

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452

NAAS Rating (2026): 4.49

Maths 2026; 11(8): 114-117

© 2026 Stats & Maths

<https://www.mathsjournal.com>

Received: 10-06-2026

Accepted: 12-07-2026

Dewi Rahardja

U.S. Federal Government, USA

Bayesian inference for misclassified binary data under a double sampling scheme: A review

Dewi Rahardja

DOI: <https://www.doi.org/10.22271/math.2026.v11.i8b.2668>

Abstract

A considerable body of literature exists on Bayesian statistical methodologies for misclassified binary data. This paper reviews Bayesian approaches within the context of misclassified binary data under a double sampling scheme. The review is structured into two main categories: one-sample and two-sample problems. Each category addresses one-type (false positives or false negatives) and both-type (both false positives and false negatives) misclassifications. A visual roadmap illustrating these Bayesian approaches is provided. Ultimately, this paper aims to assist practitioners and researchers across various fields in identifying the most appropriate methodology for their data.

2020 AMS subject classifications: 62F15 – Bayesian inference.

Keywords: Bayesian inference, proportion parameters, one-sample problem, two-sample problem, misclassification, false positives, false negatives

1. Introduction

Many real-world applications involve binary outcomes where data are collected using imperfect (fallible) measuring devices. However, a "gold standard" or infallible device is often available for a smaller subsample of the observational units. This specific framework is known as a double sampling scheme, a methodology pioneered by Tenenbein ^[1] in 1970. Although fallible devices can introduce both false-positive and false-negative errors, certain scenarios or diagnostic constraints mean that only one type of misclassification error is present ^[2-6].

A vast body of literature discusses statistical methodologies for handling misclassified binary data, spanning both frequentist and Bayesian paradigms. This paper specifically reviews Bayesian methods developed for misclassified binary data under a double sampling scheme.

The remainder of this paper is structured as follows: Section 2 focuses on one-sample Bayesian approaches, covering both one-type and both-type misclassifications (false positives and/or false negatives). Section 3 addresses two-sample Bayesian methods across three sub-categories: risk differences (proportion differences), relative risks (proportion ratios), and odds ratios, with each sub-category further broken down by one-type and both-type misclassifications. Section 4 presents a comprehensive visual roadmap summarizing these Bayesian approaches (Figure 1). Finally, Section 5 provides concluding remarks.

2. One-sample review

This section reviews the Bayesian literature on one-sample misclassified binary data, covering both one-type and both-type misclassifications (false positives and/or false negatives).

2.1. One-sample review with one-type misclassification

Several Bayesian methodologies have been developed for one-sample data subject to a single type of misclassification error (either false positives or false negatives). In 1995, York *et al.* ^[2] developed a Bayesian approach specifically to correct for false-negative errors. In 2003, Raats and Moors ^[3] proposed a Bayesian interval framework for financial audit data, handling one error type at a time. Other researchers illustrated their computational methods using the blood-testing dataset from Perry *et al.* ^[4] (2000), which exhibited only false-positive errors.

Corresponding Author:

Dewi Rahardja

U.S. Federal Government, USA

To overcome the computational difficulties of earlier methods, Lee and Byun ^[5] (2008) applied non-informative priors under a double sampling scheme subject strictly to false-positive misclassification; this approach yielded computationally simpler Bayesian credible intervals. However, because their method relied on a normal approximation of the marginal posterior density, some statistical efficiency may have been lost. Subsequently, Rahardja *et al.* ^[6] (2012) addressed this limitation by developing a closed-form, fully Bayesian algorithm that draws posterior samples of the proportion parameter directly from the exact marginal posterior distribution.

2.2. One-sample review with both types of misclassifications

Several Bayesian approaches have been developed to address one-sample data subject to concurrent false-positive and false-negative misclassification errors. In their 2003 publication, Raats and Moors ^[3] derived a Bayesian credible interval designed to accommodate both types of auditor misclassification errors simultaneously. In 2012, Rahardja and Zhao ^[7] developed a closed-form Bayesian algorithm that draws posterior samples of the proportion parameter directly from the exact marginal posterior distribution. Their simulation studies demonstrated that this algorithm is straightforward to implement and maintains nominal coverage. Subsequently, Lee ^[8] expanded upon the earlier joint work with Byun to introduce a fully Bayesian interval estimation method that simultaneously handles both types of misclassification errors in one-sample data. Most recently, Al-Kandari and Garthwaite ^[9] developed a Bayesian analysis framework for misclassified binomial data that utilizes informative priors to address both double-sampling and the zero-numerator problem.

