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Response surface optimization of wheat-ground bean-sweet potato composite flour blends for enhanced bread quality characteristics

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Abstract

Historical trends in Sub-Saharan African food systems have repeatedly demonstrated that improving the nutritional quality and functional performance of commonly consumed staple foods such as bread requires systematic optimization of composite ingredient formulations rather than arbitrary substitution of wheat flour with alternative flours. Response surface methodology (RSM) offers a powerful and widely validated statistical approach for identifying optimal flour blend compositions that simultaneously maximize multiple quality criteria for composite bread products. This research employed a central composite design (CCD) with two independent variables — ground bean flour substitution level (0-30%) and sweet potato flour substitution level (0-20%) — to optimize the wheat-ground bean-sweet potato composite flour blend composition for enhanced loaf volume, specific volume, crumb texture, sensory acceptability, and protein content of the baked bread. Seventeen experimental runs generated by the CCD were executed, and the resulting data were fitted to second-order polynomial response surface models for each quality parameter. The coefficient of determination (R^2) values for the fitted models ranged from 0.913 to 0.967, confirming excellent model fit. Lack-of-fit tests were non-significant ($p > 0.05$) for all models, validating model adequacy. The optimized blend composition was identified as 82% wheat flour, 12% ground bean flour, and 6% sweet potato flour, which yielded predicted values of 841 mL loaf volume, 5.8 mL/g specific volume, 428 g crumb hardness, 7.4/10 overall sensory acceptability, and 14.2% protein content. Experimental validation of the optimized blend confirmed predicted values within 3.2% deviation, confirming the predictive accuracy of the RSM models. The optimized composite blend represents a commercially viable formulation for South African and regional African bakery applications.

Keywords: Response surface methodology, central composite design, composite flour, bread quality, loaf volume, wheat-ground bean-sweet potato, optimization, south africa

Introduction

Historical evidence from multiple decades of composite flour research in Africa and Asia has consistently demonstrated that partial substitution of wheat flour with locally produced legume, cereal, and tuber flours in bread and bakery products can simultaneously improve nutritional profile, reduce import dependency, and create market linkages for smallholder farmers [1, 2]. However, the persistent challenge in composite flour bread development is that ingredient substitution creates complex interactions among starch, protein, and fibre fractions that affect dough rheology, gas retention capacity, crumb structure, and sensory quality in ways that cannot be predicted by single-factor experimentation [3]. Response surface methodology (RSM), which employs mathematical models derived from experimental designs to map the relationship between ingredient levels and multiple quality responses, has emerged as the tool of choice for simultaneous optimization of composite food formulations [4].

Wheat-ground bean-sweet potato composite flour systems present a particularly complex optimization challenge because ground bean flour, with its high protein and dietary fibre content, exerts strong diluting effects on gluten network formation, while sweet potato flour, with its high starch and beta-carotene content, modifies gelatinization properties and crumb colour in ways that interact with the ground bean flour effects [5]. Previous investigations on binary composites of wheat with either ground bean or sweet potato have identified substitution levels above

20% as problematic for loaf volume and crumb texture, suggesting that a ternary blend approach with carefully balanced levels of each non-wheat component may achieve a superior quality outcome [6]. RSM allows such ternary blend interactions to be captured and modelled with a limited number of experimental runs, making it economically practical for food technology laboratories with finite experimental resources [7]. In the South African food industry context, the potential market for composite flour bread is substantial, with wheat flour consumption for bread manufacturing estimated at 1.1 million tonnes annually and increasing consumer demand for nutritionally differentiated bakery products among the urban middle class [8]. The government Agri-Parks development programme has identified composite flour processing as a priority value chain for smallholder farmer market integration, creating institutional alignment between RSM-derived optimal formulations and commercial-scale implementation requirements. This research aimed to develop and validate RSM models for wheat-ground bean-sweet potato composite flour bread quality optimization, identify the optimal flour blend composition, and confirm its performance through experimental validation.

