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James Wamwenge

Department of Mathematics,
University of Nairobi, Kenya

SK Moindi

Department of Mathematics,
University of Nairobi, Kenya

W_4 Curvature tensors admitting to L.P Kenmotsu manifold

James Wamwenge and SK Moindi

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Abstract

This paper provides a comprehensive analysis of W_4 curvature tensor on l.p kenmotsu manifold provision of fundamental structure equation, formulate vigorous theorems and proofs regarding, symmetric flatness semi-symmetric and special η -Einstein and Einstein manifold under the behavior of the W_4 curvature tensor field.

Keywords: W_4 Curvature tensor, special η -Einstein, symmetric flatness, semi-symmetric, Einstein manifold

Introduction

The study of contact para contact structures and their lorentzian relation in the modern differential geometry plays a vital role in both general relativity and mathematics. FN Mburu and PW Njori developed papers on l.p kenmotsu manifold studying their properties [1-5].

Preliminaries

Let M^n be an n - dimensional differentiable manifolds equipped with l.p contact metric. (ϕ, ξ, η, g) [6].

$$\phi^2 x = x + \eta(x)\xi$$

$$\eta(\xi) = -1$$

$$\phi\xi = 0$$

$$\eta(\phi x) = 0$$

$$g(\phi x, \phi y) = g(x, y) + \eta(x)\eta(y)$$

Additionally in lp kenmotsu manifold

$$(\nabla_x \phi)y = g(\phi x, y)\xi + \eta(y)\phi x$$

$$(\nabla_x \eta)y = g(Y, y)\xi + \eta(Y)\eta(y)$$

From the set up structure above R (Riemannian curvature tensor) S (Ricci tensor) Q (Ricci operator)

$$g(Qx, y) = S(xy)$$

The following properties holds [7, 8].

Corresponding Author:

James Wamwenge

Department of Mathematics,
University of Nairobi, Kenya

$$R(\varepsilon Y)y = g(Yy)\varepsilon - \eta(y)Y$$

$$R(Yy)\varepsilon = \eta(y)Y - \eta(x)y, y$$

$$\eta(R(xy)z) = g(xz)\eta(y) - g(yz)\eta(x)$$

$$s(Y, \varepsilon) = (n-1)\eta(x)$$

$$Q\varepsilon = (n-1)\varepsilon$$

$$S(\varepsilon\varepsilon) = -n+1$$

The W_4 –Curvature tensor

From the definition Pokkari w_4 curvature tensor is defined by ^[9].

$$w_4(x, y)z = R(Yy)z + \frac{1}{n+1} [g(Yz)Qy - g(Yy)Qz]$$

The equivalent (0 4)-curvature tensor field expression is given by

$$w_4(x, Y, z, u) = g(w_4(x, y)z, u)$$

Also

$$w_4(Yy, z, u) = R(x, y, z, u) + \frac{1}{n-1} [g(Yy, z)s(Yu) - g(y, Y)s(z, u)]$$

By setting the key parameter (Z) to ε the following structure

LEMMA: 1 In a lorentzian para kenmotsu manifold w_4 satisfies the following condition

- $w_4(x, y)\varepsilon = \eta(y)x + \eta(x)y + \frac{1}{n-1}\eta(x)Q$ ^[10].
- $\eta w_4(x, y)z = 2[g(Y, z)\eta(y) - g(Y, y)\eta(z)]$

W_4 Flatness

In a lorentzian para kenmotsu manifold w_4 curvature tensor vanishes on flatness that is

$$w_4(x, y)z = 0 \quad \forall \text{ vector field.}$$

Theorem

An n dimensional lorentzian para kenmotsu manifold is w_4 flat if it is Einstein manifold of a special scalar distribution and forcing it to be negative curvature.

