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On dual Soft compact space

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Abstract

This work focuses on the definition and study of Dual Soft Sets and Dual Soft Neighborhood are employed to establish an original class of compact spaces within the framework of dual soft topology. The fundamental properties of these spaces are investigated, and their relationships with other related topological concepts are analyzed.

Keywords: Dual soft topology dual soft compact space, dual soft locally compact, dual soft neighbourhood

1. Introduction

Compactness is one of the central and most influential concepts in general topology, playing a fundamental role in the structural analysis of topological spaces and their associated properties. Owing to its theoretical significance, numerous generalizations and variations of compactness both weaker and stronger have been developed to broaden the scope of topological investigations and to deepen the understanding of relationships among different classes of spaces ^[1, 5].

Among these generalizations is H-closedness, a notion originating from the seminal work of Alexandrov and Urysohn in 1929 ^[4], which laid the groundwork for the study of spaces exhibiting closure properties beyond classical compactness. In the continued search for weaker forms of compactness. Nearly Compact spaces were presented in 1969 by M. K. Sinegal and Ashea Mathur ^[5], providing an alternative framework for analyzing spaces that do not satisfy compactness in its traditional sense. Subsequently, in 1976, Travis Thompson ^[6] proposed the concept of S-compact spaces, which has since attracted considerable attention in the topological literature.

Over time, various other forms of compactness have been explored, each contributing to a richer hierarchical structure within topology. Motivated by this ongoing development, the present work introduces and investigates a new class of topological spaces that represents a weaker form of compactness.

1.1 Objective of the Study

This study aims to investigate the fundamental properties of this new notion, establish its relationships with existing compact-type spaces, and illustrate the theory through relevant examples and supporting results. This contribution aims to extend the current understanding of compactness-related concepts and to provide a foundation for further research in this direction.

2. Preliminaries

Definition 2.1^[3]: Let V_1 and V_2 are initial universes sets and O be the set of all potential boundaries under consideration as for V_1 and V_2 . Whereas parameters are descriptions, features or properties of members of the initials universes sets.

The triple (A_D, W, G_D) is dual soft set over V_1 and V_2 where, W, G_D are functions from $\tilde{O}$ to power of V_1 and V_2 respectively, so

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$$(A_D, W, G_D) = \{(r, W(k), G(t)); \forall r \in A_D\} \cup \{(r, \emptyset, \emptyset); \forall r \in \tilde{O} - A_D\}.$$

The set consisting of all dual soft sets constitutes a dual soft space, written as.

$$Ds(V_1, V_2) \tilde{O}$$

Definition 2.2 ^[3]: A specified family of StE of $DS_{(U_1, U_2)_E}$ is termed Dual soft topology StT_E on X_{Ds} if satisfy. $\emptyset_{Ds}, X_{Ds} \in StT_E$.

1. If $DG_A, f_1 G_{1B} \in StT_E$ then $DG_A \cap D_1 G_{1B} \in StT_E$.
2. For any index Λ if $D_i G_{iA_i} \in StT_E$, then $\cup_{i \in \Lambda} D_i G_{iA_i} \in StT_E$.

So $StT_E = \{\emptyset_{Ds}, X_{Ds}, FG_A\}$ is Dual soft topology and (X_{Ds}, StT_E) is Dual soft topological space.

Definition 2.3: ^[1] Let (X_{Ds}, StT_E) be a Dual soft topological space over X_{Ds} . A Dual soft set (A_D, W, G) over X_{Ds} is said to be a Dual Soft_{Close} set in X_{Ds} , if its relative complement $(A_D, W, G)^c$ belong to StT_E .

Definition 2.4: ^[1] Let (X_{Ds}, StT_E) be a dual Soft topological space done X_{Ds} . A dual Soft set FG_B ended X_{Ds} . $e_{xy} \in X_{Ds}$. Then FG_B is said to be Dual soft neighborhood of X_{Ds} , if there exists a dual Soft_{open} set FG_A such that $e_{xy} \in FG_A \subset FG_B$.

in other words FG_A is Dual Soft neighborhood of e_{xy} if and only if there exist $FG_B \in StT_E$ such that $e_{xy} \in FG_A \subseteq FG_B$.