3. Two-sample review

The two-sample Bayesian literature spans three primary comparative metrics: the proportion difference or risk difference (Diff), the odds ratio (OR), and the risk ratio (RR). Each category is further evaluated based on whether the data are subject to one-type or both-type misclassifications.

3.1. Proportion difference or risk difference (Diff)

The literature evaluating the risk difference falls into two structural categories: one-type misclassification, where only false-positive or false-negative errors are present, and both-type misclassification, where both error types occur simultaneously.

3.1.1. Only one-type of misclassification present

There are not many Bayesian approaches that have been constructed to estimate the difference between two independent population proportions based on binomial data subject to a single type of misclassification. Rahardja ^[10] (2019) derived point and interval estimation methods using a Bayesian framework tailored for this scenario. This methodology utilizes an easy-to-implement, closed-form algorithm to sample directly from the exact posterior distributions.

3.1.2. Both (Two-type) misclassifications present

For independent two-sample binary data subject to concurrent false-positive and false-negative errors without an accompanying validation study, Evans *et al.* ^[11] (1996) developed a Bayesian approach leveraging informative priors

to make inferences on risk differences. However, it is not readily apparent how their framework can be adapted when a validation study (such as a double sampling scheme) is available instead of informative priors. Addressing a related framework, Rahardja ^[12] in 2012 developed a Bayesian credible sets method specifically designed for the risk difference in binomial data subject to both types of errors.

3.2. Odds ratio (OR)

The literature regarding the odds ratio is divided into two categories: one-type misclassification, where only false-positive or false-negative errors occur, and both-type misclassification, where both error types are present.

3.2.1. Only one-type of misclassification present

Within the two-sample literature evaluating odds ratios under a single type of misclassification, Rahardja ^[13] (2016) derived a Bayesian approach for the point and interval estimation of the OR of two binomial proportions using double-sampling data subject to only false-positive errors. To facilitate implementation, a closed-form algorithm was devised to sample directly from the exact posterior distributions.

3.2.2. Both (Two-type) misclassifications present

For two-sample designs subject to concurrent false-positive and false-negative errors, Viana ^[14] (1994) as well as Prescott and Garthwaite ^[15] (2002) proposed a two-stage method to construct a Bayesian credible interval for the odds ratio. Additionally, Lunn *et al.* ^[16] (2000) provided a Bayesian analysis framework using a logistic model. Gustafson *et al.* ^[17] (2001) have constructed Bayesian inference method for odds ratio. However, these earlier methods can be computationally burdensome, are not guaranteed to converge to the true parameter values and can be difficult for practitioners to reproduce. To address these limitations, Rahardja *et al.* ^[18] (2010) proposed a fully Bayesian model utilizing non-informative priors for analyzing misclassified binary data accompanied by a validation sub-study. This approach features a closed-form algorithm that draws all parameters directly from the joint posterior distribution, simplifying statistical inference for odds ratios.

3.3. Risk ratio (RR)

The literature evaluating the RR is similarly divided into two categories: one-type misclassification, where only false-positive or false-negative errors occur, and both-type misclassification, where both error types are present.

3.3.1. Only one-type of misclassification present

For data subject to a single type of misclassification error, Gaba and Winkler ^[19] (1992) initially developed a Bayesian method leveraging informative priors. Expanding into two-sample designs, Rahardja and Young ^[20] (2010) proposed a Bayesian approach to perform statistical inference for risk ratios based on two independent samples of binomial data subject to only false-positive misclassification.

3.3.2. Both (Two-type) misclassifications present

For two-sample scenarios involving concurrent false-positive and false-negative errors, Rahardja and Young ^[21] (2011) developed a Bayesian inference framework for the RR of two proportion parameters. Their method is tailored specifically for two independent samples of binary data subject to both types of misclassifications simultaneously.

4. Roadmap

Figure 1 presents a structured roadmap designed to guide practitioners and researchers in identifying the most

appropriate Bayesian methodology for their specific data configuration. The roadmap is organized into two primary tracks: one-sample and two-sample problem frameworks.

