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A classical polynomial set $J_n\{(x_m), y\}$ of several variables

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ABSTRACT

In this paper, a classical polynomial set $J_n\{(x_m), y\}$ of Several Variables has been defined by means of the product of generalized hypergeometric function and Lauricella function in the notations of Burchnall and Chaundy. The classical polynomial set covers as many as forty orthogonal and non-orthogonal polynomial and have been obtained with four special cases.

Keywords: Generating Function, Lauricella function, Hypergeometric Polynomial, Orthogonal Polynomial, Non-Orthogonal Polynomial

1. Introduction

We define the classical generalized hypergeometric polynomials set $J_n\{(x_m), y\}$ of several variables by means of generating relation

$$\begin{aligned} & (\xi + \lambda y^{-e} t^e)^{-\sigma} F \left[\begin{matrix} \lambda_1; (a_g); \\ (b_h); \end{matrix} \mu_1 y^{-e_1} t^{e_1} \right] \\ & \times F \left[\begin{matrix} (A_r); (C_p); (\alpha_{u_k}) \\ (B_s); (D_q); (\beta_{v_k}) \end{matrix} \mu_1^d t, \mu_2 x_2^{e_2} y^{-e_2} t^{e_2} \dots \mu_k x_k^{e_k} t^{e_k} \right] \\ & \dots \end{aligned} \quad (1.1)$$

where $\xi, \lambda, \lambda_1, \sigma, d; \mu, \mu_1, \mu_2, \dots, \mu_m$ are real and e, e_1 , are non-negative integers and $e_2, \dots, e_m$ are natural numbers.

The left hand side of (1.1) are the product of generalized hypergeometric function and Lauricella function in the notation of (Burchnall and Chaundy) [4]. The polynomial set contain a number of parameters. For simplicity we shall denote

$$\begin{aligned} & J_{n, e, e_1; e_2; e_3; \dots; e_m; (b_k); (B_s); (D_q); (\beta_{v_k})}^{\xi; \lambda; \lambda_1; \sigma; d; \mu; \mu_1; \mu_2 \dots \mu_m; (a_g); (A_r); (C_p); (\alpha_{u_k})} \{(x_m), y\} \\ & = J_n \{(x_m), y\}. \end{aligned}$$

where n denotes the order of the generalized polynomial set.

2. Notations

- I.** (i) $(n) = 1, 2, \dots, n-1, n$.
(ii) $(ap) = a_1, a_2, a_3, \dots, ap$.
(iii) $(ap; i) = a_1, a_2, a_3, \dots, ai-1, ai+1, \dots, ap$.

- II.** (i) $[(ap)] = a_1, a_2, a_3, \dots, ap$.
(ii) $\left[\begin{matrix} (a_p) \\ n \end{matrix} \right] = \prod_{i=1}^p (a_i)_n = (a_1)_n (a_2)_n (a_3)_n \dots (a_p)_n$

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$$\text{III. (i)} \quad \Delta(a; b) = \frac{b}{a}, \frac{b+1}{a} + \dots + \frac{b+a-1}{a}.$$

$$\text{(ii)} \quad \Delta(a; b) = \frac{b}{a}, \frac{b+1}{a}, \frac{b+2}{a}, \dots, \frac{b+a-2}{a}$$

$$\text{(iv)} \quad \Delta(a; b \pm c \pm d) = \Delta(a; b + c + d), \Delta(a; b + c - d), \\ \Delta(a; b - c + d), \Delta(a; b - c - d),$$

$$\text{IV. (i)} \quad \Delta_k[a; b] = \prod_{r=1}^k \left(\frac{b+r-1}{a} \right)_k = \left(\frac{b}{a} \right)_k \left(\frac{b+1}{a} \right)_k \dots \left(\frac{b+a-1}{a} \right)_k$$

$$\text{(ii)} \quad \Delta_k[a; b] = \left(\frac{b}{a} \right)_k \left(\frac{b+1}{a} \right)_k \dots \left(\frac{b+a-2}{a} \right)_k$$

$$\text{(iii)} \quad \Delta_k[m; (a_p)] = \prod_{i=1}^b \prod_{r=1}^m \left(\frac{a_i+r-1}{m} \right)_k$$

$$\text{V. (i)} \quad \Gamma[(a_p)] = \prod_{i=1}^p (a_i)$$

$$\text{(ii)} \quad \Gamma[(a_p); (s)] = \prod_{i=1}^p (a_i)$$

$$\text{(iii)} \quad \Gamma[(a; b)] = \prod_{r=1}^a \Gamma\left(\frac{b+r-1}{a}\right)$$

$$\text{(iv)} \quad \Gamma[\Delta(m); (a_p)] = \prod_{i=1}^p \prod_{r=1}^m \Gamma\left(\frac{(a_i)+r-1}{m}\right)$$

