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Selection of chain sampling plan indexed through Six Sigma quality level -1 using truncated binomial distribution

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ABSTRACT

This paper proposes a chain sampling plan of type ChSP-1 using the truncated binomial distribution (ChSP-1 TB) and indexed through Six Sigma Quality Level -1 (SSQL-1). The plan utilises the quality history of preceding lots to relax acceptance criteria when recent performance has been excellent. The truncated binomial distribution is employed to realistically model high-capability processes in which zero-defect outcomes are extremely rare. SSQL-1 corresponds to a very low defect proportion consistent with Six Sigma standards (approximately 3.4 defects per million opportunities). The acceptance probability formula is derived, and a comprehensive table of plan parameters are presented. Two detailed examples with operating characteristic (OC) curves are provided, along with practical industrial applications. The proposed plan demonstrates strong discrimination and practical efficiency for manufacturing environments.

Keywords: Chain Sampling Plan-1, Truncated Binomial distribution, Six Sigma Quality Level-1, Operating Characteristic Curve, Acceptance Sampling

Introduction

Acceptance sampling is a fundamental technique in statistical quality control that helps manufacturers decide whether to accept or reject a lot based on the inspection of a limited sample. Traditional single sampling plans often become inefficient when products are manufactured under very high quality standards, especially at Six Sigma levels. To overcome these limitations, researchers have developed various advanced sampling schemes using truncated distributions, chain sampling, group sampling, and quality-level indexing methods. Aslam, Jun, and Ahmad [1] proposed a group acceptance sampling plan for truncated life tests based on the Weibull distribution. (Aslam and Azam) [2] later developed a group chain sampling plan using truncated life tests for the Burr distribution. Aslam, Rao, and Sherwani [3] designed multiple dependent state sampling plans under truncated distributions. (Costa) [4] improved chain sampling plans based on the Cpm index. Hafeez, Aziz, and Shabbir [5] estimated Acceptable Quality Level (AQL), Limiting Quality Level (LQL), and quality regions for group chain sampling plans under the binomial distribution. (Hafeez et al.) [6] proposed a Bayesian group chain sampling plan for the Poisson distribution with a gamma prior. (Montgomery) [7] provided a comprehensive foundation for statistical quality control techniques, including acceptance sampling procedures. (Pratheeba) [8] designed a double sampling plan indexed through Six Sigma Quality Level – 1 using the truncated binomial distribution. (Pratheeba and Radhakrishnan) [9] earlier constructed a single sampling plan indexed through Six Sigma Quality Level - 1 using the same truncated binomial framework. (Radhakrishnan and Pratheeba) [10] developed double sampling plans indexed through average quality level using truncated binomial distribution. In the same year, (Radhakrishnan and Pratheeba) [11] constructed single sampling plans indexed through indifference quality level using truncated binomial distribution. (Radhakrishnan and Pratheeba) [12] further proposed link sampling plans indexed through indifference quality level under the truncated binomial model. Radhakrishnan and Sampath Kumar [13] constructed mixed sampling plans indexed through Maximum Allowable Percent Defective (MAPD) and Acceptable Quality Level (AQL) with chain sampling as the attribute plan. Radhakrishnan and Sampath Kumar [14] also compared mixed sampling plans having ChSP-(0,1) as the attribute component. (Alkarni) [15] introduced a class of truncated binomial lifetime distributions. (Schilling and Neubauer) [16] presented a detailed treatment of acceptance sampling methods in quality control. Teh, Aziz, and Zain [17] developed time-truncated group chain sampling plans for the Rayleigh distribution. Tharani and Sudamani Ramaswamy [18] designed chain sampling plans based on truncated life tests for the log-logistic distribution using the minimum angle method. Tripathi, Saha, and Dey [19] proposed a new approach to time-truncated chain sampling inspection plans. (Tsai and Wu) [20] developed Bayesian acceptance sampling plans based on truncated life tests.

Collectively, these studies show the continuous development of truncated distributions, chain sampling, group sampling, and Six Sigma quality level indexing in modern quality control. Building on this foundation, the present paper constructs a chain sampling plan of type ChSP-1 using the truncated binomial distribution and indexed through Six Sigma Quality Level -1 (SSQL-1), with the aim of providing strong consumer protection and practical applicability in high-reliability manufacturing industries.