If FG_A is a Dual soft_{open} set, then FG_A is said to be Dual Soft neighborhood of e_{xy}

i.e. FG_B is a Dual soft_{open} set for e_{xy} if and only if $e_{xy} \in FG_B \in StT_E$

Remark 2.5

1. Each $e_{xy} \in X_{Ds}$ has adual soft neighborhood
2. If FG_A, FG_B are Dual soft neighborhood the $FG_A \cap FG_B$ also adual soft neighborhood
3. If FG_A is a dual soft neighborhood and $FG_A \in FG_B$. then FG_B also adual soft neighborhood of e_{xy}
4. If FG_A is a dual soft neighborhood of e_{xy} then there exist a set FG_B such that FG_B also dual soft neighborhood of e_{xy} and FG_A is a dual soft neighborhood of each of it point of FG_B

Proof (2)

FG_A and FG_B are dual soft neighborhood of e_{xy} by definition $e_{xy} \in FG_U \subseteq FG_A$ and $e_{xy} \in FG_V \subseteq FG_B$ Since $e_{xy} \in FG_U$ and $e_{xy} \in FG_V$ then $e_{xy} \in FG_U \cap FG_V$ and $\in FG_U \subset FG_A, FG_V \subset FG_B$ so

$FG_U \cap FG_V \subseteq FG_A \cap FG_B$ Since FG_U and FG_V are dual soft open so their intersection

$FG_U \cap FG_V$ is also dual soft open, $e_{xy} \in FG_U \cap FG_V \subseteq FG_A \cap FG_B$ thus $FG_A \cap FG_B$ is a dual set dual soft neighborhood.

Proof (3): Since FG_A is dual soft neighborhood of, e_{xy} by definition $e_{xy} \in FG_U \subseteq FG_A$ but $FG_A \subseteq FG_B$ so $e_{xy} \in FG_U \cap FG_B$ then FG_B is also dual soft neighborhood of $e_{xy} \in X_{Ds}$.

Proof (4): Since FG_A is a Dual Soft Neighborhood of e_{xy} by definition $e_{xy} \in FG_B \subseteq FG_A$, Since e_{xy} is a point of FG_B and

FG_B is dual soft open according to theorem I, FG_B a dual soft Neighborhood of X_{Ds} if $e_{zv} e_{xy} \in FG_B$ because FG_B is a dual soft open again according to Theorem I, FG_B is a dual soft neighborhood FG_B

Remark 2.6: In any a a Dual Soft topological space (X_{Ds}, StT_E)

1. X_{Ds} is an dual Soft neighborhood for each $e_{xy} \in X_{Ds}$
2. Every Dual Soft_{open} set is an dual Soft open neighborhood for each element init.
3. If (X_{Ds}, StT_E) be a dual soft topological space then a Dual Soft_{open} of X_{Ds} if and only if FG_A is a dual soft neighborhood of each of it points.

Definition 2.7: Let (X_{Ds}, StT_E) be a dual Soft topological space and $e_{xy} \in X_{Ds}$ the dual soft neighborhood filter of e_{xy} denoted by $\eta_{Ds(e_{xy})} X_{Ds}$ is the collection of all dual soft neighborhood of e_{xy} .

$$\eta_{Ds(e_{xy})} X_{Ds} = \{FG_A \subseteq X_{Ds} \mid \exists FG_U \in StT_E \text{ such that } e_{xy} \in FG_U \subseteq FG_A\}$$

i.e.:- Let (X_{Ds}, StT_E) be a dual Soft topological space over X_{Ds} and $e_{xy} \in X_{Ds}$

- A dual soft neighborhood of e_{xy} is any set $FG_U \subseteq X_{Ds}$ that contains an dual Soft_{open} set containing e_{xy} .
- The a dual soft neighborhood at e_{xy} denoted $\eta_{Ds(e_{xy})} X_{Ds}$

