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## Comparing the simpson 1/3 and simpson 3/8 rules using Newton's progressive formula

**Mustafa Nadhim Salim**DOI: <https://www.doi.org/10.22271/math.2026.v11.i8b.2678>**Abstract**

This study examines the Simpson method, a numerical integration technique that is an extension of the traditional Simpson methods for approximating the values of particular integrals. The method relies on Newton's progressive formula for inclusion to derive its formula, which is based on splitting the integration period into several partial periods based on a particular condition, so that four consecutive points of the function are used in the construction of numerical approximation.

In addition to describing how the method differs from the other Simpson method in terms of how the period is divided and how many points are used in each group, the research focuses on elucidating the mathematical foundation of the method and the mechanism of its use in calculating the specific integrals. The approach is significant in mathematical, scientific, and engineering applications because it offers an efficient and straightforward way to perform numerical integration, particularly in situations where analytical integration is challenging.

**Keyword:** Simpsons' Rule 1/3, Simpsons' Rule 3/8, Derivation of Simpson's**Introduction**

end points of the two periods, where the function  $P_2(x)$  is approximated. A parabola, which is a quadratic polynomial, is represented by  $(x)$ .

**Derivation of Simpson's 1/3 Rule**

The method can be derived using Newton's forward formula for the insertion given in the following form

$$f(x_m) = f_0 + m\Delta f_0 + \frac{m(m-1)}{2!}\Delta^2 f_0 + \frac{m(m-1)(m-2)}{3!}\Delta^3 f_0 \dots$$

$$x = x_0 + m h ; m = \frac{x-x_0}{h} ; dx = h dm$$

where the integration is defined on the interval  $[x_0, x_2]$ , we will replace the integral of  $f(x)$  with a second-order polynomial using Newton's formula for insertion with the third difference truncation:

$$\begin{aligned} I &= \int_{x_0}^{x_2} f(x) dx \cong h \int_0^2 f(x_m) dm \\ &= h \int_0^2 \left[ f_0 + m\Delta f_0 + \frac{m^2-m}{2!}\Delta^2 f_0 \right] dm \\ &= h \left[ 2f_0 + \frac{4}{2}\Delta f_0 + \frac{1}{2}\left[\frac{8}{3} - \frac{4}{2}\right]\Delta^2 f_0 \right] \\ &= h \left[ 2f_0 + 2\Delta f_0 + \frac{1}{3}\Delta^2 f_0 \right] \end{aligned}$$

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We use the following relationships:

$$\Delta f_0 = f_1 - f_0$$

$$\Delta^2 f_0 = \Delta(\Delta f_0)$$

$$= \Delta(f_1 - f_0)$$

$$= f_2 - f_1 - f_1 + f_0$$

$$= f_2 - 2f_1 + f_0$$

$$I \cong h[2f_0 + 2f_1 - 2f_0 + \frac{1}{3}f_2 - \frac{2}{3}f_1 + \frac{1}{3}f_0]$$

$$= h[\frac{1}{3}f_0 + \frac{4}{3}f_1 + \frac{1}{3}f_2]$$

$$S_n = \frac{h}{3}[f_0 + 4f_1 + f_2] \quad (1)$$

Equation (1) is called the Simpson formula  $\frac{1}{3}$  For Numerical Integration

It is the general formula where it gives a fixed approximate value of the integral when the function is polynomial of the third degree and below, and by generalizing equation (1) we get the following general formula for the period  $[x_0, x_n]$ :

$$\int_{x_0}^{x_n} f(x)dx \cong \frac{h}{3}[f_0 + 4f_1 + f_2] + \frac{h}{3}[f_2 + 4f_3 + f_4] + \dots + \frac{h}{3}[f_{n-2} + 4f_{n-1} + f_n]$$

$$= \frac{h}{3} \left[ f_0 + 4 \sum_{i=1,3,5,\dots}^{n-1} f_{i+2} + 2 \sum_{j=2,4,6}^n f_j + f_n \right]$$

