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Investigating Locally DSS-Boundary, DSS-Nowhere Dense, DSS-Closed, and DSS-Open Sets in Dual Soft Topological Spaces

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Abstract

This work aims to present DSS-Nowhere Dense, DSS-closed, and DSS-Open. Additionally, the notion of dual soft local function is presented. The goal of discussing these ideas is to identify new dual soft topologies that differ from the original. This study also examines the fundamental structure, particularly a basis for such created dual soft topologies. Lastly, this article introduces the concept of Localley DSS-Boundary with soft topologies and establishes some comparable criteria related to this subject.

Keywords: Localley DSS-Boundary, DSS-Nowhere Dense, DSS-closed, DSS-Open

1. Introduction

In 1999, Molodtsov ^[3] defined (the soft set), which was considered a quality change and a crucial mathematical tool to handle problems that cannot be solved with standard techniques like fuzzy sets, particularly (mathematical function) membership. Additionally, the issue that most areas of mathematics, geometry, economics, computer science, and other sciences face draws a lot of academics to delve extensively into the characteristics and principles of (soft sets theory). Maji *et al.* ^[7] provided new definitions for soft sets, such as "subset soft set" and "the complement of a soft set," and thoroughly examined how soft set theory is applied to decision-making issues. The concept of soft topological spaces, which are defined over an initial universe with a fixed set of parameters, was first presented in ^[8]. Additionally, they examined a few fundamental ideas in soft topological spaces. In 2026 ^[2] They use the symmetric difference between the stable neutrosophic crisp open set (SNCO-set) and the neutrosophic crisp set (NC-set). It stands for the NC-set, SNCO-set complement of the quotient, and vice versa. Ali and Qays ^[1] to differentiate some new types of weakly neutrosophic crisp α -open mappings, they present the ideas of neutrosophic crisp α - α -open and semi α -open sets. In 2023 ^[6] the ideas of ideal and local function have been extended to dual soft sets and the investigation of their influence in this area. L.A. Al-Swidi and other ^[4] the term "dual soft sets" refers to the expansion of these sets. Additionally, their definition of the dual soft points differed from other sets. In 2024 ^[5] Also analyze dual $\text{Soft}_{\text{open}}$ and dual $\text{Soft}_{\text{closed}}$ mapping after they developed a new class of maps in the dual Soft topological space and examined some of their fundamental characteristics and relationships.

2. DSS-Boundary set, DSS-Nowhere Dense

Definition 2.1: FG_A be adual soft set

1. FG_A is DSS-Boundary set (DSS_b -set) if $D_{\text{ins}}(FG_A) = \emptyset$
2. FG_A is DSS- Nowhere Dense set (DSS_{nd} -set) if $D_{\text{int}}(D_{\text{cl}}FG_A) = \emptyset$

Definition 2.2: Let BH_A be adual soft set and e_{xy} be a dual soft point .Then:

1. BH_A is DSS-Boundary set (DSS_b -set) at the dual point e_{xy} if there exists $FG_A \in \text{StT}_E$

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2. BH_A is Nowhere Dense set (DSS_{nd}-set) at the dual soft point e_{xy} if there exists $FG_A \in StT_E$ such that $e_{xy} \in FG_A$ and $BH_A \cap FG_A$ is DSS_{nd}-set.

Remark 2.3: Let $FG_A, F_1G_{1A}, F_2G_{2A}, BH_A \subseteq X_{DS}$ and let e_{xy} be a dual soft point. Then:

1. FG_A is DSS- Nowhere Dense set at e_{xy} if $D_{cl}(FG_A)$ is DSS-Boundary set at e_{xy} .
2. If F_1G_{1A} and F_2G_{2A} is DSS- Nowhere Dense set at e_{xy} , then $F_1G_{1A} \cup F_2G_{2A}$ is DSS-Boundary set at e_{xy} .
3. For each $CR_A \in StT_E$ and $e_{xy} \in CR_A$ if FG_A is a DSS- set then FG_A is DSS- Nowhere Dense set at e_{xy} .
4. For any DSS- Nowhere Dense set BH_A if FG_A is a DSS- Nowhere Dense set at e_{xy} then $BH_A \cup FG_A$ and $FG_A \cap (BH_A)^c$ are also is DSS- Nowhere Dense set at e_{xy} .
5. $D_{cl}(D_{int}(FG_A))$ is the dual soft set of dual soft point at which FG_A is not Localley DSS-Boundary.
6. $D_{cl}(D_{int}(D_{cl}(FG_A)))$ is the dual soft set of dual soft point at which FG_A is not Localley DSS- Nowhere Dense set.