This is filter called the dual soft neighborhood filter at e_{xy} because

$$X_{Ds} \in \eta_{Ds(e_{xy})} X_{Ds}$$

1. If, $FG_B, FG_A \in \eta_{Ds(e_{xy})} X_{Ds}$ then $FG_B \cup FG_A \in \eta_{Ds(e_{xy})} X_{Ds}$
2. If, $FG_U \in \eta_{Ds(e_{xy})} X_{Ds}$ and, $FG_U \subseteq FG_W$ then $FG_W \in \eta_{Ds(e_{xy})} X_{Ds}$

3. dual Soft_{compact} Space

Definition 3.1: Let (X_{Ds}, StT_E) be a Dual soft topological space over X_{Ds} .

An Dual Soft_{open} cover of X_{Ds} is collection of dual Soft_{open} $\{U_\theta\}_{\theta \in FG_A}$

Such that

1. Each U_θ is dual Soft_{open} in X_{Ds}

$$X_{Ds} \subseteq \{U_\theta\}_{\theta \in FG_A}$$

2. $X_{Ds} = \cup_{\theta \in FG_A} U_\theta$

Definition 3.2: Let (X_{Ds}, StT_E) be a Dual soft topological space and $FG_A \subseteq X_{Ds}$ he set FG_A is dual Soft_{compact} if every dual Soft_{open} cover of FG_A has a finitedual Soft_{open} cover .

Definition 3.3: Let X_{Ds} be a Dual soft topological space. Most commonly X_{Ds} is called dual Soft_{locally compact} if every point e_{xy} of X_{Ds} has dual Soft_{compact} .

Theorem 3.4: Every Dual Soft_{compact} is Dual Soft_{locally compact}

Proof: Let X_{Ds} be dual Soft_{compact} suppose $\{FG_{\alpha} : \alpha \in II\}$ be an dual Soft_{open} cover of X_{Ds} because X_{Ds} is dual Soft_{compact} there exist a finite dual Soft_{open} cover $\{FG_{\alpha_i} : i = 1, 2, 3 \dots \dots n\}$ such that $X_{Ds} = \cup FG_{\alpha_i}$ we have to

prove X_{D_S} is a $\text{Soft}_{\text{locally compact}}$, for this let $e_{xy} \in X_{D_S}$ then $X_{D_S} = \bigcup F_{GA_{\alpha i}}$ we have $e_{xy} \in \bigcup F_{GA_{\alpha i}}$, $e_{xy} \in F_{GA_{\alpha i}}$ for some $i = 1, 2, 3, \dots, n$ such that $X_{D_S} = \bigcup F_{GA_{\alpha i}}$ since $e_{xy} \in F_{GA_{\alpha i}} \subseteq F_{GA_{\alpha i}}^-$ this $F_{GA_{\alpha i}}^-$ is closed dual Soft set in X_{D_S} there for $F_{GA_{\alpha i}}^-$ is dual $\text{Soft}_{\text{compact}}$ then $F_{GA_{\alpha i}}^-$ is $\text{Soft}_{\text{locally compact}}$

Remark 3.5

1. Every point of X_{D_S} has a dual $\text{Soft}_{\text{compact}}$ dual soft neighborhood.
2. Every point of X has closed dual Soft set dual $\text{Soft}_{\text{compact}}$ dual soft neighborhood

Lemma 3.6. Let X_{D_S} be a $\text{Soft}_{\text{locally compact}}$, let F_{GK} be dual $\text{Soft}_{\text{compact}}$ in X_{D_S} , and let F_{GD} be an open subset, with $F_{GK} \subset F_{GD}$. Then there exists an open set F_{GE} with:

- (i) F_{GE} be dual $\text{Soft}_{\text{compact}}$

Definition 3.7^[1]: Let (X_{D_S}, StT_E) be a Dual Soft topological space over X_{D_S} . A Dual Soft set (A_D, W, G) over X_{D_S} is said to be a Dual $\text{Soft}_{\text{Closed}}$ set in X_{D_S} , if its comparative complement $(A_D, W, G)^c$ belong to StT_E .