The last equation is called the Simpson formula  $\frac{1}{3}$

### Error of Simpson's 1/3

The truncation error at Simpson's rule is equal to

$$E_S = \frac{-1}{90}h^5 f^{(4)}(\theta), \text{ s.t } \theta \in [a, b]$$

That is the error order of the simple Simpson rule is  $O(h^5)$  The truncation error in the Simpson's vehicle base is

$$E_S = \frac{-nh^5}{180} f^{(4)}(\theta); \text{ s.t } \theta \in [a, b]$$

The amount of error for the Simpson formula is added to the value of the integration resulting from this method to get the true integration.

$$I \cong \frac{h}{3}[f_0 + 4f_1 + f_2] - \frac{1}{90}f^{(4)}(\theta)h^5$$

$$I = \underbrace{\frac{h}{3}[f_0 + 4 \sum_{i=1,3,5,\dots}^{n-1} f_i + 2 \sum_{j=2,4,6,\dots}^n f_j + f_n]}_{\text{Simpson's Rule}} - \underbrace{\frac{nh^5}{180}f^{(4)}(\theta)}_{\text{Error}} \quad (2)$$

### Simpson's Rule Simpson's Error

#### Algorithm of Simpson's 1/3

First: Enter the value of  $f(x)$ ,  $N$ ,  $a$ ,  $b$

Second, calculate the value of  $h$  from the relation  $h = \frac{b-a}{2}$

Third:  $i=0, 1, 2, \dots, N-1$ ,  $x_i = a + i h$

Fourth: Calculate the value of the integral  $S_n = \frac{h}{3}[f_0 + 2 \sum_{i=1}^n f_{2i} + 4 \sum_{j=1}^{n-1} f_{2j-1} + f_n]$

Fifth: print a value  $S_n$  and then stop.

Now Using Simpson's rule, find the value of the following integration if you know that  $n=6$

$$\int_0^1 \frac{1}{1+x^2}$$

$$h = \frac{b-a}{n} = \frac{1}{6}$$

$$S = \frac{h}{3}[f_0 + 4(f_1 + f_3 + f_5) + 2(f_2 + f_4) + f_6]$$

$$= \frac{1}{18} [1 + 4(0.97297 + 0.8 + 0.5901) + 2(0.9 + 0.6923) + 0.5]$$

$$= 0.78539$$

The following integration  $\int_1^2 e^{-\frac{1}{2}x}$  solution is Using the Simpsons Base  $\frac{1}{3}$  and through 4 intervals

Since the interval count is an even number, this means using the Simpson  $\frac{1}{3}$  rule would be appropriate to solve the question:

$$x_i = a + ih ; x_0 = a + 0h = 1$$

$$x_1 = a + 1h = 1 ; x_2 = a + 2h = 1.5$$

$$x_3 = a + 3h = 1.7 ; x_4 = a + 4h = 2$$

**Table 1:** Clinical characteristics of men with recurrent versus single-episode noninfective balanoposthitis

| $x_i$    | 1       | 1.25    | 1.5     | 1.75    | 2       |
|----------|---------|---------|---------|---------|---------|
| $f(x_i)$ | 0.60653 | 0.53526 | 0.47237 | 0.41686 | 0.36788 |

$$\int_1^2 e^{-\frac{1}{2}x} dx = \frac{h}{3} [(f_0 + f_4) + 4(f_1 + f_3) + 2f_2]$$

$$= \frac{h}{3} [(0.60653 + 0.36788) + 4(0.53526 + 0.41686) + 2(0.47237)] = 0.47703025$$

The following order shows the integration values by applying the Simpsons rule  $\frac{1}{3}$  with the use of a different number of partitions that the true integrity value of this function is  $I = \int_1^2 e^{-\frac{1}{2}x} = 0.4773024366$ , So we can see that the value of the integration is far from the true value, so we use a value of n less.

