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Designing and construction of chain sampling plan indexed through indifference quality level using truncated binomial distribution

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Abstract

This paper proposes a chain sampling plan of type Chain Sampling Plan-1 (ChSP-1) using the Truncated Binomial Distribution (TBD) and Indexed through the Indifference Quality Level (IQL). The plan leverages historical quality information from preceding lots to relax acceptance criteria when recent performance has been strong. The truncated binomial distribution is employed to better represent production processes where zero-defect outcomes are improbable due to screening or high capability. The Indifference Quality Level (IQL) serves as the quality point where acceptance probability is approximately 0.50, offering a balanced evaluation of producer and consumer risks. The proposed plan demonstrates improved transition behaviour and discrimination compared to conventional binomial-based approaches, making it suitable for industries requiring moderate defect tolerance with reliable history-based decision-making.

Keywords: Chain sampling plan-1, truncated binomial distribution, indifference quality level, operating characteristic curve and acceptance sampling

1. Introduction

Acceptance sampling remains a crucial component of statistical quality control, helping manufacturers to determine whether to accept or reject lots by inspection of few samples. Traditional single sampling and double sampling plans are no longer enough for modern high-reliability operations which work at very low levels of defects. Therefore, more attention is being paid to truncated distributions and chain-based sampling methods which can provide a good risk balance.

In their research, Al-Omari and Al-Nasser (2021) ^[1] created acceptance sampling plans based on truncated life tests under power Lomax distribution and showed the potential of using truncated distributions in quality assurance in high-reliability systems. In their study, Hafeez *et al.* (2024) ^[2] calculated the values of the Acceptable Quality Level (AQL), Limiting Quality Level (LQL) and quality regions for group chain sampling plans under the binomial distribution. As continuation of the previous study, Hafeez, Aziz, and Shabbir (2024) ^[3] proposed the calculation of AQL, LQL and quality regions in group chain sampling plans. As the next step in development of the field, Hafeez *et al.* (2025) ^[4] created a generalized Bayesian double group sampling plan which is especially aimed to be applied in manufacturing industries.

Pratheeba and Radhakrishnan (2015) ^[5] proposed single sampling plans, indexed through the Limiting Quality Level, under the truncated binomial distribution and focused on increased protection of consumers' interests at high level of defects. Previously, Radhakrishnan and Pratheeba (2011) ^[6] introduced double sampling plans, indexed through the Average Quality Level, under the truncated binomial distribution, forming the basis for the development of this field. In the same year, Radhakrishnan and Pratheeba (2011) ^[7] created single sampling plans, indexed through the Indifference Quality Level, under the truncated binomial distribution and stressed improved modelling of zero-defective process. In (2016) ^[8], Teh *et al.* Proposed time-

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truncated group chain sampling plans based on the Rayleigh distribution, demonstrating benefits of using truncation.

The current research aims to develop a Chain Sampling Plan of Type -1 (ChSP-1) under the truncated binomial distribution and indexed through the Indifference Quality Level (IQL), where the probability of acceptance equals 0.50 approximately. This approach allows providing a balance between producer and consumer risks using historical lot performances and achieving the optimal trade-off between the efforts required to conduct inspections and discrimination of the plans. The paper calculates the acceptance probability function and produces the parameter tables and gives examples of application of ChSP-1.

2. Methodology

The study constructed parameters of the ChSP-1 model by varying sample size (n), acceptance number (c), and number of preceding lots (i). IQL values were derived using the distribution to capture the expected defect levels without zero-defect occurrences. Data were tabulated for sample sizes ranging from 320 to 1000 units. The Operating Characteristic (OC) curve was generated for sample size to illustrate the probability of lot acceptance $P_a(p)$ at different defect levels p . Industrial scenarios, such as pharmaceutical quality testing and automotive defect inspections, were mapped to the proposed sampling plan to validate real-world applicability. The proposed statistical model is based on the Chain Sampling Plan of Type-1 (ChSP-1) integrated with the Truncated Binomial distribution. Unlike the conventional binomial, the distribution eliminates the zero-defect probability, which is unrealistic in industrial production where defects are rare but not absent.

3. Truncated Binomial Distribution

The Truncated Binomial Distribution is used to model the number of defectives x in a sample of size n when zero defectives are excluded: The probability Mass Function (pmf) of the distribution is given as

$$L(p) = \sum_{x=1}^i P_T(X=x) + \left(\sum_{x=1}^i P_T(X=x) \right)^k \sum_{x=i+1}^c P_T(X=x)$$

$$P(X=x) = \frac{\binom{n}{x} p^x (1-p)^{n-x}}{1 - (1-p)^n}, \quad x = 1, 2, \dots, n$$

Where, n = sample size; p = probability of a defect; x = number of defects in the sample; i = maximum number of defectives in total (current mean); c = acceptance number; k = number of previous lots to chain in memory; The denominator $1 - (1-p)^n$ adjusts the probability by excluding the zero-defect case. This adjustment is appropriate for high-capability processes where the probability of zero defects is negligible.