Theorem 3.8

- (i) Every Dual $\text{Soft}_{\text{Closed}}$ subset of Dual $\text{Soft}_{\text{Compact}}$ space is Dual $\text{Soft}_{\text{Compact}}$.
- (ii) The intersection of dual $\text{Soft}_{\text{compact}}$ subset with dual $\text{Soft}_{\text{closed}}$ sub set is Dual $\text{Soft}_{\text{Compact}}$ act.

(v) Every finite subset is Dual $\text{Soft}_{\text{Compact}}$

Proof i. let F_{GA} be a dual $F_{GA} \text{Soft}_{\text{closed}}$ subset of dual $\text{Soft}_{\text{compact}}$ X_{D_S} . Let $\{U_\theta\}_{\theta \in F_{GA}}$ be a dual $\text{Soft}_{\text{open}}$ cover of F_{GA} that is, $F_{GA} \subset \bigcup U_{\theta \in F_{GA}}$. But $X_{D_S} - F_{GA}$ is dual $\text{Soft}_{\text{open}}$ so $X_{D_S} = (X_{D_S} - F_{GA}) \cup (\bigcup U_{\theta \in F_{GA}})$. Since X_{D_S} is dual $\text{Soft}_{\text{compact}}$ then $X_{D_S} = (X_{D_S} - F_{GA}) \cup (\bigcup U_{\theta \in F_{GA}})$. and $F_{GA} \subset \bigcup U_{\theta \in F_{GA}}$. Therefore F_{GA} is dual $\text{Soft}_{\text{compact}}$.

Theorem:-3.9 Let Y_{D_S} be Dual $\text{Soft}_{\text{Open}}$ set of a dual Soft topological space X_{D_S} and $F_{GK} \subseteq Y_{D_S}$ then F_{GK} is Dual $\text{Soft}_{\text{Compact}}$ set in Y_{D_S} if and only if F_{GK} is dual $\text{Soft}_{\text{compact}}$ set in X_{D_S}

Definition 3.10:- A function $f_{D_S}: (X_{D_S}, \text{StT}_E) \rightarrow (Y_{D_S}, \text{StT}'_E)$ is said to be dual $\text{Soft}_{\text{compact}}$ if, $f_{D_S}^{-1}(HK_A)$ is Dual $\text{Soft}_{\text{compact}}$ subset in X_{D_S} , whenever (HK_A) is dual $\text{Soft}_{\text{compact}}$ subset in Y_{D_S} .

Definition 3.11:- A function $f_{D_S}: (X_{D_S}, \text{StT}_E) \rightarrow (Y_{D_S}, \text{StT}'_E)$ is said to be point inversely dual $\text{Soft}_{\text{compact}}$ if, $f_{D_S}^{-1}(Y_{D_S})$ is dual $\text{Soft}_{\text{compact}}$ subset in X_{D_S} for every $e_{xy} \in Y_{D_S}$.

Definition 3.12 [1]: Let $(X_{D_S}, \text{StT}_E, \Pi_{(U_1, U_2)})$ and $(Y_{D_S}, \text{StT}'_E, \Pi_{(U_1, U_2)})$ be two dual Soft topological spaces, $f_{D_S}: (X_{D_S}, \text{StT}_E) \rightarrow (Y_{D_S}, \text{StT}'_E)$ is a mapping.

- a) If the image $f_{D_S}(F_{GA})$ of any Dual $\text{Soft}_{\text{Open}}$ set F_{GA} over X_{D_S} is a Dual $\text{Soft}_{\text{Open}}$ set in Y_{D_S} , then f_{D_S} is called to be a dual $\text{Soft}_{\text{open}}$ mapping.