2. Methodology

The construction of the parameters of the ChSP-1 model was carried out through modification in terms of the sample size (n), acceptance number (c), and the number of previous lots (i). The SSQL-1 was calculated using the distribution in order to find the expected number of defects, except for the defects without zero defects. Tables of data were collected on samples presented in units. The OC curves were

$$L(p) = \sum_{x=1}^i P_T(X=x) + \left(\sum_{x=1}^i P_T(X=x) \right)^k \sum_{x=i+1}^c P_T(X=x)$$

$$P(X=x) = \frac{\binom{n}{x} p^x (1-p)^{n-x}}{1 - (1-p)^n}, \quad x = 1, 2, \dots, n$$

Where, n = sample size; p = probability of a defect; x = number of defects in the sample; i = maximum number of defectives in total (current mean); c = acceptance number; k = number of previous lots to chain in memory; The denominator $1 - (1-p)^n$ adjusts the probability by excluding the zero-defect case. This adjustment is appropriate for high-capability processes where the probability of zero defects is negligible. This formulation is appropriate for Six Sigma processes where the probability of zero defects is negligible.

4. Chain sampling plan structure

The Truncated Binomial Distribution based Chain Sampling Plan (ChSP-1) utilizes the inspection results of the current lot in conjunction with information from i immediately preceding lots. The decision rule is given by:

- Accept the current lot if the observed number of defectives $\leq c$.
- If any of the i preceding lots were rejected, the current lot shall be rejected.
- If the number of defectives $> c$, examine the preceding i lots.
- If all preceding i lots had defectives $\leq c$, accept the current lot.
- Otherwise, reject the current lot.

This dependency forms a cumulative inspection system that provides a better quality control by connecting the acceptance decisions on successive lots and also uses the truncated binomial distribution for a more efficient estimation of the probability of acceptance.

5. Glossary of symbols

- n = sample size
- c = acceptance number
- i = number of immediately preceding lots considered
- p = Proportion Defective / Lot Quality
- $q = 1 - p$
- β = Consumer's Risk

created for the sample size in order to show the lot acceptance probability $P_{a(p)}$ at various defect levels p . The practical applications of the sampling plan have been considered in industry applications including pharmaceutical quality inspection and automobile defect inspection.

The proposed statistical model is based on the Chain Sampling Plan of Type-1 (ChSP-1) with Truncated Binomial distribution. In contrast to the binomial, the distribution does not allow for zero-defect probability as the latter one seems to be unrealistic for industrial production.

3. Truncated binomial distribution

The Truncated Binomial Distribution is used to model the number of defectives x in a sample of size n when zero defectives are excluded: The probability Mass Function (pmf) of the distribution is given as

- $P_a(p)$ = Probability of Acceptance of the Lot Quality p .

6. Conditions for application

- The production process is in a state of statistical control.
- Lot size is large relative to the sample size.
- Accurate records of the preceding i lots are maintained.
- Inspection cost is high or testing is destructive.
- Defects occur independently and are randomly distributed.

7. Operating characteristic (OC) curve

The probability of acceptance, $P_{a(p)}$, under ChSP-1 is given as,

$$P_a(p) = \sum_{x=1}^c P(X=x)$$

(1)

Where $P(X=x)$ follows the distribution. For example, probabilities are computed at varying p (defect levels) to construct the OC curve.

8. Example 1: $n = 10000$, $c = 5$, $i = 4$ using SSQL-1 ≈ 0.000028

By fixing of the probability of acceptance of the lot, $P_{a(p)}$ as value with truncated binomial distribution as the base distribution and from equation (1), the values of the SSQL-1 are obtained for the various combination of n , c and i using an excel package and are presented in Table 1. The parameters of chain sampling plan-1, n , c , and i are recorded for various combination of SSQL-1. For a given SSQL-1 = 0.000028, the values of n , c and i are obtained from Table 1 as $n = 10000$, $c = 5$ and $i = 4$ which are associated in parameters of chain sampling plan with the specified SSQL-1 = 0.000028. The OC curve for this plan is presented in Figure 1.