Lemma 2.4: If adual soft set FG_A is locally DSS-Boundary at adual soft point e_{xy} , then $e_{xy} \notin D_{int}(FG_A)$.

Proof: Assume that FG_A is locally DSS -boundary at the dual soft -point e_{xy} . Then there exists an dual soft open set $BH_A \in StT_E$ such that $e_{xy} \in BH_A$ and $BH_A \cap FG_A$ is a DSS-Boundary. By definition of DSS-Boundary set, $D_{int}(BH_A \cap FG_A) = \emptyset_{DS}$. Suppose for contradiction that $e_{xy} \in D_{int}(FG_A)$. Then there exists an dual soft open set $CR_A \in StT_E$ such that that $e_{xy} \in CR_A \subseteq FG_A$. Since $e_{xy} \in BH_A$, we here $e_{xy} \in BH_A \cap CR_A$ moreover, $BH_A \cap CR_A \subseteq FG_A \cap BH_A$. This implies that, $D_{int}(BH_A \cap FG_A) = \emptyset_{DS}$, which contradiction the assumption that $BH_A \cap FG_A$ is DSS-Boundary set. Therefore, $e_{xy} \notin D_{int}(FG_A)$.

Proposition 2.5: Every DSS-Boundary set is locally DSS-Boundary.

Proof: Let FG_A be a DSS-Boundary set. Then by definition $D_{int}(FG_A) = \emptyset_{DS}$. Let e_{xy} be any dual soft point in X_{DS} choose $BH_A = X_{DS}$, where $X_{DS} \in StT_E$. Then $(BH_A \cap FG_A) = FG_A$. Since BH_A is DSS-Boundary Hence, FG_A is locally DSS-Boundary at every dual soft point.

Proposition 2.6

1. The DSS-set of all dual soft point, where FG_A locally DSS-Boundary DSS-Boundary set.
2. The DSS-set of all dual soft point, where $D_{cl}(FG_A)$ is locally DSS-Boundary is DSS- Nowhere Dense set.

Proof

i) Let $BH_A = \{e_{xy} \in X_{DS} : e_{xy} \in FG_A \text{ and } \exists CR_A \in StT_E, e_{xy} \in CR_A, D_{int}(BH_A \cap FG_A) = \emptyset_{DS}\}$. Assume, $D_{int}(BH_A) = \emptyset_{DS}$. Then $\exists SW_A \in X_{DS}$, $SW_A = \emptyset_{DS}$, $SW_A \subseteq BH_A$, pick $e_{xy} \in BH_A$, there exist $CR_A \in X_{DS}$ with $e_{xy} \in CR_A$, $D_{int}(BH_A \cap FG_A) = \emptyset_{DS}$. But $e_{xy} \in CW_A \cap CR_A$, $CW_A \cap CR_A \in X_{DS}$, $CR_A \neq \emptyset_{DS}$ and since $SW_A \subseteq BH_A \subseteq FG_A$, $CW_A \cap CR_A \subseteq FG_A \cap CR_A$ then $D_{int}(BH_A \cap FG_A) \neq \emptyset_{DS}$ a contradiction. Hence $D_{int}(BH_A) = \emptyset_{DS}$, so BH_A is a DSS-Boundary set.

Let $BH_A = e_{xy} \in X_0 : \exists CR_A \in SET_E, e_{xy} \in CR_A, D_{int}(D_{cl}(FG_A) \cap CR_A) = \emptyset_{DS}$.

Assume $D_{int}(D_{cl}(BH_A)) \neq \emptyset_{DS}$. Then $\exists SW_A \in SET_E, SW_A \subseteq D_{cl}(BH_A)$ choose $e_{xy} \in (SW_A \cap BH_A)$. Then $e_{xy} \in CR_A, e_{xy} \in CR_A, D_{int}(D_{cl}(FG_A) \cap CR_A) = \emptyset_{DS}$. But $e_{xy} \in (SW_A \cap CR_A)$, $SW_A \cap CR_A \in SET_E$, $SW_A \cap CR_A \neq \emptyset_{DS}$ and $SW_A \subseteq D_{cl}(BH_A)$ implies $SW_A \cap CR_A \subseteq D_{cl}(BH_A) \cap CR_A$ in particular $SW_A \cap CR_A \in SET_E$ is an nonempty dual soft open sub set of $(D_{cl}(FG_A) \cap CR_A)$, hence $D_{int}(D_{cl}(FG_A) \cap CR_A) \neq \emptyset_{DS}$ contradiction. Therefore $D_{int}(D_{cl}(BH_A)) = \emptyset_{DS}$, so BH_A is a DSS- Nowhere Dense set.