If the image $f_{D_S}(F_{GB})$ of any dual $\text{Soft}_{\text{closed}}$ set F_{GB} over X_{D_S} is a Dual $\text{Soft}_{\text{Closed}}$ set in Y_{D_S} , then f_{D_S} is said to be a Dual $\text{Soft}_{\text{closed}}$ mapping

Theorem 3.13: Let $f_{D_S}: (X_{D_S}, \text{StT}_E) \rightarrow (Y_{D_S}, \text{StT}'_E)$ be a Dual $\text{Soft}_{\text{Closed}}$ mapping function **Then** f_{D_S} is a Dual $\text{Soft}_{\text{Compact}}$ subset of Y_{D_S}

Theorem 3.14: let $f_{D_1 S_1}: (X_{D_S}, \text{StT}_E) \rightarrow (Y_{D_S}, \text{StT}'_E)$ and $f_{D_2 S_2}: (Y_{D_S}, \text{StT}'_E) \rightarrow (Z_{D_S}, \text{StT}'_E)$ be function If $f_{D_1 S_1}$ is dual $\text{Soft}_{\text{compact}}$ and $f_{D_2 S_2}$ is Dual $\text{Soft}_{\text{Compact}}$ then $f_{D_1 S_1} \circ f_{D_2 S_2}$ is Dual $\text{Soft}_{\text{Compact}}$

Proof: Let F_{GB} be dual $\text{Soft}_{\text{compact}}$ of Z_{D_S} then $(f_{D_1 S_1} \circ f_{D_2 S_2})^{-1}(F_{GB})$ is dual $\text{Soft}_{\text{compact}}$ in X_{D_S} so $f_{D_1 S_1}^{-1}(f_{D_2 S_2}^{-1}(F_{GB}))$ is dual $\text{Soft}_{\text{compact}}$ in Y_{D_S} . Also $f_{D_1 S_1}^{-1}(f_{D_2 S_2}^{-1}(F_{GB})) = f_{D_1 S_1}^{-1}(f_{D_2 S_2}^{-1}(F_{GB})) = f_{D_1 S_1}^{-1}(f_{D_2 S_2}^{-1}(F_{GB})) = f_{D_2 S_2}^{-1}(F_{GB})$

Definition 3.15^[1]

Let $(X_{D_S}, \text{StT}_E, \Pi_{(U_1, U_2)})$ and $(Y_{D_S}, \text{StT}'_E, \Pi_{(U_1, U_2)})$ be two Dual soft topological spaces, $f_{D_S}: (X_{D_S}, \text{StT}_E) \rightarrow (Y_{D_S}, \text{StT}'_E)$ be a mapping, if f_{D_S} is a bijective, dual Soft continuous and $f_{D_S}^{-1}$ is a Dual Soft continuous mapping, then f_{D_S} is said to be a Dual Soft Homeomorphism from X_{D_S} to Y_{D_S} , when a Dual Soft Homeomorphism f_{D_S} exists between X_{D_S} and Y_{D_S} we say that X_{D_S} is dual Soft homeomorphism to Y_{D_S} .

Theorem 3.16

$f_{D_S}: (X_{D_S}, \text{StT}_E) \rightarrow (Y_{D_S}, \text{StT}'_E)$ be a dual Soft homeomorphism then if F_{GK} is dual $\text{Soft}_{\text{compact}}$ subset of X_{D_S} so $f(F_{GK})$ is also dual $\text{Soft}_{\text{compact}}$.

Proof: suppose $\{V_\alpha: \alpha \in \Lambda\}$ be an dual $\text{Soft}_{\text{open}}$ cover of $f(F_{GK})$, $f(F_{GK}) \subset \bigcup V_\alpha$ which is dual $\text{Soft}_{\text{open}}$. Then $f(F_{GK}) \subset \bigcup V_\alpha^0$ so $F_{GK} \subset f^{-1}(\bigcup V_\alpha^0) = f^{-1}(\bigcup V_\alpha^0)$ so $f^{-1}(V_\alpha)$ is a dual $\text{Soft}_{\text{open}}$ but $F_{GK} = \bigcup_{i=1}^n f^{-1}(V_{\alpha_i})$ implies $f(F_{GK}) = f(\bigcup_{i=1}^n f^{-1}(V_{\alpha_i})) = \bigcup_{i=1}^n f f^{-1}(V_{\alpha_i}) = \bigcup_{i=1}^n (V_{\alpha_i})$ There for $f(F_{GK})$ dual $\text{Soft}_{\text{compact}}$.