**Table 2:** Frequency of recurrent episodes by clinical subtype and HbA1c category

| N | H   | F      |
|---|-----|--------|
| 3 | 1/3 | 0.8272 |
| 5 | 1/5 | 0.9733 |
| 7 | 1/7 | 1.0408 |

### Rule 3/8 Simpson's

The Simpson  $\frac{3}{8}$  method is considered one of the methods of numerical integration and is considered an extension of the Simpson  $\frac{1}{3}$  method with the difference being observed, in the Simpson  $\frac{1}{3}$  method we divide the  $[a, b]$  into an even number of partial periods, while in the Simpson  $\frac{3}{8}$  method, the period is divided into n of partial periods so that n is divisible by 3, i.e. n is an odd number, that is, in the Simpson  $\frac{3}{8}$  method, each partial period contains four points, i.e. four values of the function, as it can be derived from Newton's progressive formula for the entry of the period  $[x_0, x_3]$

In addition, the difference between the Simpson  $\frac{1}{3}$  method and the Simpson  $\frac{3}{8}$  method is that the amputation error rank is equal in both formulas, but the coefficient of the first is lower than the coefficient of the second, which makes the Simpson  $\frac{1}{3}$  method a desirable method in numerical integration.

### Derivation of Simpson's 3/8 Rule

We derive this rule using Newton's formula for forward differences, where the period from  $x_0$  to  $x_3$  (i.e., for the period from  $[0, 3]$  relative to n) includes four values of the function ( $f_0, f_1, f_2, f_3$ ) as well as four points where the polynomial is written as follows:

$$f(x_m) = f_0 + m\Delta f_0 + \frac{m(m-1)}{2!} \Delta^2 f_0 + \frac{m(m-1)(m-2)}{3!} \Delta^3 f_0 + \dots$$

$$x = x_0 + m h ; m = \frac{x-x_0}{h} ; dx = h dm$$

$$I = \int_{x_0}^{x_3} f(x) dx = \int_0^3 f(x_m) h dm$$

$$= h \int_0^3 [f(x_0) + m\Delta f_0 + \frac{m(m-1)}{2!} \Delta^2 f_0 + \dots] dm$$

Cutting off the boundaries containing the fourth forward differences and beyond, by taking the integration period from  $[0, 3]$  and using the following relations:

$$\Delta f_0 = f_1 - f_0$$

$$\Delta^2 f_0 = f_2 - 2f_1 + f_0$$

$$\Delta^3 f_0 = f_3 - 3f_2 + 3f_1 - f_0$$

$$\int_0^3 f(x) dx \cong h \left[ mf_0 + \frac{m^2}{2} \Delta f_0 + \left( \frac{m^3}{8} - \frac{m^2}{4} \right) \Delta^2 f_0 + \left( \frac{m^4}{24} - \frac{m^3}{6} + \frac{m^2}{6} \right) \Delta^3 f_0 \right] \Big|_0^3$$

$$= h \left[ 3f_0 + \frac{9}{2} \Delta f_0 + \frac{9}{4} \Delta^2 f_0 + \frac{3}{8} \Delta^3 f_0 \right]$$

Compensation for  $\Delta f_0, \Delta^2 f_0, \Delta^3 f_0$

$$\int_0^3 f(x) dx \cong h \left[ \frac{3}{8} f_0 + \frac{3}{8} (3f_1) + \frac{3}{8} (3f_2) + \frac{3}{8} f_3 \right]$$

$$S_{\frac{3}{8}} = \frac{3h}{8} [f_0 + 3f_1 + 3f_2 + f_3] \dots (3)$$

Equation (3) The previous one represents the Simpson  $\frac{3}{8}$  formula and by generalizing the equation along the period  $[x_0, x_n]$  we get:

$$\int_0^3 f(x) dx \cong \frac{3h}{8} [(f_0 + 3f_1 + 3f_2 + f_3) + (f_3 + 3f_4 + 3f_5 + f_6) + \dots + (f_{n-3} + 3f_{n-2} + 3f_{n-1} + f_n)] \dots (4)$$

Whereas equation (4) represents the Simpson  $\frac{3}{8}$  formula.

