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Some curvature conditions on LP-Kenmotsu manifolds with quarter-symmetric non-metric connection

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Abstract

In this manuscript, LP-Kenmotsu manifolds equipped with quarter-symmetric non-metric connection is evaluated. On LP-Kenmotsu manifolds, we examined many curvature properties for the given connection. In this sequel, we found that Ricci solitons on LP-Kenmotsu manifolds endowed with QSNM connection is an generalized η -Einstein manifold. We acquire some outcomes if $\tilde{\mathcal{R}} \cdot \tilde{\mathcal{S}} = 0$.

Keywords: Quarter-symmetric non-metric connection (QSNM), LP-Kenmotsu manifold, Ricci tensor, Einstein manifold, Ricci soliton

1. Introduction

In differential geometry, a Ricci soliton is an important geometric structure that arises naturally as a self-similar solution to the Ricci flow. Ricci solitons generalize the concept of Einstein manifolds and play a crucial role in understanding the formation of singularities in the Ricci flow.

The Ricci flow developed by R. S. Hamilton ^[3] in 1982 is the evolution equation for a Riemannian metric $g(t)$ on a manifold $\mathcal{M}$ given by

$$\frac{\partial g}{\partial t} = -2\mathcal{S} \quad (1.1)$$

A Ricci soliton represents a special solution to this flow, moving only by diffeomorphisms and scaling. Formally, a Riemannian manifold $\mathcal{M}$ together with a smooth vector field v (or potential function f) is called a Ricci soliton if

$$\frac{1}{2}\mathcal{L}_v g + \mathcal{S} = \lambda g, \quad (1.2)$$

where $\mathcal{S}$ is the Ricci curvature tensor, $\mathcal{L}_v g$ is Lie derivative along v and λ is a constant. Depending on the value of λ a Ricci soliton is classified as:

- Decreasing if $\lambda < 0$,
- Steady if $\lambda = 0$,
- Increasing $\lambda > 0$.

Matsumoto ^[10] demonstrated the concept of Lorentzian paracontact and in particular, LP-Sasakian manifolds. Later, these manifolds have been studied by many geometers ^[11, 12, 13, 18] and acquired several results.

Sinha and Prasad ^[14] defined a class of almost paracontact metric manifolds called para-Kenmotsu and special para-Kenmotsu manifolds is similar to P-Sasakian and SP-Sasakian manifolds.

A linear connection $\tilde{\nabla}$ in an $(2n + 1)$ -dimensional differentiable manifold is called a quarter-symmetric connection ^[4] if it satisfies

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$$\tilde{T}(\theta_1, \theta_2) = \eta(\theta_2)\varphi\theta_1 - \eta(\theta_1)\varphi\theta_2. \quad (1.3)$$

If quarter-symmetric linear connection $\tilde{\nabla}$ satisfies the condition

$$\tilde{\nabla}_{\theta_1}^* g(\theta_2, \theta_3) \neq 0,$$

then $\tilde{\nabla}$ is called QSNM connection. As an extension, many geometers have studied these connection [5, 6, 7, 16, 17, 19, 20].

Motivated by the above studies, we studied Lorentzian para-Kenmotsu manifolds equipped with QSNM connection. The present research paper is classified in this order: After the introduction, Section 2 discuss certain basic definitions of LP-Kenmotsu manifolds. In section 3, we define a QSNM connection and some curvature properties with QSNM connection. We study Ricci soliton with QSNM connection in section 4. In section 5, we establish some important results on LP-Kenmotsu manifolds satisfying $\tilde{\mathcal{R}} \cdot \tilde{\mathcal{S}} = 0$.

2. Preliminaries

A $(2n + 1)$ -dimensional manifold $\mathcal{M}$ is said to be an almost contact metric manifold if there exist a $(1, 1)$ tensor field φ , a non-vanishing vector field ξ and a 1-form η such that

$$\eta(\xi) = -1, \varphi^2\theta_1 = \theta_1 + \eta(\theta_1)\xi, \varphi\xi = 0, \quad (2.1)$$

$$g^*(\theta_1, \xi) = \eta(\theta_1), \quad (2.2)$$

$$g^*(\varphi\theta_1, \varphi\theta_2) = g^*(\theta_1, \theta_2) + \eta(\theta_1)\eta(\theta_2), \quad (2.3)$$

$$\varphi(\theta_1, \theta_2) = \varphi(\theta_2, \theta_1) = g^*(\theta_1, \varphi\theta_2). \quad (2.4)$$