4. Chain Sampling Plan Structure

The Truncated Binomial Distribution based Chain Sampling Plan (ChSP-1) utilizes the inspection results of the current lot in conjunction with information from i immediately preceding lots. The decision procedure is defined as follows

- Accept the current lot if the observed number of defectives $\leq c$.
- If not, examine the records of the i preceding lots. If all of these lots were accepted under the same acceptance

criterion (defectives $\leq c$), then the current lot shall also be accepted.

- If any of the i preceding lots were rejected, the current lot shall be rejected.

This dependency creates a cumulative inspection system, providing enhanced quality control by linking acceptance decisions across successive lots while utilizing the truncated binomial distribution to estimate the probability of acceptance more efficiently.

5. Glossary of Symbols

- n = sample size
- c = acceptance number
- i = number of immediately preceding lots considered
- p = Proportion Defective / Lot Quality
- $q = 1 - p$
- β = Consumer's Risk
- $P_a(p)$ = Probability of Acceptance of the Lot Quality p .

6. Conditions for Application

- The production process must be in a state of statistical control, ensuring that the probability of defect occurrence remains constant over time.
- The lot size should be large, and the sampling fraction relatively small, justifying the use of the truncated binomial model as an approximation.
- Reliable inspection and record-keeping of the preceding i lots is essential to maintain the chain structure.
- The plan is suitable where inspection costs are high or destructive testing is involved, necessitating reduced sample sizes without compromising decision reliability.
- The method assumes independent defect occurrences and random distribution of defects within lots.

It should not be applied when the production process is unstable, lots are non-homogeneous, or when preceding lot information is incomplete or unavailable.

7. Construction of ChSP-1 TB Indexed Through IQL

- Define IQL as the p where $P_a(p) \approx 0.50$.
- Select chain length i and acceptance number c .
- Determine n such that $P_a(IQL) \approx 0.50$.
- Use iterative calculation for parameter generation.
- Tabulate combinations for practical selection.

8. Operating Characteristic (OC) Curve

The probability of acceptance, $P_a(p)$, under ChSP-1 is given as,

$$P_a(p) = \sum_{x=1}^c P(X=x)$$

Where $P(X=x)$ follows the distribution. For example, probabilities are computed at varying p (defect levels) to construct the OC curve.

9. Example 1: $n = 600$, $c = 4$, $i = 3$ (IQL ≈ 0.0838)

By fixing of the probability of acceptance of the lot, $P_a(p)$ as value with truncated binomial distribution as the base distribution and from equation (1), the values of the IQL are obtained for the various combination of n , c and i using an

excel package and are presented in Table 1. The parameters of chain sampling plan-1, n , c , and i are recorded for various combination of IQL. For a given IQL = 0.0838, the values of n , c and i are obtained from Table 1 as $n = 600$, $c = 4$ and $i = 3$

which are associated in parameters of chain sampling plan with the specified IQL = 0.0838. The OC curve for this plan is presented in Figure 1.

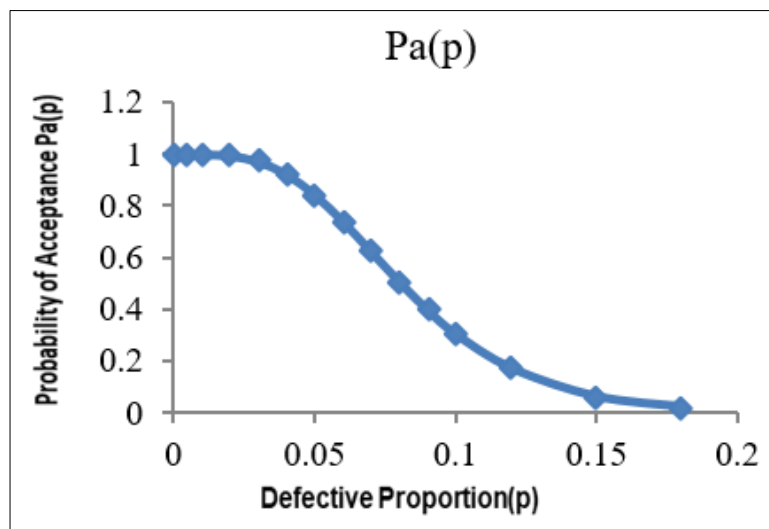


Fig 1: OC Curve for ChSP-1($n=600$; $c=4$; $i=3$)

9.1 Practical Application

This plan is highly suitable for automotive safety electronics such as ABS sensors, airbag controllers, or ADAS camera modules supplied to companies like Bosch, Continental, or Tata Motors. Even moderate defects can lead to critical vehicle malfunctions. Sample 600 units and perform functional + visual testing. Accept up to 4 defectives only if the last 3 lots were perfect. At IQL ≈ 0.0838 (8.38% defective), acceptance probability is balanced at 50%, providing fair evaluation of suppliers while ensuring strong protection against safety risks. The S-shaped OC curve ensures smooth transition and quick rejection when quality deteriorates.

