator of population mean. Expressions for bias and mean square error (MSE) of proposed a class of estimators up to the first order approximation have been obtained using SRSWOR. Again, our proposed class of estimators is compared with several other estimators namely simple mean estimator (t_0), ratio estimator (t_1), product estimator (t_2) and the different classes of estimators due to Srivastava (t_3) Chakrabarty (t_4) Srivenkataramana (t_5), Singh and Espejo (t_6) Swain (t_7), Jerajuddin and Jai Kishun (t_8), Gupta and Yadav (t_9), Muili and Audu (t_{10}), Singh and Nigam (t_{11}) and Lall and Dash (t_{12}) both theoretically and numerically by considering all possible samples of sizes 5 of 6 natural populations available in different text books. Again, the study reveals that our proposed class of estimators is asymptotically unbiased with a high percent relative efficiency and the sampling distribution is symmetrical and mesokurtic. The conditions under which our proposed class of estimators is preferable than others are also derived. From both the theoretical and simulation study favours the proposed class of estimators than other competing estimators.

Funding statement

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Data availability statement

No Data was used for the research described in the article

Disclosure statement

No potential conflict of interest was reported by the author(s).

Acknowledgments

The authors are highly thankful to the reviewers and the editors for their valuable suggestions for improving the paper

References

1. Cochran WG. Sampling Techniques. New York: John Wiley; 1977.
2. Murthy MN. Sampling Theory and Methods. Calcutta: Statistical Publishing Society; 1967.
3. Srivastava SK. An estimator using auxiliary information in sample surveys. Cal. Stat. Assoc. Bull. 1967;16:121-132.
4. Walsh JE. Generalization of ratio estimator for population total. Sankhya A. 1970;32:99-106.
5. Chakrabarty RP. Some ratio-type estimators. Journal of Indian Society and Agricultural Statistics. 1979;31:49-62.
6. Bandopadhyaya S. Improved ratio and product estimators. Sankhya. 1980;42:45-49.
7. Srivenkataramana T. A dual to ratio estimator in sample surveys. Biometrika. 1980;67:199-204.
8. Upadhyaya LN, Singh HP. Use of transformed auxiliary variable in estimating the finite population mean. Biometrical Journal. 1999;41:627-636.
9. Singh HP, Espejo MR. On linear regression and ratio-product estimation of a finite population mean. Journal of the Royal Statistical Society: Series D (The Statistician). 2003;52(1):59-67.
10. Choudhury S, Singh BK. An efficient class of dual to product-cum-dual to ratio estimators of finite population mean in sample surveys. Global Journal of Science Frontier of Research Mathematics & Decision Sciences. 2012;12:25-33.
11. Jerajuddin M, Kishun J. Modified Ratio Estimators for Population Mean Using Size of the Sample Selected from Population. IJSRSET. 2016;2(2):10-16.
12. Gupta RK, Yadav SK. Improved Estimation of Population Mean Using Information on Size of the Sample American Journal of Mathematics and Statistics. 2018;8(2):27-3.
13. Lall N, Dash PR. A Class of Efficient Estimators of Finite Population Mean Using Unbiased Estimators of One Utilizing the Auxiliary Information. International Journal of Statistics and Reliability Engineering. 2024;11:58-69.
14. Sifawa AA, Shehu MZ. Modified Ratio - Cum- Product Type Estimator for Population Mean in Simple Random Sampling Scheme. Himalayan Journal of Applied Medical Sciences and Research. 2021;2(5):1-5.
15. Singh HP, Nigam P. A General Class of Dual to Ratio Estimators. Pakistan Journal of Statistics and Operation Research. 2020;16(33):421-431.
16. Bahl S, Tuteja R. Ratio and product type exponential estimators. Journal of Information and Optimization Sciences. 1991;12(1):159-164.
17. Reddy VN. Ratio and product method of estimation. Sankhya, B. 1973;35:307-316.
18. Reddy VN. On a transformed ratio method of estimation. Sankhya, C. 1974;36:59-70.

19. Sharma B, Tailor R. A new ratio-cum dual to ratio estimator of finite population mean in simple random sampling. *Global Journal of Science Frontier Research*. 2010;10(1):27-31.
20. Sisodia BVS, Diwedi VK. A Class of Ratio-Cum-Product-Type Estimator. *Biometrical Journal*. 1981;23:133-139.
21. Sukhatme PV, Sukhatme BV, Sukhatme S, Asok C. *Sampling theory of surveys with applications*. Ind. Soc. Ag. Stat., New Delhi; 1984.
22. Swain AKPC. *Finite Population Sampling: Theory and Methods*. South Asia Publishers, New Delhi; 2003.
23. Swain AKPC. On Some Modified Ratio and Product Type Estimators. *Revista Investigaci'on Operacional*. 2003;34(1):35-57.
24. Swain AKPC. On an improved ratio type estimator of finite population mean in sample surveys. *Revista Invest. Opera*. 2014;35(1):49-57.
25. Singh HP, Tailor R, Tailor R, Kakran MS. An Improved Estimator of Population Mean Using Power Transformation. *Journal of the Indian Society of Agricultural Statistics*. 2004;58(2):223-230.
26. Searls DT. The utilization of known coefficient of variation in the estimation procedure. *Journal of American Statistical Association*. 1964;59:1125-1126.
27. Subramani J, Kumarapandiyan G. A class of modified ratio estimators using deciles of an auxiliary variable. *International Journal of Statistical Application*. 2012;2:101-107.
28. Subzar M, Raja TA, Maqbool S, Nazir N. New alternative to ratio estimator of population mean. *International Journal of Agricultural and Statistics sciences*. 2016;12:221-225.
29. Subzar M, Maqbool S, Raja TA, Shabeer M. A new ratio estimators for estimation of population mean using conventional location parameters. *World Applied Sciences Journal*. 2017;35(3):377-384.
30. Subzar M, Maqbool S, Raja TA, Sharma P. A new ratio estimator: An alternative to regression estimator in survey sampling using auxiliary information. *Statistics in Transition new series*. 2019;20:181-189.
31. Yan Z, Tian B. Ratio method to the mean estimation using coefficient of skewness of auxiliary variable. *ICICA 2010(II), CCIS 106*. 2010:103-111.

How to cite this article: Behera B, Dash P, Sreelatha C. Exploring auxiliary information for estimating finite population mean by an improved class of estimators. *International Journal of Statistics and Applied Mathematics*. 2026;11(9):172-186. doi: <https://www.doi.org/10.22271/math.2026.v11.i9b.2700>