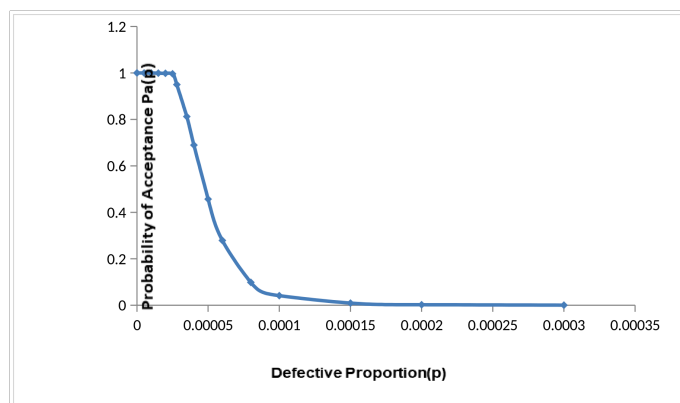


Figure 1. OC Curve for ChSP-1 ($n = 10000$, $c = 5$, $i = 4$)

Practical application

A semiconductor company that makes microchips receives a large lot of chips. The quality team takes a sample of 10,000 chips and counts the number of defectives. This is called d . If d is less than or equal to 5 ($d \leq 5$), they look at the previous 4 lots. So if all those 4 lots were taken then this lot is also accepted. But if $d > 5$, or if any of the last 4 lots was rejected, the whole lot is rejected and returned to the supplier. This allows the company to only accept good quality chips and prevent problems in mobile phones and computers.

9. Example 2: $n = 50000$, $c = 6$, $i = 5$ using SSQL-1 ≈ 0.0000056

By fixing of the probability of acceptance of the lot, $P_{a(p)}$ as value with truncated binomial distribution as the base distribution and from equation (1), the values of the SSQL-1 are obtained for the various combination of n , c and i using an excel package and are presented in Table 1. The parameters of chain sampling plan-1, n , c , and i are recorded for various combination of SSQL-1. For a given SSQL-1 = 0.0000056, the values of n , c and i are obtained from Table 1 as $n = 50000$, $c = 6$ and $i = 5$ which are associated in parameters of chain sampling plan with the specified SSQL-1 = 0.0000056. The OC curve for this plan is presented in Figure 2.

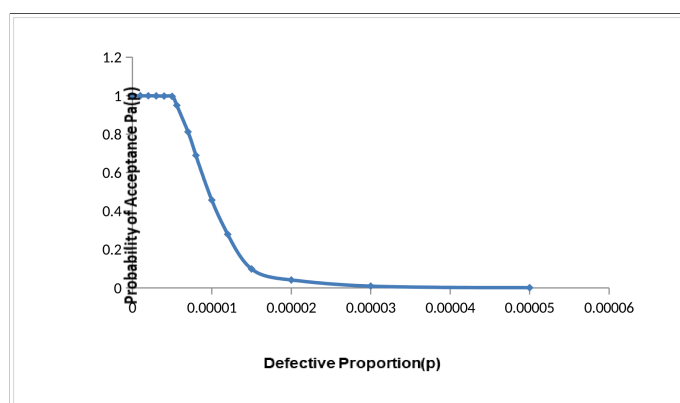


Figure 2. OC Curve for ChSP-1 ($n = 50000$, $c = 6$, $i = 5$)

Practical application

Inspectors at an aerospace plant that makes turbine blades pull a sample of 50,000 blades from a large lot of incoming blades. They count the number of defective blades, d . If $d \leq 6$ they look at the last 5 lots. Present lot is accepted if all five earlier lots accepted with good quality. However, to protect aircraft safety, the whole lot is rejected if $d > 6$ ($d > 6$) or if any of the last 5 lots are rejected. This simple chain rule helps the company to accept only reliable blades and reject risky ones.

Table 1. Parameters of ChSP-1 for a specified SSQL-1

Sample size (n)	c	i	SSQL-1
1000	2	1	0.000680
1000	3	2	0.000580
1000	4	3	0.000390
1000	5	4	0.000270
2000	3	2	0.000290
2000	4	3	0.000195
3000	4	3	0.000130
4000	4	3	0.000098
5000	4	3	0.000078
5000	5	4	0.000055
7500	5	4	0.000037
10000	5	4	0.000028
15000	5	4	0.0000185
20000	6	5	0.000014
25000	6	5	0.0000112
30000	6	5	0.0000093
40000	6	5	0.0000070
50000	6	5	0.0000056
60000	7	6	0.0000047
70000	7	6	0.0000040
80000	7	6	0.0000035
90000	7	6	0.0000031
100000	7	6	0.0000028
100000	8	7	0.0000021

10. Conclusion

The chain sampling plan using truncated binomial distribution and indexed through Six Sigma Quality Level-1 is very useful for companies and industries that deal with second quality lots, especially in high-reliability sectors like semiconductors, electronics, aerospace, and medical devices. By looking at the quality history of previous lots, the plan allows good suppliers to get their lots accepted more easily while quickly rejecting poor quality material, which helps companies save inspection time and cost. At the same time, consumers benefit because only lots with extremely low defect rates are allowed to pass, reducing the chance of receiving faulty products and improving overall product safety and reliability.