4. Results and Discussion

4.1 Results

This Analysis of a dual soft compact space (X_{D_S}, StT_E) confirms the structure of dual soft topology to classical topological properties as established in the triple (AD, W, GD) defined by the authors. The results shown in Section 3 are consistent with the foundational structure of compactness being preserved under dual soft union and intersection therefore establishing the axiomatic consistency of dual soft topology. Additionally, Theorem 3.4 states that all dual soft compact spaces are also dual soft locally compact by using properties of closure on a finite subcover generated from the original compactness property. Thus since the closure of the dual soft open neighborhood is also compact as shown in Section 4 meets the criteria of Remark 3.5 and the classical method that a global property of compactness will provide a pointwise property of local compactness which provides continuity for future uses of the theory for use with convergent and filter bases in X_{D_S}

4.2 Discussion

The dual soft compact property is demonstrated by Theorem 3.8, which shows that dual soft closed subsets inherit their parent spaces' compactness. The method used to prove this includes taking the relative complement of the closed subset in an open cover (similar to how proof of the standard compactness works) using dual soft closed sets defined as the complements of all elements in the topology StESTe ^[6] is essential in the demonstration of this fact. Therefore, this result asserts that a dual soft compact set's intersection with a dual soft closed set will also be a dual soft compact space (Theorem 3.8, Part ii). The consequence of this is that these intersections are vital when attempting to establish separation axioms for dual soft spaces; e.g., if the dual soft space satisfies a higher degree of separation (dual soft Hausdorff), then all of its dual soft compact subsets will also be dual soft closed.

The structure of the mappings was used to study the behaviour of dual soft compactness under morphisms by considering compositions of the mappings. Theorem 3.14 demonstrated that the composition of two dual soft compact functions is therefore still dual soft compact, which gives a categorical consistency in the mapping structures between the spaces $(\text{XD}s, \text{StTE})$ and $(\text{YD}s, \text{StTE}')$ ^[7]. Theorem 3.16 showed that dual soft compactness is also respected under dual soft homeomorphisms as defined in ^[8]. The proof procedures involved using properties of the continuity of bijective mappings to return to the original open covers. This shows that the image of a dual soft compact subset under a dual soft homeomorphism will also be dual soft compact. However, further review of the presented theorems revealed limitations in that the entire exposition in the current format does not explicitly support the more rigorous and general theorem that the continuous images of a dual soft compact set must continue to have the property of dual soft compactness. This would be a fundamental result according to the general and/or classical theory of compactness and suggests a premise for further theoretical expansion. Future research should look at developing and refining Theorem 3.13 to demonstrate the general behaviour of continuous surjections as opposed to the case of homeomorphisms, thereby completing the characterization of dual soft compactness as a proper topological invariant under the mapping structures defined by ^[9].

4 Conclusion and future study

The purpose of this document is to present a new class of Topological Spaces referred to as dual soft compact spaces. The study of dual soft compact spaces has been thoroughly researched in regards to dual soft topology. We utilize the fundamental notions of dual soft sets and dual soft neighborhoods in order to describe a "weakened" version of compactness that generalizes the traditional compactness definition into a wider/directly related dual soft topology equivalence. This work has produced many new and interesting results. For example, it can be shown that all dual soft compact spaces are dual soft locally compact spaces, thus these spaces can be combined with previously existing classes of compact spaces. Furthermore, all closed dual soft subsets of dual soft compact spaces are also dual soft compact, while finite sets are always counted as dual soft compact. In addition, significant work has been completed exploring how dual soft compactness operates with respect to mappings,