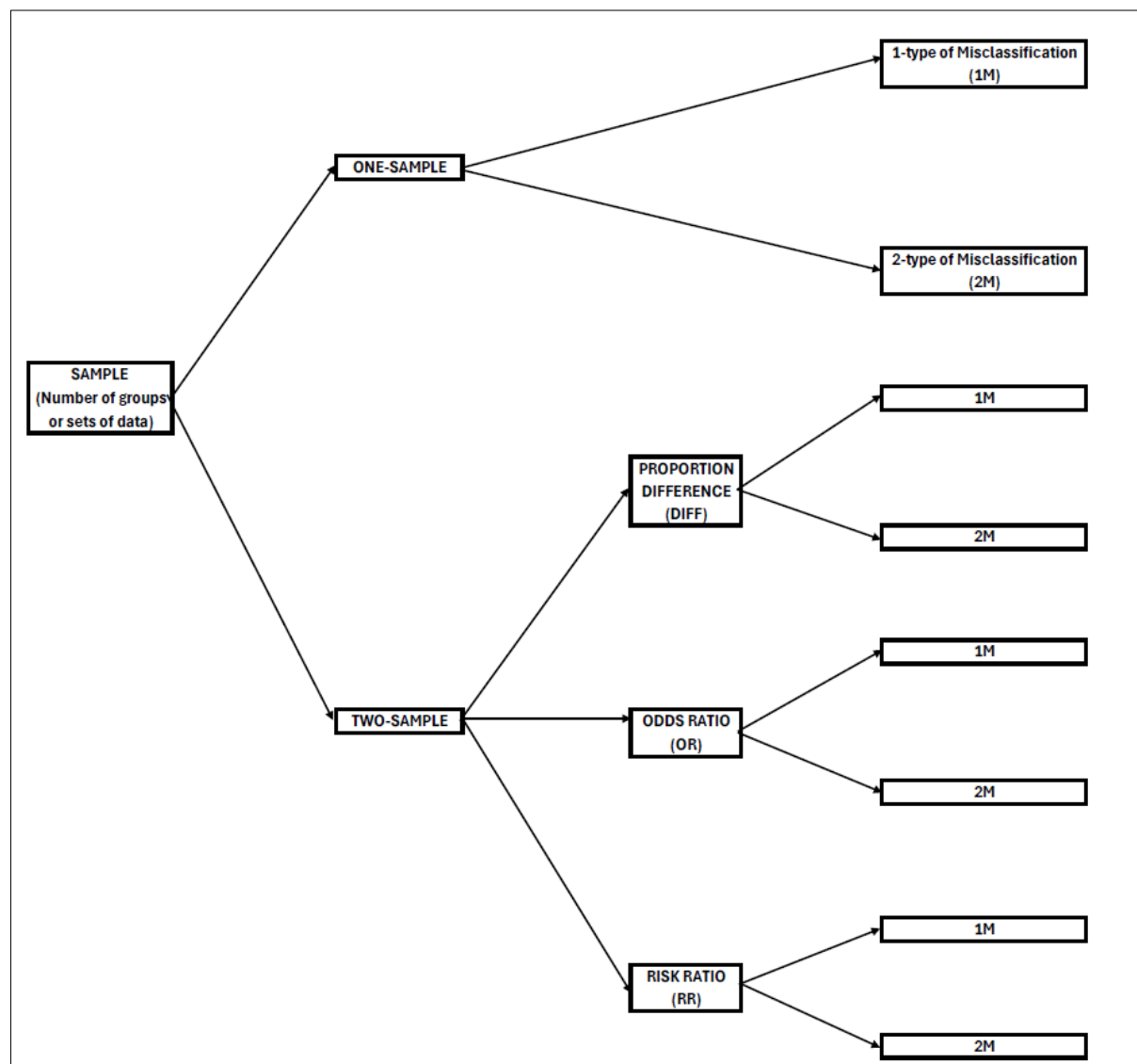


Fig 1: Methodological Roadmap of the Bayesian Literature on Misclassified Binary Data.

Within the one-sample track, the roadmap branches based on whether the data exhibit a single type or both types of binary misclassification errors (false positives and/or false negatives).

Within the two-sample track, the roadmap delineates into three comparative sub-categories: proportion differences, odds ratios, and risk ratios. Each of these sub-categories is further subdivided to address either one-type or both-type misclassifications.

5. Summary

The misclassification of binary data is a widespread challenge across numerous real-world disciplines, including medicine, epidemiology, transportation safety, financial auditing, etc. While a vast body of literature addresses this issue, this paper specifically provides a literature review of practical, implementable Bayesian statistical methodologies within a double sampling framework. By categorizing the existing literature into distinct one-sample and two-sample paradigms and further stratifying by error types and comparative metrics this review simplifies methodological selection. Ultimately, the synthesis and visual roadmap presented in this paper serve

as a valuable reference to help researchers and practitioners across diverse fields identify and implement the most appropriate Bayesian statistical models for their misclassified data.