$$\text{VI. (i)} \quad \Gamma(a \pm b) = \Gamma(a+b)\Gamma(a-b)$$

$$\text{(ii)} \quad \Gamma_{**}(a+b) = \Gamma(a+b)\Gamma(a+b)$$

$$M = \frac{[(A_r)]_n [(C_p)]_n (\lambda_1^{e_d})^n}{[(B_s)]_n [(D_q)]_n n!}$$

3. Representation of polynomial set $J_n\{(x_m), y\}$ in series from

Theorem : If the classical polynomial set $J_n\{(x_m), y\}$ is defined by (1.1) then

$$J_n\{(x_m), y\} = \xi^\sigma \sum_{m=0}^{\left\lfloor \frac{n}{e} \right\rfloor} \sum_{m_1=0}^{\left\lfloor \frac{n-em}{e_1} \right\rfloor} \sum_{m_2=0}^{\left\lfloor \frac{n-em-e_1m_1}{e_2} \right\rfloor} \dots \sum_{m_k=0}^{\left\lfloor \frac{n-em-e_1m_1-e_2m_2-\dots-e_{k-1}m_{k-1}}{e_k} \right\rfloor}$$

$$\times \frac{[(A_r)]_{n-em-e_1m_1-(e_2-1)m_2-\dots-(e_k-1)m_k} [(C_p)]_{n-ek-e_1k_1-e_2k_2-\dots-e_km_k}}{[(B_s)]_{n-em-e_1m_1-(e_2-1)m_2-\dots-(e_k-1)m_k} [(D_q)]_{n-ek-e_1k_1-e_2k_2-\dots-e_km_k}}$$

$$\times \frac{[(\alpha_{u_k})]_{m_k} [(a_g)]_{m_1} (\sigma)_m (-\lambda)^m (-\lambda_1)_{m_1} \mu^{m_1} (\mu_2 x_2^{e_2})^{m_2}}{[(\beta_{v_k})]_{m_k} [(b_h)]_{m_1} \xi^m m! m_1! y^{em+e_1m_1+e_2m_2} m_2!}$$

Proof : We obtain from (1.1)

$$\dots \quad (3.1)$$

$$\sum_{n=0}^{\infty} J_n\{(x_m), y\} t^n = \xi^\sigma \sum_{n=0}^{\infty} \sum_{m, m_1, m_2, \dots, m_k=0}^{\infty} \frac{(\sigma)_m (-\lambda)^m t^{me}}{\xi^m y^{em} m!}$$

$$\times \frac{(\lambda_1)_{m_1} \mu_1^{m_1} t^{e_1m_1} [(a_g)]_{m_1} [(A_r)]_{n+m_2+\dots+m_k} [(C_p)]_n}{m_1! y^{e_1m_1} [(b_h)]_{m_1} [(B_s)]_{n+m_2+\dots+m_k} [(D_q)]_n}$$

$$\times \frac{[(\alpha_{u_k})]_{m_k} (\mu_2 x_2^{e_2})^{m_2} t^{e_2m_2} (\mu_k x_k^{e_k})^{m_k} t^{e_km_k} (\mu x_1^d)^n t^n}{[(\beta_{v_k})]_{m_k} m_2! y^{em+e_1m_1+e_2m_2} m_k! n!}$$

$$= \xi^\sigma \sum_{n=0}^{\infty} \sum_{m, m_1, m_2, \dots, m_k=0}^{\infty} \frac{[(A_r)]_{n+m_2+\dots+m_k} [(C_p)]_n [(\alpha_{u_k})]_{m_k}}{[(B_s)]_{n+m_2+\dots+m_k} [(D_q)]_n [(\beta_{v_k})]_{m_k}}$$

$$\times \frac{\left[(a_g) \right]_{m_1} (\sigma)_m (\lambda_1)_{m_1} (-\lambda)^m \mu_1^{m_1} (\mu_2 x_2^{e_2})^{m_2}}{\left[(b_h) \right]_{m_1} m! m_1! \xi^m y^{em+e_1 m_1+e_2 m_2} m_2!}$$

$$\dots \quad (3.2)$$

$$= \sum_{n=0}^{\infty} \sum_{m=0}^{\left[\frac{n}{e} \right]} \sum_{m_1=0}^{\left[\frac{n-em}{e_1} \right]} \sum_{m_2=0}^{\left[\frac{n-em-e_1 m_1}{e_2} \right]} \dots \sum_{m_k=0}^{\left[\frac{n-em-e_1 m_1-e_2 m_2-\dots-e_{k-1} m_{k-1}}{e_k} \right]}$$

$$\times \frac{\left[(A_r) \right]_{n-em-e_1 m_1-(e_2-1)m_2-\dots-(e_k-1)m_k}}{\left[(B_s) \right]_{n-em-e_1 m_1-(e_2-1)m_2-\dots-(e_k-1)m_k}} \quad \dots \quad (3.3)$$