3. DSS-closed domain sets and DSS-open domain sets

Definition 3.1

Let (X_0, SET_E) be a dual soft topological space. A dual soft set $FG_A \subseteq X_{DS}$ is called a DSS-closed domain set (DSSCD-set) if:

1. FG_A is DSS-closed; and
2. FG_A is not DSS-nowhere dense at any dual soft point of FG_A . The complement of a DSSCD-set is called a DSS-open domain set (DSSOD-set).

Remark 3.2

Let $FG_A \subseteq X_{DS}$. Then:

- 1) FG_A is DSSCD-set if and only if $FG_A = D_{cl}(D_{int}(FG_A))$
- 2) FG_A is DSSOD-set if and only if $FG_A = D_{int}(D_{cl}(FG_A))$
- 3) FG_A is DSSCD-set then $(FG_A)^c$ is a DSSOD - set

Proprsition 3.3: FG_A is a DSSCD-set if and only if $FG_A = D_{cl}(CR_A)$ for some dual soft open set $CR_A \subseteq SET_E$.

Proof: Assume that FG_A is a DSSCD-set. Then by definition 3.1 $FG_A = D_{int}(D_{cl}(FG_A))$.

Let

$CR_A = D_{int}(D_{cl}(FG_A))$ Since CR_A is dual soft open, we obtain $FG_A = (D_{cl}(CR_A))$ conversely, suppose that there exists adual

soft open set $CR_A \in SET_E$ such that $FG_A = D_{cl}(CR_A)$. Then $D_{int}((FG_A)) \supseteq CR_A$. Hence $D_{cl}(D_{int}(FG_A)) \supseteq D_{cl}(FG_A) = FG_A$. On the other hand, $D_{cl}(D_{int}(FG_A)) \subseteq D_{cl}(FG_A) = FG_A$. Therefore, $FG_A = D_{cl}(D_{int}(FG_A))$ and so FG_A is DSSCD-set.

Definition 3.4: Let (X_{DS}, τ_E) be a dual soft topological space over X_{DS} and $FG_A \subseteq SE_E$. Then, the dual soft boundary of a dual soft set FG_A over X_{DS} is denoted by $DSSF_r(FG_A)$ and is defined as: $DSSF_r(FG_A) = D_{cl}(FG_A) \cap D_{cl}(FG_A^c)$. It must be noted that: $DSSF_r(FG_A) = DSSF_r((FG_A))$

Proposition 3.5: FG_A is a DSSCD – set if and only if: $DSSF_r(FG_A) \subseteq D_{cl}(D_{int}(FG_A))$

Proof: Assume first that FG_A is a DSSCD-set. Then by definition 3.1: $FG_A = D_{cl}(D_{int}(FG_A))$. Since: $DSSF_r(FG_A) = D_{cl}(FG_A) \cap D_{cl}(FG_A^c)$ and FG_A is DSS-closed, we have: $D_{cl}(FG_A) = FG_A$. Thus: $DSSF_r(FG_A) = FG_A \cap D_{cl}(FG_A^c) \subseteq FG_A$. Conversely, assume that: $DSSF_r(FG_A) \subseteq D_{cl}(D_{int}(FG_A))$. Since: $D_{int}(FG_A) \cup DSSF_r(FG_A) = D_{cl}(FG_A)$ and FG_A is DSS-closed, we obtain: $D_{cl}(FG_A) = FG_A$. Hence $FG_A = D_{int}(FG_A) \cup DSSF_r(FG_A) \subseteq D_{cl}(D_{int}(FG_A))$. But clearly: $D_{cl}(D_{int}(FG_A)) \subseteq D_{cl}(FG_A) = FG_A$. Therefore: $FG_A = D_{cl}(D_{int}(FG_A))$, which means that FG_A is a DSSCD-set.

Corollary 3.6: If FG_A is a DSSCD-set, then: $DSSF_r(D_{cl}((FG_A)^c)) = DSSF_r(D_{int}(FG_A))$.

Proposition 3.7: If $\{F_j G_{jA}\}$ is a family of DSSCD-sets, then $D_{cl}(\bigcup_j F_j G_{jA})$ is a DSSCD-set.

Proof: Since for each j , there exists $B_j, H_j \in S\tau_E$ such that: $F_j G_{jA} = D_{cl}(B_j, H_{jA})$.

Hence: $D_{cl}(\bigcup_j F_j G_{jA}) = D_{cl}(\bigcup_j D_{cl}(B_j, H_{jA})) = D_{cl}(\bigcup_j (B_j, H_{jA}))$. Hence $\bigcup_j F_j G_{jA}$ is a DSSCD-set.