### Rule 3/8 of Simpson's Error

The amputation error of the Simpson formula is

$$E_t = \frac{-3}{8} h^5 f^{(4)}(\theta) ; \theta \in [0, 3]$$

$$S_{\frac{3}{8}} = \frac{3h}{8} [f_0 + 3f_1 + 3f_2 + f_3] - \frac{3}{80} h^5 f^{(4)}(\theta)$$

Represents the approximate amount of integration after adding the amount of error to the integration value

### Algorithm of Simpson's 3/8

First: Enter the value of  $f(x)$ ,  $N$ ,  $a$ ,  $b$

Second: calculate the value of  $h$  from the relation  $h = \frac{b-a}{2}$

Third:  $i=0, 1, 2, \dots, N-1$ ,  $x_i = a + i h$

Fourth: Calculate the value of the integral  $S_{\frac{3}{8}} = \frac{3h}{8} [f(a) + 3 \sum_{i=1}^{n-1} f(x_i) + f(b)], x_i \dots + f, b.]$

Fifth: print the value of  $S_{\frac{3}{8}}$

Use a Simpson rule of 3/8 to find the value of the integral, if you know that  $n=6$

$$\int_0^1 x^4 dx$$

To find the number of points we use

$$x_i = a + ih ; x_1 = 0 + \frac{1}{6} = \frac{1}{6} ; x_2 = 0 + 2 \left( \frac{1}{6} \right) = \frac{2}{6} ; x_3 = 0 + 3 \left( \frac{1}{6} \right) = \frac{3}{6}$$

$$x_4 = 0 + 4 \left( \frac{1}{6} \right) = \frac{4}{6} ; x_5 = 0 + 5 \left( \frac{1}{6} \right) = \frac{5}{6} ; x_6 = 0 + 6 \left( \frac{1}{6} \right) = 1$$

$$S_{\frac{3}{8}} = \frac{3h}{8} [(f_0 + 3f_1 + 3f_2 + f_3) + (f_3 + 3f_4 + 3f_5 + f_6)]$$

$$= 0.2002243$$

**Table 3:** Spearman rank correlation between continuous variables and episode count

| N | H             | F      |
|---|---------------|--------|
| 2 | $\frac{1}{2}$ | 0.2227 |
| 4 | $\frac{1}{4}$ | 0.2014 |
| 6 | $\frac{1}{6}$ | 0.2041 |
| 8 | $\frac{1}{8}$ | 0.2074 |

From the previous table, we can see that the value of the integration is closer to the true value the larger n is and especially if the value of n is even where the more divisions the faster the solution is reached, as the truth value of the integration is

$$I = \int_0^1 x^4 dx = \frac{x^5}{5} \Big|_0^6 = 0.2$$

Using the Simpsons 3/8 formula to find the integration for the following polynomial

$$f(x) = (0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5)$$

$$a=0 \text{ to } b=0.8$$

When using The Simpson 3/8 Mini we need 4 points

$$h = \frac{0.8-0}{3} = 0.2667$$

$$x_0 = 0 + 0 = 0; x_1 = 0 + 0.2667 = 0.2667$$

$$x_2 = 0 + 2(0.2667) = 0.5333; x_3 = 0 + 3(0.2667) = 0.8$$

$$f(0) = 0.2; f(0.2667) = 1.432724; f(0.533) = 3.487177; f(0.8) = 0.232$$

$$S_{\frac{3}{8}} = 0.8 \left[ \frac{0.2 + 3(1.432724 + 3.487177) + 0.232}{8} \right] = 1.519170$$

$$\text{The value of true integration is } I = \int_0^{0.8} f(x) dx = 1.640533$$

The true error equal to the difference between the true error and the approximate error deduced from the Simpson 3/8 formula is as follows:

$$E_t = 1.640533 - 1.519170 = 0.1213630$$

When the higher derivatives of the function are known, one of Newton-Coates' higher-order rules (i.e., one that contains a greater number of function values) is chosen. Accordingly, the Simpson rule 1/3 is more accurate than the Simpson rule 3/8, which has a high error order and a larger number of divisions, giving accurate results compared to the trapezoidal rule.

The number of points determines which special methods are used, and while they take more time and work, they produce accurate results that are extremely near to the genuine results.

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