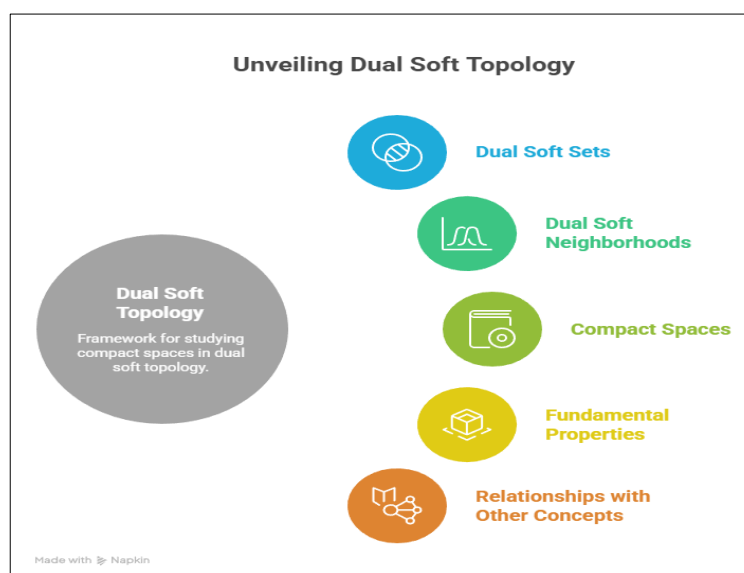
thus, it was demonstrated that the dual soft compactness of a set is preserved in the image of a dually soft homeomorphism. Furthermore, it was shown that the composition of two or more dual soft compact functions results in a dual soft compact function. Lastly, it was established how the dual soft compactness of an open subspace influences the dual soft compactness of the surrounding space. This research adds to our understanding of behaviours based on compactness within dual soft topological spaces and provides a solid basis for future theory development. The research and results presented have relied upon much of the work of other mathematicians, whilst offering new insights about structural features in soft set theory and its applications ^[10, 12].

The introduction of dual soft compactness opens up a variety of new possibilities in research. Subsequent studies may compare the differences in weak vs. strong forms of compactness in the context of dual soft topology (e.g., dual soft Lindelöf spaces) such as dual soft countably compact spaces, dual soft sequentially compact spaces and how those spaces relate to one another. One additional area for future research concerns the study of dual soft compactness, in conjunction with the different types of dual soft separation axioms (e.g., dual soft Hausdorff, dual soft regular, or dual soft normal). There is a need to determine whether dual soft compactness will lead to the establishment of dual soft closure in spaces that are dual soft Hausdorff. Another broad region for future research relates to discovering an appropriate definition of product topology on dual soft spaces, and examining Tychonoff-type theorems associated with dual soft compactness. Finally, we can use dual soft compactification in combination with the one-point compactification and the Stone-Čech compactification, as well as investigate both uniqueness and universality characteristics. It appears further studies should be done on potential uses that are beyond theoretical progress made for problems with decision-making or analyses of data/uncertainty modelling as a consequence of the outcome produced from dual soft compact type spaces of two interrelated world types. Future work may also look at ways to sustain dual soft compactness through different function types (example: perfect mappings, irreducible mappings, quotient mappings) using dual soft sets ^[18].

Lastly, the concept of dual soft compact could be expanded by including dual fuzzy soft topological spaces; dual intuitionistic soft topological spaces; and dual neutrosophic soft topological spaces. In other words, this would include both the concept of soft topology and more applications for soft topology.

Graphical abstract

This project presents and develops a new concept of dual soft topology, which extends the classical theory of soft sets (created by Molodtsov in 1999) to dual topological contexts. The research is divided into four major areas: (1) study of dual soft sets; (2) description of dual soft neighborhoods; (3) their role in defining compact spaces in dual soft topology, and (4) an examination of the main properties of the new structure, as well as its connections to more traditional topological and soft set ideas. A major result of the research will be the establishment of a framework for studying compact spaces within the dual soft topological framework. The work was done in cooperation with P. Knight, as shown in the summary.



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