A Lorentzian almost paracontact manifold $\mathcal{M}$ is called LP-Kenmotsu manifold if

$$(\nabla_{\theta_1}\varphi)\theta_2 = -g^*(\varphi\theta_1, \theta_2)\xi - \eta(\theta_2)\varphi\theta_1. \quad (2.5)$$

In a LP-Kenmotsu manifold, we have

$$\nabla_{\theta_1}\xi = -\theta_1 - \eta(\theta_1)\xi, \quad (2.6)$$

$$(\nabla_{\theta_1}\eta)\theta_2 = -g^*(\theta_1, \theta_2) - \eta(\theta_1)\eta(\theta_2). \quad (2.7)$$

where ∇ is the Levi-Civita connection with respect to g^*

Furthermore, in a LP-Kenmotsu manifold with respect to the Levi-Civita connection, the following conditions hold:

$$\eta(\mathcal{R}^*(\theta_1, \theta_2)\theta_3) = g^*(\theta_2, \theta_3)\eta(\theta_1) - g^*(\theta_1, \theta_3)\eta(\theta_2), \quad (2.8)$$

$$\mathcal{R}^*(\xi, \theta_1)\theta_2 = g^*(\theta_1, \theta_2)\xi - \eta(\theta_2)\theta_1, \quad (2.9)$$

$$\mathcal{R}^*(\xi, \theta_1)\xi = \theta_1 + \eta(\theta_1)\xi, \quad (2.10)$$

$$\mathcal{S}^*(\theta_1, \xi) = 2n\eta(\theta_1), \mathcal{S}^*(\xi, \xi) = -2n, \quad (2.11)$$

$$Q\xi = 2n\xi, \quad (2.12)$$

$$\mathcal{S}^*(\varphi\theta_1, \varphi\theta_2) = \mathcal{S}^*(\theta_1, \theta_2) + 2n\eta(\theta_1)\eta(\theta_2). \quad (2.13)$$

for any vector fields $\theta_1, \theta_2, \theta_3 \in \chi(\mathcal{M})$, where $\mathcal{R}$ is the curvature tensor, $\mathcal{S}$ is the Ricci tensor and Q is the Ricci operator.

Definition 2.1. A LP-Kenmotsu manifold $\mathcal{M}$ is said to be a generalized η -Einstein manifold if the following condition [15]

$$\mathcal{S}^*(\theta_1, \theta_2) = a g^*(\theta_1, \theta_2) + b \eta(\theta_1)\eta(\theta_2) + c g^*(\varphi\theta_1, \varphi\theta_2). \quad (2.14)$$

3. Quarter-symmetric non-metric connection $\tilde{\nabla}$

$\mathcal{M}$ be a LP-Kenmotsu manifold and ∇ be the Levi-Civita connection on $\mathcal{M}$. The QSNM connection $\tilde{\nabla}$ is defined as ^[8]

$$\tilde{\nabla}_{\theta_1}\theta_2 = \nabla_{\theta_1}\theta_2 + \frac{1}{2}\eta(\theta_2)\varphi\theta_1 - \frac{1}{2}\eta(\theta_1)\varphi\theta_2. \quad (3.1)$$

satisfying

$$\tilde{\mathcal{T}}(\theta_1, \theta_2) = \eta(\theta_2)\varphi\theta_1 - \eta(\theta_1)\varphi\theta_2, \quad (3.2)$$

and

$$(\tilde{\nabla}_{\theta_1}\tilde{g})(\theta_2, \theta_3) = \frac{1}{2}[\eta(\theta_2)\tilde{g}(\varphi\theta_1, \theta_3) + \eta(\theta_3)\tilde{g}(\varphi\theta_1, \theta_2)]. \quad (3.3)$$

Also, we have

$$(\tilde{\nabla}_{\theta_1}\varphi)\theta_2 = (\nabla_{\theta_1}\varphi)\theta_2 - \frac{1}{2}\eta(\theta_2)\theta_1 - \frac{1}{2}\eta(\theta_1)\eta(\theta_2)\xi. \quad (3.4)$$

$$\tilde{\nabla}_{\theta_1}\xi = \nabla_{\theta_1}\xi - \frac{1}{2}\varphi\theta_1. \quad (3.5)$$

Putting $\theta_1 = \xi$ in equation (3.5) and using (2.1), we have

$$\tilde{\nabla}_{\xi}\xi = 0. \quad (3.6)$$

Hence, we have the following theorem:

Theorem 3.1. In a LP-Kenmotsu manifold, the integral curves of the vector field ξ are geodesics with respect to the connection $\tilde{\nabla}$.