Proposition 3.8: If $FG_A \subseteq K_{LA}$, $e_{xy} \in K_{LA}$ and K_{LA} is a DSSCD-set, then FG_A is a DSS-set at e_{xy} if and only if FG_A is a DSS boundary-set relatively to K_{LA} at e_{xy} .

Proof

$FG_A \subseteq K_{LA} = D_{cl}(D_{int}(K_{LA}))$ and $e_{xy} \in K_{LA} = D_{cl}(D_{int}(K_{LA}))$. So $FG_A = FG_A \cap D_{cl}(D_{int}(K_{LA}))$ is DSS-set relative to $D_{cl}(D_{int}(K_{LA})) = K_{LA}$. Conversely, let FG_A be DSS boundary-set relative to K_{LA} . Then $D_{int}(K_{LA}(FG_A)) = \emptyset$, but $D_{int}(FG_A) \subseteq D_{int}(K_{LA}(FG_A))$. Therefore FG_A is DSS boundary-set at $e_{xy} \in K_{LA}$.

Remark 3.9

1. FG_A is a DSSOD – set if and only if $(FG_A)^c$ is a DSSCD – set.
2. FG_A is a DSSOD-set if and only if FG_A is a DSS-open set and $DSSF_r(FG_A) \subseteq D_{cl}(D_{int}((FG_A)^c))$.
3. If FG_A is a DSSOD-set, then $DSSF_r(D_{cl}(FG_A)) = DSSF_r(D_{cl}((FG_A)^c))$.
4. The intersection of finite many DSSOD-set is DSSOD-set.
5. Let $DSI(FG_A)$ denote the dual soft -set of all dual soft point .at which FG_A is locally DSS-now here .Then $DSI(FG_A)$ is a a DSSCD – set.
6. For any dual soft -set FG_A , $FG_A \cap DSI(FG_A) \cap (FG_A \cap DSI(FG_A))^c = \emptyset_{DS}$

Proposition 3.10. The following statements are equivalent with SET_E is advance DSS-topological space

- i. Every $FG_A \in SET_E$ is DSS-Nowhere dense set.
- ii. Every DSS-nowhere dense set BH_A , we have $D_{int}(BH_A) = \emptyset_{DS}$.
- iii. Every DSS-nowhere dense set $BH_A \in SET_E$ we have $D(FG_A) \cap FG_A = \emptyset_{DS}$ where $D(FG_A)$ denoted the set of dual soft -points at which FG_A is locally DSS-boundary

proof: i $\rightarrow$ ii Assume that every nonempty open dual soft set is DSS-nowhere dense .Let BH_A Dss-nowhere dense set suppose for contradiction, that $D_{int}(BH_A) \neq \emptyset_{DS}$. Then there exists a nonempty dual soft open $SW_A \in SET_E$ such that $SW_A \subseteq BH_A$ since SW_A is dual soft open by assumption it is DSS-nowhere dense But this contradicts the fact that a dual soft open set can not have empty interior Hence, $D_{int}(BH_A) = \emptyset_{DS}$. $D_{int}(BH_A) = \emptyset_{DS}$.

ii $\rightarrow$ iii Assume that for every DSS-nowhere dense set BH_A , $D_{int}(BH_A) = \emptyset_{DS}$. Let $FG_A \in SET_E$. suppose that there exists $e_{xy} \in D_{cl}(FG_A) \cap FG_A$. By definition of $D_{cl}(FG_A)$, (FG_A) is locally DSS-boundary at e_{xy} . Hence exists an dual soft open set SW_A such that $D_{int}(FG_A) \cap SW_A = \emptyset_{DS}$. But since FG_A is open and contains e_{xy} contradicts assumption .Therefore $D(FG_A) \cap FG_A = \emptyset_{DS}$.

iii $\rightarrow$ i Assume that for every dual soft open FG_A , $D(FG_A) \cap FG_A = \emptyset_{DS}$ suppose, for contradiction that there exists a nonempty dual soft open set SW_A that is not DSS-nowhere dense .Then $D_{int}(D_{cl}(SW_A)) \neq \emptyset_{DS}$. This implies the existence of a dual soft point $e_{xy} \in SW_A$

where SW_A is locally DSS-boundary set, Thus $e_{xy} \in D_{cl}(SW_A) \cap (SW_A)$ which contradicts assumption (3) .Hence every dual soft open set is DSS-nowhere dense .

4. Conclusion

We introduced locally DSS-boundary sets, DSS-nowhere dense sets, DSS-closed dual soft domain sets, and DSS-open dual soft sets in dual soft topological space. The obtained results provide new insights into the local structure of dual soft topologies and open the door for further applications in analysis and related fields.

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