10. Example 2: $n=1000$, $c=5$, $i=4$ (IQL ≈ 0.0149)

By fixing the probability of acceptance of the lot, $Pa(p)$ as value with truncated binomial distribution as the base distribution and from equation (1), the values of the IQL are obtained for the various combination of n , c and i using an excel package and are presented in Table 1. The parameters of chain sampling plan-1, n , c , and i are recorded for various combination of IQL. For a given IQL = 0.0149, the values of n , c and i are obtained from Table 1 as $n = 1000$, $c = 5$ and $i = 4$ which are associated in parameters of chain sampling plan with the specified IQL = 0.0149. The OC curve for this plan is presented in Figure 2.

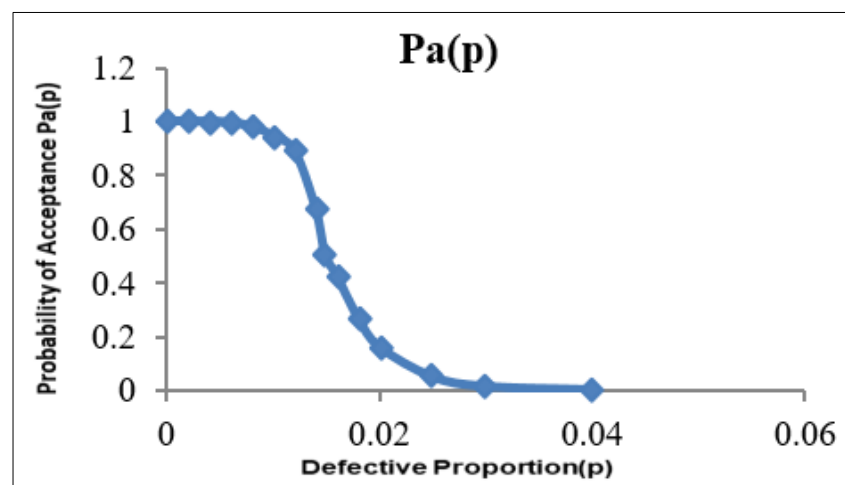


Fig 2: OC Curve for ChSP-1($n=1000$; $c=5$; $i=4$)

10.1 Practical Application

This plan is ideal for medical device manufacturing, particularly for critical components like pacemaker leads, insulin pump tubing, or surgical implants. Defects in these components can have life-threatening consequences. Sample 1000 units using rigorous testing (dimensional, functional, and biocompatibility checks). Accept up to 5 defectives only

if the last 4 lots were perfect. At IQL ≈ 0.0149 (1.49% defective), the plan maintains a balanced 50% acceptance probability. The clear S-shaped OC curve provides excellent discrimination, ensuring patient safety while avoiding unnecessary rejections of good-quality batches from reliable suppliers. It aligns well with ISO 13485 and FDA requirements.

Table 1: Parameters of ChSP-1_{TB} for a specified IQL

Sample size (n)	c	i	IQL
320	2	1	0.0823
320	3	2	0.0709
320	4	3	0.3475
500	2	1	0.0180
500	3	2	0.0169
500	4	3	0.1523
600	2	1	0.0077
600	3	2	0.0074
600	4	3	0.0838
800	2	1	0.0013
800	3	2	0.0013
800	4	3	0.0024
1000	2	1	0.0002
1000	3	2	0.0002
1000	4	3	0.0051
1000	5	4	0.0149

11. Conclusion

The chain mechanism further enhances the plan's intelligence: it rewards consistent quality by relaxing acceptance criteria when preceding lots demonstrate strong performance, while automatically imposing stricter scrutiny when any deviation occurs. This dynamic behaviour is clearly reflected in the Operating Characteristics (OC) curves and parameter tables, where larger sample sizes and longer chain lengths provide tighter control around the Indifference Quality Level (IQL) region without unnecessarily increasing inspection burden for reliable suppliers. The examples presented - particularly for automotive electronics and medical device components - illustrate the plan's practical value in real-world settings where moderate defect tolerance must coexist with reliable history-based decisions.

Compared to traditional single or double sampling plans, Chain Sampling Plan-1 (ChSP-1) using the Truncated Binomial Distribution (TBD) delivers several key advantages: improved discrimination at transitional quality levels reduced average inspection effort under stable conditions, and built-in responsiveness to emerging quality issues. These plans are very useful for the companies which has at least one defective unit in their lot and also useful to the companies which are using second quality lots.

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