References

1. Aslam M, Jun CH, Ahmad M. A group acceptance sampling plan for a truncated life test based on the Weibull distribution. *Journal of Statistical Computation and Simulation*. 2010;80(1):67–76.
2. Aslam M, Azam M. Group chain sampling plan using truncated life tests for the Burr distribution. *Quality and Reliability Engineering International*. 2015;31(4):607–614.
3. Aslam M, Rao GS, Sherwani RA. Design of multiple dependent state sampling plans under truncated distributions. *Quality Engineering*. 2019;31(2):184–195.
4. Costa AFB. Improving the chain sampling plans based on Cpm index. *Quality and Reliability Engineering International*. 2025.
5. Hafeez W, Aziz N, Shabbir J. Estimation of AQL, LQL and quality regions for group chain sampling plan with binomial distribution. *Thailand Statistician*. 2024;22(3):674–687.
6. Hafeez W, Aziz N, Zain Z, Kamarudin NA. Bayesian group chain sampling plan for Poisson distribution with gamma prior. *Computers, Materials & Continua*. 2022;70(2).
7. Montgomery DC. *Introduction to Statistical Quality Control* (8th ed.). John Wiley and Sons; 2019.

8. Pratheeba S. Designing of double sampling plan indexed through Six Sigma Quality Level – 1 using truncated binomial distribution. *Educational Administration: Theory and Practice*. 2024;30(5):2889–2893.
9. Pratheeba S, Radhakrishnan R. Construction of single sampling plan indexed through Six Sigma Quality Level - 1 using truncated binomial distribution. *International Journal of Emerging Trends and Technology in Computer Science*. 2014;3(5):141–142.
10. Radhakrishnan R, Pratheeba S. Construction of double sampling plan indexed through average quality level using truncated binomial distribution. *International Journal of Computational and Mathematical Sciences*. 2011;4(1):63–68.
11. Radhakrishnan R, Pratheeba S. Construction of single sampling plan indexed through indifference quality level using truncated binomial distribution. *Research Journal of Applied Sciences, Engineering and Technology*. 2011;3(5):456–460.
12. Radhakrishnan R, Pratheeba S. Construction of link sampling plan indexed through indifference quality level using truncated binomial distribution. *Global Journal of Engineering & Applied Sciences*. 2012;2(2).
13. Radhakrishnan R, Sampath Kumar R. Construction of mixed sampling plans indexed through MAPD and AQL with chain sampling plan as attribute plan. *STARS International Journal*. 2006;7(1):14–22.
14. Radhakrishnan R, Sampath Kumar R. Construction and comparison of mixed sampling plans having ChSP-(0,1) plan as attribute plan. *International Journal of Statistics and Management Systems*. 2009;2(1-2):88–97.
15. Said hofan Alkarni. A class of truncated binomial lifetime distribution. *open Journal of Statistics*. 2013;3(5):37644.
16. Schilling EG, Neubauer DV. *Acceptance Sampling in Quality Control* (3rd ed.). CRC Press; 2017.
17. Teh MAP, Aziz N, Zain Z. Time truncated group chain sampling plans for Rayleigh distribution. *Global Journal of Pure and Applied Mathematics*. 2016;12(4):3693–3699.
18. Tharani K, Sudamani Ramaswamy AR. Designing chain sampling plan based on truncated life test for log-logistic distribution using minimum angle method. *International Journal of Mathematics Trends and Technology*. 2019;65(7):1–7.
19. Tripathi H, Saha M, Dey S. A new approach of time truncated chain sampling inspection plan and its applications. *International Journal of System Assurance Engineering and Management*. 2022;13:2307–2326.
20. Tsai TR, Wu SC. Bayesian acceptance sampling plans based on truncated life tests. *European Journal of Operational Research*. 2006;175(1):337–348.

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