Proof

Consider $w_4(x, y)z = 0$ therefore

$$R(x, y)z + \frac{1}{n-1} [g(y)(xz)Qy - g(xy)Qz] = 0$$

Taking the inner product w.r.t k

$$R(x, y, z, k) = \frac{1}{n-1} [g(x, y)s(z, k) - g(x, z)s(y, k)]$$

$$\text{Let } x = k = \varepsilon$$

$$R(\varepsilon, y, z, \varepsilon) = \frac{1}{n-1} [g(\varepsilon, y)s(z, \varepsilon) - g(\varepsilon, z)s(y, \varepsilon)]$$

Under properties

$$g(R(\varepsilon, y)z) = R(\varepsilon, y, z, \varepsilon)$$

$$\Rightarrow g(g(y, z)\varepsilon - \eta(z)y)$$

Thus

$$-g(yz)-\eta(z)\eta(y)=\frac{1}{n-1}[\eta(y)(n-1)\eta(z)-g(\varepsilon z)s(y\varepsilon)]$$

$$\Rightarrow \eta(y)\eta(z)-\eta(z)\eta(y)=0$$

This gives an algebraic contraction indicating $s(y,z)$ should align proportionally from orthonormal from $\{e_i\}$ we have

$$R(e_i, y, z, e_i) + \frac{1}{n-1} \sum_{i=1}^n [g(e_i, z)s(y, e_i) - g(e_i, y)s(z, e_i)] = 0$$

$$\frac{s(yz)Q + s(yz) - s(yz)}{Q} = 0$$

$$= s(yz) = 0$$

Considering $s(yz)=0$ then $Qy=0$

Therefore $R(x,y)z=0$

Hence a completely w_4 flat lorentzian para kenmotsu manifold cannot exist unless its dimensional drop.

W_4 Symmetric L.p kenmotsu manifold

An lp kenmotsu manifolds is referred W_4 symmetric if its W_4 curvarture tensor is parallel within covariant differentiation that is $\nabla_u w_4 = 0$.

Theorem: W_4 symmetric lorentzian para kenmotsu manifold is an Einstien manifold

Proof

Let $\nabla_u w_4 = 0$. This indicate that the scalar values given by contraction are invariant.

consider $\nabla_u w_4(x,y)\varepsilon = 0$

from

$$\nabla_u w_4(x,y)\varepsilon = (\nabla_u w_4)(x,y)\varepsilon + w_4(\nabla_u x, y)\varepsilon + w_4(x, \nabla_u y)\varepsilon + w_4(x,y)\nabla_u \varepsilon$$

Let $\nabla_u = R$ and by inducing a local coordinates system where $Rx = Ry = 0$ at a given point, and using $R\varepsilon = \nabla_u \Rightarrow \nabla_u \varepsilon = u + \eta(u)\varepsilon$ Therefore

$$(Rw_4)(x,y)\varepsilon = R(w_4(x,y)\varepsilon - w_4(x,y)u + \eta(u)\varepsilon$$

Let $\nabla_u w_4 = R = 0$

Then

$$\nabla_u (w_4(x,y)\varepsilon) = w_4(x,y)u + \eta(u)w_4(x,y)\varepsilon$$

An inner product with ε and through properties $Rs=0$, Taking the Ricci tensor to be c a steady multiple of the metric tensor field g .
hence satisfies $s(x,y) = \lambda g(x,y)$ Thus an Einstein manifolds

W_4 - Curvarture tensor on semi-symmetric manifolds

A lorentzian Para Kenmotsu manifold is refereed to as w_4 - semi symmetric when the curvature operator $R(x,y)$ acts as a derivative on the w_4 curvarture tensor. That is

$$R(x,y). w_4(z,u)v = 0$$

By expansion ^[11].

$$R(x,y)(w_4(z,u)v) - w_4(R(x,y)z, u)v - w_4(z, R(x,y)u)v - w_4(z,u)R(x,y)v = 0$$

Theorem

A w_4 -semi symmetric lorentzian para kenmotsu manifold is an η -Einstein manifolds

Proof

Let $Y = \varepsilon$

$$R(\varepsilon, y)(w_4(z,u)v - w_4(\varepsilon, y)z, u)v - w_4(z, R(\varepsilon, y)u)v - w_4(z,u)R(\varepsilon, y)v = 0$$

By $R(\varepsilon y)z = g(yz)\varepsilon - \eta(z)y$

Then

- $[g(y, w_4(z, u)v)\varepsilon - \eta(w_4(zu)v)y]$
- $[g(yz)w_4(\varepsilon u)v - \eta(z)w_4(yu)v]$
- $[g(yu)w_4(z, \varepsilon)v - \eta(u)w_4(zv)v]$
- $[g(yv)w_4(z, u)\varepsilon - \eta(v)w_4(zv)y] = 0$

By the inner product of the equation with ε . Also $g(\varepsilon, \varepsilon) = -1$,

$$\eta(w_4(\dots))\eta = g(w_4(\dots)\varepsilon),$$

The multiple terms vanishes let $z = \varepsilon$ then $s(uv)$ gives a combination of $ag(u, v)$ and $b\eta(u)\eta(v)$