On comparing the co-efficient of m from both sides, we achieve (3.1).

$$\times \frac{\left[(C_p) \right]_{n-ek-e_1 k_1-e_2 k_2-\dots-e_k m_k} \left[(\alpha_{u_k}) \right]_{m_k} \left[(a_g) \right]_{m_1}}{\left[(D_q) \right]_{n-ek-e_1 k_1-e_2 k_2-\dots-e_k m_k} \left[(\beta_{v_k}) \right]_{m_k} \left[(b_h) \right]_{m_1}}$$

Hence, the proof.

$$\times \frac{(\sigma)_m (\lambda_1)_{m_1} (-\lambda)^m \mu_1^{m_1} (\mu_2 x_2^{e_2})^{m_2} (\mu_m x_m^{e_k})^{m_k}}{m! m_1! \xi^m y^{em+e_1 m_1+e_2 m_2} m_2! m_k!}$$

After little simplification (3.3) can be thrown in to the form

$$J_n \{ (x_m), y \} = M \sum_{m, m_1, m_2, \dots, m_k=0}^{\infty} \frac{\left[1 - (B_s) - n \right]_{em+e_1 m_1+(e_2-1)m_2+\dots+(e_k-1)m_k}}{\left[1 - (A_r) - n \right]_{em+e_1 m_1+(e_2-1)m_2+\dots+(e_k-1)m_k}}$$

$$\times \frac{\left[1 - (D_q) - n \right]_{em+e_1 m_1+e_2 m_2+\dots+e_k m_k} \left[(\alpha_{u_k}) \right]_{m_k} \left[(a_g) \right]_{m_1}}{\left[1 - (C_p) - n \right]_{em+e_1 m_1+e_2 m_2+\dots+e_k m_k} \left[(\beta_{v_k}) \right]_{m_k} \left[(b_h) \right]_{m_1}}$$

$$\times \frac{(\sigma)_m (\lambda_1)_{m_1} (-\lambda)^m \mu_1^{m_1} (\mu_2 x_2^{e_2})^{m_2} \dots (\mu_m x_m^{e_k})^{m_k}}{m! m_1! \xi^m y^{em+e_1 m_1+e_2 m_2} m_2! m_k!}$$

...

$$(3.4) \quad \text{The single terminating factor makes all summation in (3.4) runs up to } \infty.$$

$$\text{where } M = \frac{\left[(A_r) \right]_n \left[(C_p) \right]_n (\mu x_1^d)^n}{\left[(B_s) \right]_n \left[(D_q) \right]_n n!}$$

$$\times \frac{(-n)_{em+e_1m_1+e_2m_2+\dots+e_km_k}}{(\mu x_1^d)^{em+e_1m_1+e_2m_2+\dots+e_km_k}}$$

4. Lauricella form of the polynomial set $J_n\{(x_m), y\}$

Various forms of (3.4) have been obtain on specializing the parameters $r, s, e, e_1, e_2, \dots, e_k$ which are given in the form of corollaries.

Corollary I : If $e_2 > 1, \dots, e_k > 1$, we achieve

$$J_n\{(x_m), y\} = M F_{s+q+v+h; \beta_{v_1}, \beta_{v_2}, \dots, \beta_{v_k}}^{r+p+u+g+1; \alpha_{u_1}, \alpha_{u_2}, \dots, \alpha_{u_k}} \left[\begin{matrix} [(-n); e, e_1, e_2, \dots, e_k] \\ \hline \end{matrix} \right];$$

$$\left[(1 - (B_s) - n): e, e_1, e_2 - 1, \dots, e_{k-1} \right], \left[(1 - (D_q) - n): e, e_1, e_2, \dots, e_k \right], \\ \left[(1 - (A_r) - n): e, e_1, e_2 - 1, \dots, e_{k-1} \right], \left[(1 - (C_p) - n): e, e_1, e_2, \dots, e_k \right],$$

$$\left[(\alpha_{u_1}): 1 \right], \left[(\alpha_{u_2}): 1 \right] \dots \left[(\alpha_{u_k}): 1 \right], \left[(a_g): 1 \right] [\sigma: 1], [\lambda_1: 1]; \\ \left[(\beta_{v_1}): 1 \right], \left[(\beta_{v_2}): 1 \right] \dots \left[(\beta_{v_k}): 1 \right], [(b_h): 1]$$

$$\frac{(-\lambda)(-1)^{(r+s+p+q+g+h+u+v+1)e}}{\xi(\mu y x_1^d)^e}, \frac{\mu_1(-1)^{(r+s+p+q+g+h+u+v+1)e_1}}{(\mu x_1^d y)^{e_1}},$$

$$\left[\frac{\mu_2 x_2^{e_2} (-1)^{e_2(r+s+p+q+u+v+g+h+1)+r+s}}{(\mu x_1^d y)^{e_2}}, \dots, \frac{\mu_k x_k^{e_k} (-1)^{e_k(r+s+p+q+g+h+u+v+1)+r+s}}{(\mu x_1^d)^{e_k}} \right]$$

...