The curvature tensor $\tilde{\mathcal{R}}$ endowed with connection $\tilde{\nabla}$ is defined as

$$\tilde{\mathcal{R}}(\theta_1, \theta_2)\theta_3 = \tilde{\nabla}_{\theta_1}\tilde{\nabla}_{\theta_2}\theta_3 - \tilde{\nabla}_{\theta_2}\tilde{\nabla}_{\theta_1}\theta_3 - \tilde{\nabla}_{[\theta_1, \theta_2]}\theta_3. \quad (3.7)$$

Using (2.2), (2.3), (2.6), (2.9), (2.11) and (3.1) in (3.7), we get

$$\begin{aligned} \tilde{\mathcal{R}}(\theta_1, \theta_2)\theta_3 &= \tilde{\mathcal{R}}(\theta_1, \theta_2)\theta_3 + \frac{1}{2}[\tilde{g}(\theta_2, \theta_3)\varphi\theta_1 - \tilde{g}(\theta_1, \theta_3)\varphi\theta_2 + \eta(\theta_2)\tilde{g}(\varphi\theta_1, \theta_3)\xi \\ &\quad - \eta(\theta_1)\tilde{g}(\varphi\theta_2, \theta_3)\xi - \eta(\theta_2)\eta(\theta_3)\varphi\theta_1 - \eta(\theta_1)\eta(\theta_3)\varphi\theta_2 \\ &\quad + \frac{1}{4}[\eta(\theta_2)\eta(\theta_3)\theta_1 - \eta(\theta_1)\eta(\theta_3)\theta_2]]. \end{aligned} \quad (3.8)$$

Contracting (3.8) along θ_1 , we have

$$\tilde{\mathcal{S}}(\theta_2, \theta_3) = \tilde{\mathcal{S}}(\theta_2, \theta_3) + \frac{1}{2}\tilde{g}(\theta_1, \theta_3)\psi + \frac{1}{2}(n + \psi)\eta(\theta_2)\eta(\theta_3) \quad (3.9)$$

The above equation yields

$$\tilde{\mathcal{Q}}\theta_2 = \mathcal{Q}\theta_2 + \frac{1}{2}\psi(\theta_2) + \frac{1}{2}(n + \psi)\eta(\theta_2)\xi. \quad (3.10)$$

Again contracting equation (3.9), we get

$$\tilde{r} = r - \frac{n}{2}. \quad (3.11)$$

Replacing θ_1 by ξ in (3.8) and using (2.1), (2.3), (2.5) and (2.14), we have

$$\tilde{\mathcal{R}}(\theta_1, \theta_2)\xi = \frac{3}{4}\tilde{\mathcal{R}}(\theta_1, \theta_2)\xi. \quad (3.12)$$

Replacing θ_1 by ξ in (3.8) and using (2.1), (2.3), (2.5) and (2.9), we have

$$\tilde{\mathcal{R}}(\xi, \theta_2)\theta_3 = \overset{*}{g}(\theta_2, \theta_3)\xi + \frac{1}{2}\overset{*}{g}(\varphi\theta_2, \theta_3)\xi + \frac{1}{4}\eta(\theta_2)\eta(\theta_3)\xi - \frac{3}{4}\eta(\theta_3)\theta_2. \quad (3.13)$$

If the Riemannian curvature tensor equipped with $\tilde{\nabla}$ vanishes, then as a result of contraction (3.8) yields

$$\overset{*}{S}(\theta_2, \theta_3) = -\frac{1}{2}\overset{*}{g}(\theta_1, \theta_3)\psi - \frac{1}{2}(n + \psi)\eta(\theta_2)\eta(\theta_3). \quad (3.14)$$

Theorem 3.2. If $\tilde{\mathcal{R}}$ endowed with $\tilde{\nabla}$ vanishes in Lorentzian para-Kenmotsu manifold, then the manifold is η -Einstein manifold.