That is $s(uv) = ag(u, v) + b\eta(u)\eta(v)$

This fulfills an η -Einstein manifolds

Characterization of einstein and η -einstein manifolds

For Ricci tensor (s) an l.p kenmotsu manifolds is refereed to as η -Einstein manifolds if

$$s(x, y) = ag(xy) + b\eta(x)\eta(y) \quad [12]$$

For a and b are smooth function on the manifold when $b=0$ its manifold becomes to Einstein manifold

Theorem

If on n - dimensional l.p kenmotsu manifold $w_4(xy)z = R(xy)z$ The manifold is Ricci flat

That is $s=0$

Proof

Consider the condition $w_4(xy)z = R(xy)z$

Then definition of w_4

$$R(x, y)z + \frac{1}{n-1} [g(xz)Qy - g(Y, X)Qz] = w_4(x, y)z = R(x, y)z$$

Then

$$\frac{1}{n-1} [g(xz)Qy - g(xy)Qz] = 0$$

For $n \geq 3$,

$$g(xz)Qy - g(xy)Qz = 0$$

Setting $x=z$ for x is a unit vector orthogonal to

$$y(g(xy)) = 0, \quad g(xx) = 1$$

Therefore $Qy - 0(Qz) = 0$

Thus $Qy = 0$

Q is the Ricci operator

That is $s(x, y)$

Therefore $s(xy) = 0 \quad \forall$ vector fields hence proven

W_4 Curvatures tensor on special η -einstein manifolds

A special η -Einstein manifold enhances geometric condition for a and b constrained generally by the dimension constraints of the systems.

Theorem

Let M be an η -Einstein lorentzian para kenmotsu manifolds if w_4 - curvature tensor fulfills the condition

$\eta(w_4(Yy)z) = 0$ then M manifold is of special η -Einstein manifold type where $a = -n+1$ and $b=0$, confirming its Einstein form

Proof

$$\eta(w_4(xy)z) = \frac{1}{n-1} [g(xz)s(y\varepsilon) - g(xy)s(z\varepsilon)] + \eta(R(Yy)z)$$

From $s(x\varepsilon) = (n-1)\eta(x)$

$$\eta(R(xy)z=g(xz)\eta(y)-g(Yz)\eta(x))$$

Therefore

$$\begin{aligned}\eta(w_4(xy)z) &= \frac{1}{n-1} [g(xz)(n-1)\eta(y)-g(xy)\eta(z)] + g(xz)\eta(y)-g(yz)\eta(x) \\ &\Rightarrow 2[g(xz)\eta(y)-g(xy)\eta(z)]\end{aligned}$$

From the condition $\eta(w_4(xy)z)=0$

Then

$$2[g(xz)\eta(y)-g(xy)\eta(z)]$$

$$\text{Let } y=\varepsilon$$

$$g(xz)(-1)-g(x\varepsilon)\eta(z)=0$$

$$\text{Thus } -g(xz)-\eta(x)\eta(z)=0$$

$$g(xz)=-\eta(x)\eta(z)$$

This is a degenerate state for lorentzian metric framework unless restricted its distribution.

$W_4.W_4$ Curvature tensors a lp kenmotsu manifold

Consider $w_4.w_4=0$

$$w_4(xy).w_4(zu)v=0$$

Expanding

$$= w_4(xy)w_4(zu)v-w_4(w_4(xy)zu)v-w_4(z,w_4(xy)u)v-w_4(zu)w_4(xy)v=0$$

Theorem

An n - dimensional lp kenmotsu manifold fulfilling the condition $w_4.w_4=0$ is w_4 - flat.

Proof

Let $x=\varepsilon$ then

$$\begin{aligned}w_4(\varepsilon y)z &= R(\varepsilon y)z + \frac{1}{n-1} [g(\varepsilon z)Qy-g(\varepsilon y)Qz] \\ &\Rightarrow g(yz)\varepsilon-\eta(z)y + \frac{1}{n-1} [\eta(z)Qy-\eta(y)Qz]\end{aligned}$$

Taking inner product with ε

$$(n-1)||w_4(zu)v||^2=0$$

Therefore $w_4(zu)v=0$

Thus collapsing the tensor field while leading the manifold locally w_4 - flat.

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