(4.1)

Corollary II : For $e_2 = 1$, we obtain

$$J_n\{(x_m), y\} = M F_{s+q+v+h; \beta_{v_1}, \beta_{v_2}, \dots, \beta_{v_k}}^{r+p+u+g+1; \alpha_{u_1}, \alpha_{u_2}, \dots, \alpha_{u_k}} \left[\begin{matrix} [(-n); e, e_1, 1, \dots, e_k] \\ \hline \end{matrix} \right];$$

$$\left[(1 - (B_s) - n): e, e_1, 0, \dots, e_{k-1} \right], \left[(1 - (D_q) - n): e, e_1, 1, \dots, e_k \right], \\ \left[(1 - (A_r) - n): e, e_1, 0, \dots, e_{k-1} \right], \left[(1 - (C_p) - n): e, e_1, 1, \dots, e_k \right],$$

$$\left[(\alpha_{u_1}): 1 \right], \left[(\alpha_{u_2}): 1 \right] \dots \left[(\alpha_{u_k}): 1 \right], \left[(a_g): 1 \right] [\sigma: 1], [\lambda_1: 1]; \\ \left[(\beta_{v_1}): 1 \right], \left[(\beta_{v_2}): 1 \right] \dots \left[(\beta_{v_k}): 1 \right], [(b_h): 1]$$

...

(4.2)

Corollary III : For $e_k = 1$, we achieve

$$\frac{(-\lambda)(-1)^{e(r+s+p+q+g+h+u+v+1)}}{\xi(\mu y x_1^d)^e}, \frac{\mu_1(-1)^{e_1(r+s+p+q+u+v+g+h+1)}}{(\mu x_1^d y)^{e_1}},$$

$$J_n \left\{ (x_m), y \right\} = M F_{s+q+v+h; \beta_{v_1}, \beta_{v_2}, \dots, \beta_{v_k}}^{r+p+u+g+1; \alpha_{u_1}, \alpha_{u_2}, \dots, \alpha_{u_k}} \left[\frac{[(-n); e, e_1, e_2, \dots, 1]}{\dots}; \right.$$

$$\left[(1 - (B_s) - n): e, e_1, e_2 - 1, \dots, 0 \right], \left[(1 - (D_q) - n): e, e_1, e_2, \dots, 1 \right], \\ \left[(1 - (A_r) - n): e, e_1, e_2 - 1, \dots, 0 \right], \left[(1 - (C_p) - n): e, e_1, e_2, \dots, 1 \right],$$

$$\left[(\alpha_{u_1}): 1 \right], \left[(\alpha_{u_2}): 1 \right] \dots \left[(\alpha_{u_k}): 1 \right], \left[(a_g): 1 \right] [\sigma: 1], [\lambda_1: 1]; \\ \left[(\beta_{v_1}): 1 \right], \left[(\beta_{v_2}): 1 \right] \dots \left[(\beta_{v_k}): 1 \right], \left[(b_h): 1 \right]$$

$$\frac{-\lambda(-1)^{e(r+s+p+q+g+h+u+v+1)}}{\xi(\mu y x_1^d)^e}, \frac{\mu_1(-1)^{e_1(r+s+p+q+g+h+u+v+1)}}{(\mu x_1^d y)^{e_1}},$$

$$\left[\frac{\mu_2 x_2^{e_2} (-1)^{e_2(r+s+p+q+u+v+g+h+1)+r+s}}{(\mu x_1^d y)^{e_2}}, \dots, \frac{\mu_k x_k^{e_k} (-1)^{2(r+s)+g+h+p+q+u+v+1}}{(\mu x_1^d)^{e_k}} \right]$$