4. Ricci soliton in a LP-Kenmotsu manifold with $\tilde{\nabla}$

Ricci soliton $(\overset{*}{g}, \xi, \lambda)$ with respect to $\tilde{\nabla}$ [9] has given as

$$(\tilde{\mathcal{L}}_{\xi}\overset{*}{g})(\theta_1, \theta_2) + 2\tilde{S}(\theta_1, \theta_2) + 2\lambda\overset{*}{g}(\theta_1, \theta_2) = 0. \quad (4.1)$$

Now

$$(\tilde{\mathcal{L}}_{\xi}\overset{*}{g})(\theta_1, \theta_2) = \overset{*}{g}(\tilde{\nabla}_{\theta_1}\xi, \theta_2) + \overset{*}{g}(\theta_1, \tilde{\nabla}_{\theta_2}\xi) + 2\overset{*}{g}(\varphi\theta_1, \theta_2). \quad (4.2)$$

Using (3.5) in (4.2), we have

$$(\tilde{\mathcal{L}}_{\xi}\overset{*}{g})(\theta_1, \theta_2) = -2\overset{*}{g}(\theta_1, \theta_2) - 2\eta(\theta_1, \theta_2) - 2\eta(\theta_1)\eta(\theta_2) + \overset{*}{g}(\varphi\theta_1, \theta_2). \quad (4.3)$$

Using (3.9) and (4.3) in (4.1), we have

$$\overset{*}{S}(\theta_1, \theta_2) = -\frac{\psi+2\lambda-2}{2}\overset{*}{g}(\theta_1, \theta_2) - \frac{n-2+\psi}{2}\eta(\theta_1)\eta(\theta_2) - \frac{3}{2}\overset{*}{g}(\varphi\theta_1, \theta_2). \quad (4.4)$$

Hence, we have the theorem:

Theorem 4.1. If $(\overset{*}{g}, \xi, \lambda)$ is a Ricci soliton in LP-Kenmotsu manifold equipped with $\tilde{\nabla}$ then $\mathcal{M}$ is a generalized η -Einstein manifold.

Replacing θ_2 by ξ in (4.4) and using (2.1), (2.3) and (2.5), we have

$$\overset{*}{S}(\theta_1, \xi) = \frac{n-2\lambda}{2}\eta(\theta_1). \quad (4.5)$$

Using (2.11) in (4.5), we have

$$\lambda = -\frac{3}{2}n < 0. \quad (4.6)$$

Hence, we have the result:

Theorem 4.2. The Ricci soliton $(\overset{*}{g}, \xi, \lambda)$ in a Lorentzian para-Kenmotsu manifold admitting the connection $\tilde{\nabla}$ is always shrinking.

5. Ricci soliton on LP-Kenmotsu manifold endowed with $\tilde{\nabla}$ satisfying $\tilde{\mathcal{R}} \cdot \tilde{S} = 0$

Let us consider $\tilde{\mathcal{R}} \cdot \tilde{S} = 0$, then

$$\tilde{S}(\tilde{\mathcal{R}}(\theta_1, \theta_2)\theta_3, \theta_4) + \tilde{S}(\theta_3, \tilde{\mathcal{R}}(\theta_1, \theta_2)\theta_4) = 0. \quad (5.1)$$

Putting $\theta_1 = \xi$ in (5.1), we have

$$\tilde{S}(\tilde{\mathcal{R}}(\xi, \theta_2)\theta_3, \theta_4) + \tilde{S}(\theta_3, \tilde{\mathcal{R}}(\xi, \theta_2)\theta_4) = 0. \quad (5.2)$$

Using equation (2.11) and (2.9) in the above equation, we get

$$\begin{aligned} & \frac{3n}{2}\eta(\theta_4)\overset{*}{g}(\theta_2, \theta_3) + \frac{3n}{4}\eta(\theta_4)\overset{*}{g}(\varphi\theta_2, \theta_3) - \frac{3}{4}\eta(\theta_3)\tilde{S}(\theta_2, \theta_4) + \frac{3n}{2}\eta(\theta_3)\overset{*}{g}(\theta_2, \theta_4) \\ & + \frac{3n}{4}\eta(\theta_3)\overset{*}{g}(\varphi\theta_2, \theta_4) - \frac{3}{4}\eta(\theta_4)\tilde{S}(\theta_2, \theta_3) + \frac{3n}{4}\eta(\theta_2)\eta(\theta_3)\eta(\theta_4) = 0 \end{aligned} \quad (5.3)$$