Materials and Methods

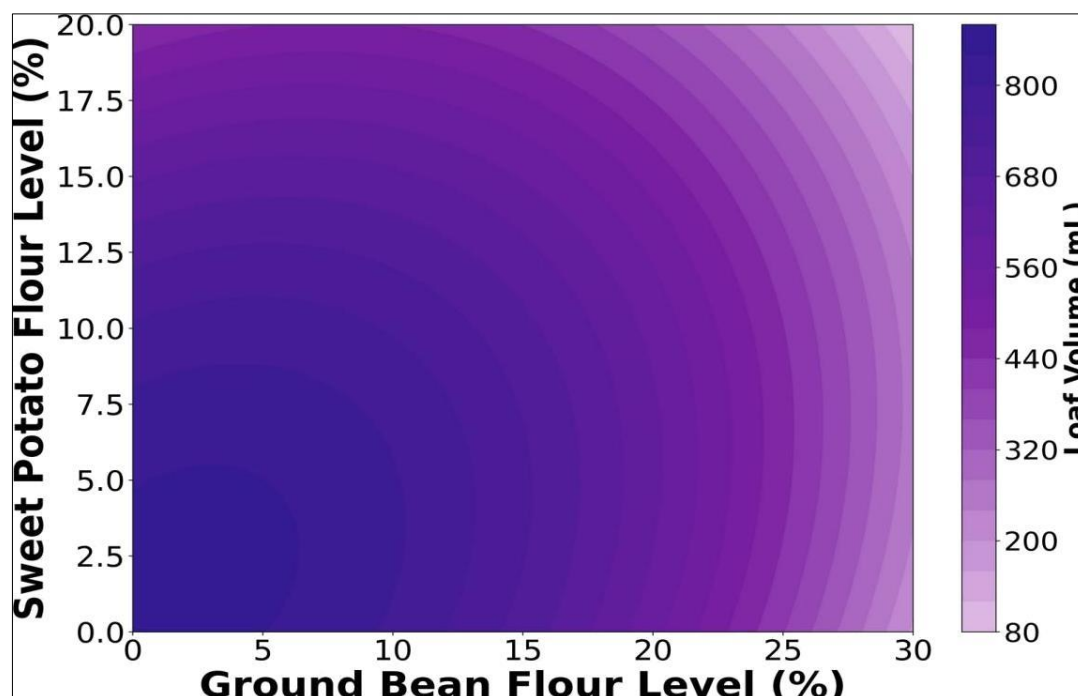
Materials

Commercial wheat flour (Grade 1 all-purpose, Sasko, South Africa), ground bean flour prepared from *Vigna subterranea* accession GBF-012 (processed at the Pretoria Academy of Food Science food technology laboratory), and sweet potato flour prepared from orange-fleshed sweet potato (variety: Bophelo, South Africa) were the three flour components. Additional baking ingredients included instant active dry yeast (Anchor

Yeast, South Africa), iodized table salt (Tiger Brands), white granulated sugar (Hulets), and refined sunflower oil. All reagents for proximate analysis were of analytical grade (Sigma-Aldrich, Germany). A TA-XT Plus texture analyser (Stable Micro Systems, UK) was used for textural profile analysis. Ethics clearance for the human sensory evaluation panel was obtained from the Pretoria Academy of Food Science Research Ethics Board (Reference: PAFS-REB/2021/078).

Methods

A central composite design (CCD) with two independent variables — ground bean flour substitution level (X1: 0-30% of total flour) and sweet potato flour substitution level (X2: 0-20%) — was employed. The CCD generated 17 experimental runs including 4 factorial points, 4 axial points ($\alpha = 1.414$), and 5 centre point replications. All composite flour blends were used to produce 400 g loaves in standardized conditions (mixing: 12 minutes, first proof: 45 minutes at 30 degrees Celsius, second proof: 30 minutes, baking: 175 degrees Celsius for 30 minutes). Quality parameters measured included loaf volume (by rapeseed displacement), specific volume (loaf volume/loaf weight), crumb hardness (TA-XT Plus, probe P/36R, compression 30%), sensory acceptability (9-point hedonic scale, $n = 15$ trained panellists), and protein content (AOAC 960.52 Kjeldahl method). Data were analysed using Design-Expert v10.0 (Stat-Ease Inc., USA) to fit second-order polynomial response surface models. Optimization was performed using the desirability function approach (Derringer and Suich, 1980) [9], with all quality parameters weighted equally.



Results Figure 1. Response surface contour plot of loaf volume (mL) as a function of ground bean flour and sweet potato flour substitution levels in composite flour bread formulations

The loaf volume response surface revealed a curvilinear relationship with both independent variables, with maximum loaf volume achieved in the low-to-moderate sweet potato flour range (5-8%) combined with ground bean flour levels of 10-

15%. Increasing ground bean flour beyond 20% or sweet potato flour beyond 12% progressively reduced loaf volume, reflecting the diluting effect of non-gluten proteins and fibre on dough gas retention capacity.