... Corollary IV : For $e_2 = 1 = \dots = e_k$, we arrive at

(4.3)

$$J_n \left\{ (x_m), y \right\} = M F_{s+q+v+h; \beta_{v_1}, \beta_{v_2}, \dots, \beta_{v_k}}^{r+p+u+g+1; \alpha_{u_1}, \alpha_{u_2}, \dots, \alpha_{u_k}} \left[\frac{[(-n); e, e_1, 1, \dots, 1]}{\dots}; \right.$$

$$\left[(1 - (B_s) - n): e, e_1, 0, \dots, 0 \right], \left[(1 - (D_q) - n): e, e_1, 1, \dots, 1 \right], \\ \left[(1 - (A_r) - n): e, e_1, 0, \dots, 0 \right], \left[(1 - (C_p) - n): e, e_1, 1, \dots, 1 \right],$$

$$\left[(\alpha_{u_1}): 1 \right], \left[(\alpha_{u_2}): 1 \right] \dots \left[(\alpha_{u_k}): 1 \right], [\sigma: 1], [\lambda_1: 1]; \\ \left[(\beta_{v_1}): 1 \right], \left[(\beta_{v_2}): 1 \right] \dots \left[(\beta_{v_k}): 1 \right],$$

$$\frac{-\lambda(-1)^{e(r+s+p+q+g+h+u+v+1)}}{\xi(\mu x_1^d y)^e}, \frac{\mu_1(-1)^{e_1(r+s+p+q+g+h+u+v+1)}}{(\mu x_1^d y)^{e_1}},$$

$$\left[\frac{\mu_2 x_2 (-1)^{2(r+s)+p+q+u+v+g+h+1}}{\mu x_1^d y}, \dots, \frac{\mu_k x_k (-1)^{2(r+s)+p+q+g+h+u+v+1}}{\mu x_1^d} \right]$$

... (4.4) **Special cases**
Case I : On taking $m_1 = 0 = m_2 = \dots = m_k \Rightarrow x_1 = 0 = x_2 = \dots = x_k$ in (4.1), we arrive at

$$J_n(y) = M \sum_{m=0}^{\infty} \frac{(-n)_{em} [1 - (B_s) - n]_{em} [1 - (D_q) - n]_{em}}{[1 - (A_r) - n]_{em} [1 - (C_p) - n]_{em} m!}$$

... (4.5) **Case II :** If we putting $m_2 = \dots = m_k \Rightarrow x_2 = \dots = x_k$ in (4.1), we get

$$J_n(x_1, 0, 0 \dots y) = M \sum_{m, m_1=0}^{\infty} \frac{(-n)_{em+e_1 m_1} [1 - (B_s) - n]_{em+e_1 m_1}}{[1 - (A_r) - n]_{em+e_1 m_1}}$$

... (4.6) **Case III :** On making the substitution $m_1 = \dots = m_k ; x_1 = 1 \Rightarrow x_3 = 0 = x_4 = \dots = x_k$ in

$$\times \frac{[1 - (D_q) - n]_{em+e_1 m_1} [(a_g)]_m [(\alpha_{\mu_k})]_{m_1} (\sigma)_m (\lambda_1)^m}{[1 - (C_p) - n]_{em+e_1 m_1} [(b_h)]_m [(\beta_{v_k})]_{m_1} m! m_1!}$$

(4.1), we achieve

$$J_n(x_2, y) = M \sum_{m, m_2=0}^{\infty} \frac{(-n)_{em+e_2 m_2} [1 - (B_s) - n]_{em+(e_2-1)m_2}}{[1 - (A_r) - n]_{em+(e_2-1)m_2}}$$

$$\times \frac{[1 - (D_q) - n]_{em+e_2 m_2} [(\alpha_{\mu_k})]_{m_2} (\sigma)_m (-\lambda)^m}{[1 - (C_p) - n]_{em+e_2 m_2} [(\beta_{v_k})]_{m_2} m! \xi^m y^{em+e_2 m_2}}$$

... (4.7) **Case IV :** On making the substitution $m_2 = 0 = \dots = m_{k-1} \Rightarrow x_2 = 0 = \dots = x_{k-1}$, we arrive at

$$J_n(x_1 \dots x_k, y) = M \sum_{m, m_k=0}^{\infty} \frac{(-n)_{em+e_k m_k} [1 - (B_s) - n]_{em+(e_k-1)m_k}}{[1 - (A_r) - n]_{em+(e_k-1)m_k}}$$

$$\times \frac{[1 - (D_q) - n]_{em+e_k m_k} [(\alpha_{\mu_k})]_{m_k} (\sigma)_m (-\lambda)^m}{[1 - (C_p) - n]_{em+e_k m_k} [(\beta_{v_k})]_{m_k} m! \xi^m y^{em}}$$

...

(4.8)

5. Particular cases

On specializing the values of the respective parameters involved in Lauricella form a number of known and unknown polynomials can be deduced as a particular cases of the newly defined a classical polynomial set. Some of them, which are most useful and well known are listed below:

(A)

Separating the term corresponding to (4.5) and specializing the remaining parameters, we have:

(i) Sylvester polynomials

If we take $r = 0 = s = p = q = d$; $\sigma = x$; $\xi = m = 1 = \lambda = g = h = e = x1$; $y = x = \mu$ in (4.5), we get

$$J_n(x) = \frac{x^n}{n!} {}_2F_1 \left[\begin{matrix} -n, x; \\ \frac{1}{x} \end{matrix} \right] = \Phi_n(x)$$

where $\Phi_n(x)$ are the Sylvester Polynomials.