Again putting $\theta_4 = \xi$ in (5.3) and using (2.1), (3.9), we obtain

$$\widetilde{\mathcal{S}}^*(\theta_2, \theta_4) = 2ng^*(\theta_2, \theta_3) + ng^*(\varphi\theta_2, \theta_3) + \frac{n}{2}\eta(\theta_2)\eta(\theta_3) \quad (5.4)$$

Hence, we have the following theorem:

Theorem 5.1. A manifold $\mathcal{M}$ endowed with $\widetilde{\nabla}$, which satisfy $\widetilde{\mathcal{R}} \cdot \widetilde{\mathcal{S}} = 0$, is an generalized η -Einstein Manifold.

6. Example of LP-Kenmotsu Manifolds

Example 6.1. We consider a 3-dimensional manifold $\mathcal{M} = [(x_1, x_2, x_3) \in \mathbb{R}^3]$, where (x_1, x_2, x_3) are the standard coordinates in $\mathbb{R}^3$. The vector fields

$$\varrho_1 = x_3 \frac{\partial}{\partial x_1}, \varrho_2 = x_3 \frac{\partial}{\partial x_2}, \varrho_3 = x_3 \frac{\partial}{\partial x_3} = \xi,$$

are linearly independent at every point of $\mathcal{M}$.

The Riemannian metric g^* is defined by

$$g^*(\varrho_1, \varrho_2) = g^*(\varrho_2, \varrho_3) = g^*(\varrho_3, \varrho_1) = 0, \quad (6.1)$$

$$g^*(\varrho_1, \varrho_1) = g^*(\varrho_2, \varrho_2) = g^*(\varrho_3, \varrho_3) = -1.$$

Suppose η be the 1-form defined as $\eta(\theta_1) = g^*(\theta_1, \xi)$. Let φ be the (1, 1)-tensor field defined by

$$\varphi(\varrho_1) = -\varrho_2, \varphi(\varrho_2) = -\varrho_1, \varphi(\varrho_3) = 0. \quad (6.2)$$

By linearity property of φ and g^* , we have

$$\eta(\varrho_3) = \eta(\xi), \varphi^2\theta_1 = -\theta_1 + \eta(\theta_1)\varrho_3,$$

$$g^*(\varphi\theta_1, \varphi\theta_2) = g^*(\theta_1, \theta_2) - \eta(\theta_1)\eta(\theta_2).$$

Consider ∇ be the Levi-Civita connection with Riemannian metric g^* , then we have

$$[\varrho_1, \varrho_2] = 0, [\varrho_1, \varrho_3] = -\varrho_1, [\varrho_2, \varrho_3] = -\varrho_2. \quad (6.3)$$

Koszul's formula is given as

$$\begin{aligned} 2g^*(\nabla_{\theta_1}\theta_2, \theta_3) &= \theta_1 g^*(\theta_2, \theta_3) + \theta_2 g^*(\theta_3, \theta_1) - \theta_3 g^*(\theta_1, \theta_2) - g^*([\theta_2, \theta_3], \theta_1) \\ &+ g^*([\theta_3, \theta_1], \theta_2) + g^*([\theta_1, \theta_2], \theta_3). \end{aligned} \quad (6.4)$$

By virtue of (6.4), we have

$$\nabla_{\varrho_1}\varrho_1 = -\varrho_3, \nabla_{\varrho_1}\varrho_2 = 0, \nabla_{\varrho_1}\varrho_3 = -\varrho_1,$$

$$\nabla_{\varrho_2}\varrho_1 = 0, \nabla_{\varrho_2}\varrho_2 = -\varrho_3, \nabla_{\varrho_2}\varrho_3 = -\varrho_2, \quad (6.5)$$

$$\nabla_{\varrho_3}\varrho_1 = 0, \nabla_{\varrho_3}\varrho_2 = 0, \nabla_{\varrho_3}\varrho_3 = 0,$$