References

1. Tenenbein A. A double sampling scheme for estimating from binomial data with misclassifications. *Journal of the American Statistical Association*. 1970;65(331):1350-1361. DOI:10.1080/01621459.1970.10481170.
2. York J, Madigan D, Heuch I, Lie RT. Birth defects registered by double sampling: a Bayesian approach incorporating covariates and model uncertainty. *Applied Statistics*. 1995;44(2):227-242. DOI:10.2307/2986341.
3. Raats VM, Moors JJA. Double-checking auditors: a Bayesian approach. *The Statistician*. 2003;52(3):351-365. DOI:10.1111/1467-9884.00364.
4. Perry M, Vakil N, Cutler A. Admixture with whole blood does not explain false-negative urease tests. *Journal of Clinical Gastroenterology*. 2000;30(1):64-65. DOI:10.1097/00004836-200001000-00012.

5. Lee SC, Byun JS. A Bayesian approach to obtain confidence intervals for binomial proportion in a double sampling scheme subject to false-positive misclassification. *Journal of Korean Statistical Society*. 2008;37(4):393-403. DOI:10.1016/j.jkss.2008.05.001.
6. Rahardja D, Zhao YD, Zhang H. Bayesian credible sets for a binomial proportion based on one-sample binary data subject to one type of misclassification. *Journal of Data Science*. 2012;10(1):51-59. Available from: <https://jds-online.org/journal/JDS/article/1207>.
7. Rahardja D, Zhao YD. Bayesian inference of a binomial proportion using one-sample misclassified binary data. *Model Assisted Statistics and Applications*. 2012;7(1):17-22. DOI:10.3233/MAS-2011-0197.
8. Lee SC. Bayesian confidence intervals of proportion with misclassified binary data. *Journal of Korean Statistical Society*. 2013;42(3):291-299. DOI:10.1016/j.jkss.2012.09.001.
9. Al-Kandari NM, Garthwaite PH. Bayesian analysis of misclassified binomial data: double-sampling and the zero-numerator problem. *Communications in Statistics - Simulation and Computation*. 2023;52(2):334-348. DOI:10.1080/03610918.2020.1855448.
10. Rahardja D. Bayesian inference for the difference of two proportion parameters in over-reported two-sample binomial data using the doubly sample. *Stats*. 2019;2(1):111-120. DOI:10.3390/stats2010009.
11. Evans M, Guttman I, Haitovsky Y, Swartz T. Bayesian analysis of binary data subject to misclassification. In: Berry D, Chaloner K, Geweke J, editors. *Bayesian Analysis in Statistics and Econometrics: Essays in Honor of Arnold Zellner*. New York: John Wiley; 1996. p. 67-77.
12. Rahardja D, Zhao YD. Bayesian credible sets for the risk difference in binomial data with a fallible classifier and a gold standard. *Journal of Statistical Computation and Simulation*. 2012;82(1):63-71. DOI:10.1080/00949655.2010.526610.
13. Rahardja D. Bayesian credible sets for odds ratio of two binomial proportions using double sampling with false-positive misclassification. *American Journal of Mathematics and Management Sciences*. 2016;35(3):271-279. DOI:10.1080/01966324.2016.1174093.
14. Viana MAG. Bayesian small-sample estimation of misclassified multinomial data. *Biometrics*. 1994;50(1):237-243. DOI:10.2307/2533215.
15. Prescott GJ, Garthwaite PH. A simple Bayesian analysis of misclassified binary data with a validation substudy. *Biometrics*. 2002;58(2):454-458. DOI:10.1111/j.0006-341X.2002.00454.x.
16. Lunn DJ, Thomas A, Best N, Spiegelhalter D. WinBUGS - a Bayesian modelling framework: concepts, structure, and extensibility. *Statistics and Computing*. 2000;10(4):325-337. DOI:10.1023/A:1008929526011.
17. Gustafson P, Le ND, Saskin R. Case-control analysis with partial knowledge of exposure misclassification probabilities. *Biometrics*. 2001;57(2):598-609. DOI:10.1111/j.0006-341X.2001.00598.x.
18. Rahardja D, Zhao YD, Zhang HH. Bayesian inference of odds ratios in misclassified binary data with a validation substudy. *Communications in Statistics - Simulation and Computation*. 2010;39(10):1845-1854. DOI:10.1080/03610918.2010.518271.
19. Gaba A, Winkler RL. Implications of errors in survey data: a Bayesian model. *Management Science*. 1992;38(7):913-925. DOI:10.1287/mnsc.38.7.913.
20. Rahardja D, Young DM. Credible sets for risk ratios in over-reported two-sample binomial data using the double-sampling scheme. *Computational Statistics and Data Analysis*. 2010;54(5):1281-1287.
21. Rahardja D, Young DM. Bayesian inference of risk ratio of two proportions using a double sampling scheme. *Journal of Biopharmaceutical Statistics*. 2011;21(3):393-404.