(ii) Srivastava's Polynomials [10]

On making the substitution $r = 0 = s = p = q$; $d = 1 = e = \xi = x1 = \lambda = m$; $\sigma = 1 + \lambda 2$; $\mu = y$ in (4.5) we get

$$J_n(y, 1) = \frac{y^n}{n!} {}_2F_0 \left[\begin{matrix} -n; 1 + \lambda_2; \\ \frac{1}{y} \end{matrix} \right] = A_n^{\lambda_2}(y)$$

where $L_n^{(\alpha)}(y)$ are the Laguerre Polynomials.

$$(1 + \alpha)_n J_n \left(\frac{1}{y} \right) = \frac{(1 + \alpha)_n}{n! y^n} F \left[\begin{matrix} -n; \\ 1 + \alpha; \end{matrix} y \right] = \frac{1}{y^n} L_n^{(\alpha)}(y)$$

where $A_n^{\lambda_2}(y)$ are the Srivastava P.N. Polynomials.

(iii) Srivastava's K.N. Polynomials [11]

For $r = 0 = s = q = g = h$; $p = 1 = m = \xi = d = e = \sigma = x1$; $y = \frac{1}{y}$; $C1 = 1 + \lambda$; $\mu = \frac{1}{y}$ then (4.5) becomes

$$J_n \left(\frac{1}{y} \right) = \frac{(-1)^n (1 + \lambda)_n}{n!} F \left[\begin{matrix} -n; \\ -\lambda - n; \end{matrix} -y \right]$$

where $A_n^{\lambda}(y)$ are the Srivastava Polynomials.

$$= y^{-n} A_n^{\lambda}(y)$$

(B)

Separating the term corresponding to (4.6) and specializing the remaining parameters, we have:

(iv) Abdul-Halim and Al-Salam Polynomials [1]

On taking $p = 0 = q = r = s = u = v = m$; $g = 1 = h = e = x1 = d = \mu = e1 = m1 = \lambda = \mu1$; $y = \frac{1}{x}$ $g = p$, $h = q$ in (4.6), we achieve

$$n! J_n \left(\frac{1}{x} \right) = F \left[\begin{matrix} -n; (a_p); \\ (b_q); \end{matrix} x \right] = {}_1F_1 \left[\begin{matrix} -n; \\ b; \end{matrix} x \right]$$

where ${}_1F_1[-n, b; x]$ are the Abdul-Halim and Al-Salam Polynomials.

(v) Laguerre polynomials

On making the substitution $p = 0 = q = r = s = g = uk = vk$; $m = m1 = h = 0 = \lambda1 = -\lambda = \mu1 = y = \mu$; $b1 = 1 + \alpha$, and $x_1 = \frac{1}{y}$, in (4.6) we get

(vi) Tchebicheffe polynomials of first kind

On taking $r = 0 = s = p = g = uk = vk$; $q = h = y = d = e = e1 = v1 = \mu1 = \sigma = \lambda1 = -\lambda = \mu = \frac{1}{2}$; $D_1 = \frac{1}{2} = b_1$; and putting $x_1 = \frac{x-1}{x+1}$ in (4.6), we arrive at

$$\left(\frac{1}{2} \right)_n n! J_n \left(\frac{x-1}{x+1}; 1 \right) = (x+1)^{-n} \left(\frac{x-1}{2} \right)^n F \left[\begin{matrix} -n, \frac{1}{2} - n; \\ \frac{x+1}{x-1} \end{matrix} \frac{x+1}{x-1} \right]$$

$$= (x+1)^{-n} T_n(x)$$

where, $T_n(x)$ are the Tchebicheffe Polynomials of First Kind.

(vii) Tchebicheffe polynomials of second kind

If we take $r = 0 = s = p = g = uk = vk$; $q = 1 = h = d = e = e1 = \xi = \sigma = \lambda1 = -\lambda = y$; $\mu_1 = \frac{1}{2} = \mu = D_1 = \frac{3}{2} = b_1$; and writing $\frac{x-1}{x+1}$, for x in (4.6), we achieve

$$\left(\frac{3}{2} \right)_n (n+1)! J_n \left(\frac{x-1}{x+1}, 1 \right) = \frac{(x+1)^n (n+1)!}{n!} \left(\frac{x-1}{2} \right)^n$$

where $U_n(x)$ are the Tchebicheffe Polynomials of Second Kind.

$$\times {}_2F_1 \left[\begin{matrix} -n, -\frac{1}{2} - n; \\ \frac{x+1}{x-1} \end{matrix} \frac{3}{2} \right] = (x-1)^n U_n(x)$$