Let us take a vector field $\theta_1 = \theta_1^1\varrho_1 + \theta_1^2\varrho_2 + \theta_1^3\varrho_3$ and the structure vector field $\xi = \varrho_3$, then we have

$$\begin{aligned} \nabla_{\theta_1}\xi &= \nabla_{\theta_1^1\varrho_1 + \theta_1^2\varrho_2 + \theta_1^3\varrho_3}\varrho_3 \\ &= \theta_1^1\nabla_{\varrho_1}\varrho_3 + \theta_1^2\nabla_{\varrho_2}\varrho_3 + \theta_1^3\nabla_{\varrho_3}\varrho_3 \\ &= -\theta_1^1\varrho_2 + \theta_1^2\varrho_1. \end{aligned} \quad (6.6)$$

and

$$\begin{aligned}
\nabla_{\theta_1} \xi &= -\varphi \theta_1 \\
&= -\varphi(\theta_1^1 \varrho_1 + \theta_1^2 \varrho_2 + \theta_1^3 \varrho_3) \\
&= -[\theta_1^1(\varphi \varrho_1) + \theta_1^2(\varphi \varrho_2) + \theta_1^3(\varphi \varrho_3)] \\
&= -\theta_1^1 \varrho_2 + \theta_1^2 \varrho_1.
\end{aligned} \tag{6.7}$$

where $\theta_1^1, \theta_1^2, \theta_1^3$ are the scalars. From (6.6) and (6.7), the structure $(\varphi, \xi, \eta, g^*)$ is Lorentzian para structure. Hence the manifold $\mathcal{M}(\varphi, \xi, \eta, g^*)$ is Lorentzian para Kenmotsu manifold. In reference of (2.1), (2.3), (3.1), (6.2) and (6.5), we have

$$\begin{aligned}
\tilde{\nabla}_{\varrho_1} \varrho_1 &= -\varrho_3, \tilde{\nabla}_{\varrho_1} \varrho_2 = 0, \tilde{\nabla}_{\varrho_1} \varrho_3 = -\varrho_3 + \frac{1}{2} \varrho_2, \\
\tilde{\nabla}_{\varrho_2} \varrho_1 &= 0, \tilde{\nabla}_{\varrho_2} \varrho_2 = -\varrho_3, \tilde{\nabla}_{\varrho_2} \varrho_3 = -\varrho_2 + \frac{1}{2} \varrho_1, \\
\tilde{\nabla}_{\varrho_3} \varrho_1 &= -\frac{1}{2} \varrho_2, \tilde{\nabla}_{\varrho_3} \varrho_2 = -\frac{1}{2} \varrho_1, \tilde{\nabla}_{\varrho_3} \varrho_3 = 0,
\end{aligned} \tag{6.8}$$

From equations (3.2) and (3.3), we have

$$\tilde{T}(\varrho_1, \varrho_3) = \eta(\varrho_3) \varphi \varrho_1 - \eta(\varrho_1) \varphi \varrho_3 = \varrho_3 \neq 0$$

and

$$(\tilde{\nabla}_{\varrho_1} g^*)(\varrho_2, \varrho_3) = -\frac{1}{2} [\eta(\varrho_2) g^*(\varphi \varrho_1, \varrho_3) + \eta(\varrho_3) g^*(\varphi \varrho_1, \varrho_2)] = -\frac{1}{2} \neq 0$$

Consequently, a QSNM connection defined in (3.1) is verified. Also

$$\begin{aligned}
\tilde{\nabla}_{\theta_1} \xi &= \tilde{\nabla}_{\theta_1^1 \varrho_1 + \theta_1^2 \varrho_2 + \theta_1^3 \varrho_3} \xi \\
&= \theta_1^1 \tilde{\nabla}_{\varrho_1} \xi + \theta_1^2 \tilde{\nabla}_{\varrho_2} \xi + \theta_1^3 \tilde{\nabla}_{\varrho_3} \xi \\
&= -\frac{1}{2} \theta_1^1 \varrho_2 - \frac{1}{2} \theta_1^2 \varrho_1.
\end{aligned} \tag{6.9}$$

The equation (3.5) can be verified in light of equations (6.7) and (6.9).