(viii) Simple Laguerre polynomials

If we take $r = 0 = s = p = q = g = \alpha_{u_k} = \beta_{v_k}$; $\sigma = 1 + \beta$; $h = 1 = uk = \lambda 1 = \frac{1}{1}$
 $-\lambda = y = e = e1 = \xi = \mu 1$; $\mu = -1$; $b1 = 1$, $\beta 1 = 1 + \alpha$ and writing $\mathcal{X}$
 for $x1$ in (4.6), we get

$$J_n\left(\frac{1}{x}, \dots, 1\right) = \frac{x^{-n}}{n!} {}_2F_2\left[\begin{matrix} -n, 1+\beta; \\ 1, 1+\alpha; \end{matrix} x\right]$$

$$J_n\left(\frac{1}{x}, \dots, 1\right) = \frac{x^{-n}}{n!} {}_1F_2\left[\begin{matrix} -n; \\ 1+\alpha, 1+\beta; \end{matrix} x\right] = \frac{x^{-n}}{n!} f_n(x)$$

(x) Pseudo-Laguerre Polynomials [09; P.45(12)]

On taking $r = 0 = s = q = h = \alpha_{u_k} = \beta_{v_k}$; $\mu = 1 = \mu 1$; $p = 1 = \xi = \sigma = \frac{1}{1}$
 $-\lambda = \lambda = e = e1 = y = d$; $c1 = -\lambda$; $\mu 1 = -1$, we get when $\mathcal{X}$ for $x1$ in
 (4.6)

$$J_n\left(\frac{1}{x}, \dots, 1\right) = \frac{x^{-n} (-\lambda)_n}{n!} {}_1F_1\left[\begin{matrix} -n; \\ 1+\lambda-n; \end{matrix} x\right]$$

where $f_n(x)$ are the Pseudo-Laguerre Polynomials.

$$= x^{-n} f_n(x)$$

(C)

Separating the term corresponding to (4.7) and specializing the remaining parameters we get :

(xi) Gupta and Jain Polynomials [6]

On putting $r = 0 = s = p = q = uk = vk$; $e2 = m = x2$; $\sigma = 1 = \xi = \lambda = e = y$; $\mu 2 = \beta$; $\mu = \alpha$ in (4.7), we get

$$J_n(1, 1) = \frac{\alpha^n}{n!} F\left[\begin{matrix} \Delta(m; -n); \\ \beta\left(-\frac{m}{\alpha}\right)^m; \\ -; \end{matrix}\right] = H_{n,m}(\alpha, \beta)$$

where $H_{n,m}(\alpha, \beta)$ are the Gupta and Jain polynomial.

$$J_n(x, x) = \frac{\left(\frac{1}{2}\right)_n (3x)^n}{n!} F\left[\begin{matrix} \Delta(3; -n); \\ \frac{1}{4x^2}; \\ \Delta\left(2; \frac{1}{2} - n\right); \end{matrix}\right] = g_n^*(x)$$

where $g_n^*(x)$ are the Pinchavle polynomials.

where $\phi n(x)$ are [156; 235 (12)] reduces to Simple Laguerre Polynomials.

$$= \frac{x^{-n}}{n} \phi_n(x)$$

(ix) The Generalized Polynomials [09; P.243(4)]

For $r = 0 = s = p = q = g = \alpha_{u_k} = \beta_{v_k}$; $h = 1 = uk = e = e1 = y = d = \xi = \frac{1}{1}$
 $-\lambda = \sigma$; $b1 = 1 + \alpha$, $\beta 1 = 1 + \beta$ and $\mathcal{X}$ for $x1$ in (4.6), we achieve

(xii) Brafman Polynomials [2]

On taking $r = 0 = s = p = q$; $e2 = p$; $e = 1 = \sigma = \lambda = \xi = x$; $\mu 2 = pp$; $\mu = -p$ and $uk = u1$, $vk = v$; $y = \frac{1}{x}$, in (4.7) we get

$$J_n(1, x) = \frac{(-p)^n}{n!} F\left[\begin{matrix} \Delta(p; -n), (\alpha_u); \\ (\beta_v); \end{matrix} x\right] = \frac{(-p)^n B_n^p(x)}{n!}$$

where $B_n^p(x)$ are the generalization Hermite polynomials by (Brafman).

(D)

Separating the term corresponding to (4.8) and specializing the remaining parameters we get :

(xiii) Humbert polynomials

For $p = 0 = q = s = uk = vk$; $r = 1 = \sigma = \xi = \lambda = y = d = e = xk = b1 = v$, $ek = 3 = \mu$; $\mu_3 = \frac{3}{4}$ and writing x for $x1$ in (4.8), we have

$$J_n(x, 1) = \frac{(v)_n (3x)^n}{n!} F\left[\begin{matrix} \Delta(3; -x); \\ \frac{1}{4x^3}; \\ \Delta(2; 1 - v - n); \end{matrix}\right] = h_n(x)$$

where $h_n(x)$ are the Humbert Polynomials.

(xvi) Pinchavle Polynomials [05; Vol. III. P. 246(7)]

On putting $A_1 = \frac{1}{2}$ in (4.8), we obtain;

(xv) Rekha Panda Polynomials [8]

On putting $r = 0 = s = p = q$; $ek = m = xk = e = 1 = d = \sigma = \lambda = \xi$; $\mu = v$, $\mu k = \square$ and for $\alpha_{u_k} = \alpha_r$; $\beta_{v_k} = b_s$; $x1 = x$, in (4.8) we get

$$J_n(x, 1) = \frac{(vx)^n}{n!} F\left[\begin{matrix} \Delta(m; -n), (\alpha_r); \\ \mu\left(-\frac{m}{vx}\right)^m; \\ (bs); \end{matrix}\right]$$

$$= A_n(x)$$

where $A_n(x)$ are the Panda, R polynomials.

(xvi) Bragg Polynomials [3]

On making the substitution $r = 0 = s = p = q = uk = vk$; $ek = p = xk = \mu$ in (4.8), we get

$$J_n(y;1) = \frac{(py)^n}{n!} {}_pF_0 \left[\begin{matrix} \Delta(p; -n); \\ -; \end{matrix} \middle| -\left(-\frac{1}{y}\right)^p \right]$$

$$= \frac{1}{n!} g_n^p(y)$$

where $g_n^p(y)$ are the Bragg Polynomials.

(xvii) Lahiri Polynomials [7]

On taking $r = 0 = s = p = q = uk = vk$; $e = 1 = d = \xi = \sigma = \lambda = y = xk$; $\mu k = m = ek = m$; $\mu = v$ and writing x for $x1$ in (4.8) we get

$$J_n(x,1) = \frac{(vx)^n}{n!} {}_mF_0 \left[\begin{matrix} \Delta(m; -n); \\ -; \end{matrix} \middle| -\left(-\frac{m}{vx}\right)^m \right]$$

$$= \frac{1}{n!} H_{n,m,v}(x)$$

where $H_{n,m,v}(x)$ are the Maya Lahiri polynomials.

(xviii) Brafman Polynomials [2]

On putting $r = 0 = s = p = q$; $ek = p$, $e = 1 = d = \sigma = \xi = \lambda = xk = x1$; $\mu = -p$; $\mu = -p = \mu k$; $y = \frac{1}{x}$, in (4.8) we get

$$J_n\left(1, \frac{1}{x}\right) = \frac{(-p)^n}{n!} F \left[\begin{matrix} \Delta(p; -n), (\alpha_u); \\ (\beta_v); \end{matrix} \middle| x \right]$$

$$= \frac{(-p)^n}{n!} H_n^p(x)$$

where $H_n^p(x)$ are the generalization of the Hermite polynomials by Brafman.

Conclusion and future scope

In this article we have obtained many interesting new results for the classical polynomial set $J_n[(xm), y]$ followed by important and interesting particular cases. These are at most important for Scientist, Engineers and physical Sciences, because these occur in the solution of integral equation and analytic function which describe physical problems

References

1. Abdul Halim N, Al-Salam WA. A characterization of Laguerre polynomials. Rend, Sem, Univ., padova. 1964;34:176-179.
2. Brafman F. Some generating functions for Laguerre and Hermite polynomials. Conad, J. Math. 1957;9:180-187.
3. Bragg LR. Products of certain generalized Hermite polynomials; Associated relations. Boll.Un.Mat, Ital. Ser. (4) I. 1968;347-355.
4. Burchnell JL, Chaundy TW. Expansions of Appell's double hypergeometric functions (ii). Quart. J. Math. Oxford ser. 1941;12:112-128.
5. Erdélyi A, et al. Higher transcendental Functions, vol. I, II & III, Bateman Manuscript Project, California Institute of Technology. 1953.
6. Gupta RP, Jain GC. A generalized Hermite distribution and its properties. SIAM J. Appl Math. 1974;27:359-363.
7. Lahiri M. Generalization of hermite Polynomials. proc. Amer. Math. Soc.(l). 1971;27:117-121.
8. Panda R. Thesis on, A generalized polynomial $An(x)$. B.H.U. (U.P.). 1967.
9. Rainville ED. Special functions. New York: Mac Millan Co.; 1960.
10. Srivastava PN. Classical polynomials- A unified presentation. Pub. Inst. Math. (Beograd) (N.S.) tome. 1978;23(37):169-177.
11. Srivastava KN. Some polynomials related to the Laguerre polynomial jour. Indian math. Soc. 1964;28(2):43-59.

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