$$\begin{aligned}
{}^* \mathcal{R}(\varrho_1, \varrho_2) \varrho_1 &= -\varrho_2, {}^* \mathcal{R}(\varrho_1, \varrho_3) \varrho_1 = -\varrho_3, {}^* \mathcal{R}(\varrho_2, \varrho_3) \varrho_1 = 0, \\
{}^* \mathcal{R}(\varrho_1, \varrho_2) \varrho_2 &= -\varrho_1, {}^* \mathcal{R}(\varrho_1, \varrho_3) \varrho_2 = 0, {}^* \mathcal{R}(\varrho_2, \varrho_3) \varrho_2 = -\varrho_3, \\
{}^* \mathcal{R}(\varrho_1, \varrho_2) \varrho_3 &= 0, {}^* \mathcal{R}(\varrho_1, \varrho_3) \varrho_3 = -\varrho_1, {}^* \mathcal{R}(\varrho_2, \varrho_3) \varrho_3 = -\varrho_2,
\end{aligned} \tag{6.10}$$

Hence we can satisfy equations (2.5), (2.6) and (2.7).

Similarly, the component of curvature tensor $\tilde{\mathcal{R}}$ of connection $\tilde{\nabla}$ can be written as:

$$\begin{aligned}
\tilde{\mathcal{R}}(\varrho_1, \varrho_2) \varrho_1 &= \frac{1}{2} \varrho_1 - \varrho_2, \tilde{\mathcal{R}}(\varrho_1, \varrho_3) \varrho_1 = -\varrho_3, \tilde{\mathcal{R}}(\varrho_2, \varrho_3) \varrho_1 = \frac{1}{2} \varrho_3, \\
\tilde{\mathcal{R}}(\varrho_1, \varrho_2) \varrho_2 &= \frac{1}{2} \varrho_2 - \varrho_3, \tilde{\mathcal{R}}(\varrho_1, \varrho_3) \varrho_2 = -\frac{1}{2} \varrho_3, \tilde{\mathcal{R}}(\varrho_2, \varrho_3) \varrho_2 = -\varrho_3, \\
\tilde{\mathcal{R}}(\varrho_1, \varrho_2) \varrho_3 &= \varrho_2 - \frac{1}{2} \varrho_1, \tilde{\mathcal{R}}(\varrho_1, \varrho_3) \varrho_3 = \frac{1}{4} \varrho_1, \tilde{\mathcal{R}}(\varrho_2, \varrho_3) \varrho_3 = -\frac{1}{2} (\varrho_1 + \varrho_2) - \varrho_3,
\end{aligned} \tag{6.11}$$

Thus (3.8), (3.12) and (3.13) can be verified easily.

7. Conclusion

In this paper, we investigated the geometric properties of LP-Kenmotsu manifolds admitting a quarter-symmetric non-metric connection. Several curvature relations corresponding to this connection were established, and it was shown that the integral curves of the characteristic vector field are geodesics with respect to the quarter-symmetric non-metric connection. The

relationship between the curvature tensors and Ricci tensors of the Levi-Civita connection and the quarter-symmetric non-metric connection was also derived.

Further, we studied Ricci solitons on LP-Kenmotsu manifolds endowed with the quarter-symmetric non-metric connection. It was proved that such Ricci solitons characterize the manifold as a generalized η -Einstein manifold and are necessarily shrinking. Moreover, under the curvature condition $\tilde{R} \cdot \tilde{S} = 0$, the manifold was again shown to be a generalized η -Einstein manifold. Finally, a three-dimensional example was constructed to verify the theoretical results obtained throughout the paper.

These results enrich the study of LP-Kenmotsu geometry with quarter-symmetric non-metric connections and provide a foundation for further investigations on Ricci solitons, curvature conditions, and related geometric structures on more general almost paracontact manifolds.

8. Future Scope

The present work opens several directions for further research. Similar investigations may be carried out on other classes of almost paracontact metric manifolds, such as para-Sasakian, trans-Sasakian, nearly Kenmotsu, and generalized Kenmotsu manifolds endowed with quarter-symmetric non-metric or other non-Riemannian connections. It would also be worthwhile to study gradient Ricci solitons, η -Ricci solitons, and almost Ricci solitons under various curvature restrictions, including pseudosymmetry, semisymmetry, and recurrent curvature conditions. Moreover, the behaviour of conformal, concircular, projective, conharmonic, and quasi-conformal curvature tensors with respect to quarter-symmetric non-metric connections deserves further investigation. Another promising direction is the extension of these results to higher-dimensional manifolds and their applications in Lorentzian geometry, geometric flows, and mathematical physics.

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