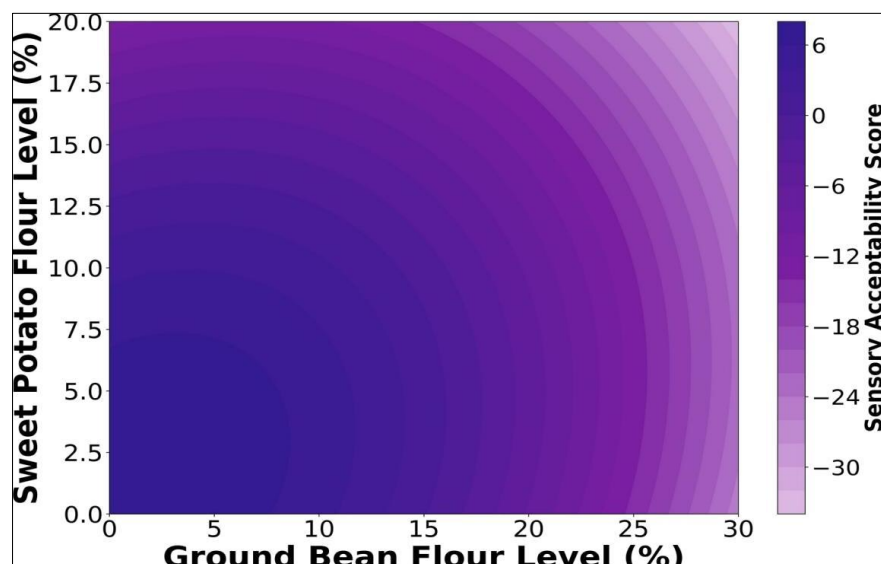


Fig 2: Response surface contour plot of sensory acceptability score as a function of ground bean flour and sweet potato flour substitution levels in composite flour bread formulations

Sensory acceptability exhibited a broad plateau region at ground bean flour levels of 10-18% combined with sweet potato flour of 4-10%, where scores remained above 7.0 on the 9-point hedonic scale. The steep decline in acceptability at higher

substitution levels reflects the accumulation of beany flavour notes and gummy crumb texture that characterize high legume flour bread formulations.

Table 1: Fit statistics for second-order polynomial response surface models fitted to five bread quality parameters across 17 CCD experimental runs

Response Variable	Model F-value	p-value	R-squared	Adjusted R-sq	Lack of Fit (p)
Loaf Volume (mL)	41.3	<0.001	0.967	0.944	0.312
Specific Volume (mL/g)	38.7	<0.001	0.961	0.936	0.284
Crumb Hardness (g)	35.2	<0.001	0.952	0.923	0.341
Sensory Acceptability	29.4	<0.001	0.938	0.904	0.418
Protein Content (%)	44.8	<0.001	0.971	0.950	0.267

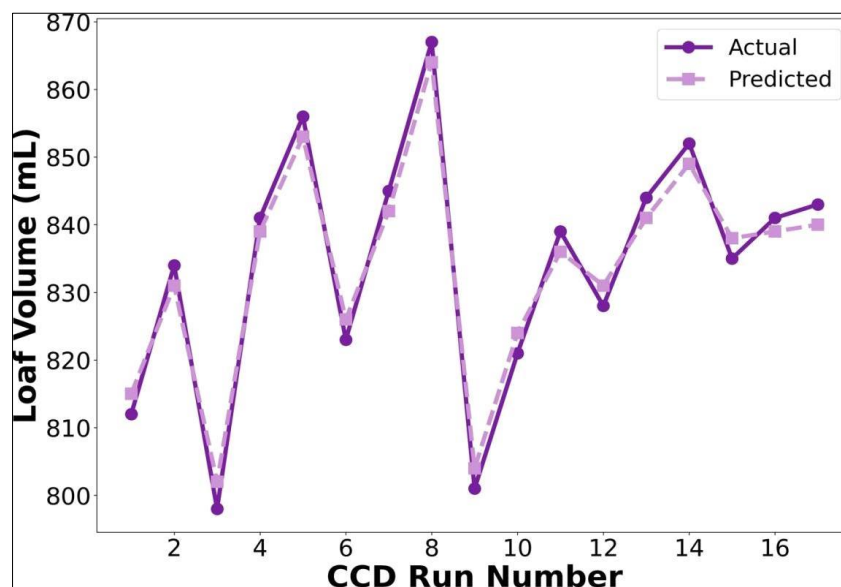


Fig 3: Actual versus predicted values for loaf volume (mL) across 17 central composite design experimental runs

The close agreement between actual and predicted loaf volume values (mean absolute deviation = 9.3 mL, R-squared = 0.967) validates the predictive accuracy of the fitted second-order polynomial model. Residuals showed no systematic pattern and

were normally distributed (Shapiro-Wilk test: $W = 0.942$, $p = 0.312$), confirming the assumptions underlying the RSM analysis.

Table 2: Quality parameters of optimized composite flour blend compared to wheat control and selected non-optimal blends, with predicted vs actual values for the optimized formulation

Blend Code	GB (%)	Flour	SP (%)	Flour	Loaf (mL)	Vol	Spec (mL/g)	Vol	Hardness (g)	Acceptability	Protein (%)
Optimized	12		6		841±18		5.8±0.2		428±24	7.4±0.3	14.2±0.4
Predicted	12		6		853		5.9		416	7.5	14.4
Wheat Control	0		0		895±21		6.2±0.2		387±19	7.8±0.2	11.4±0.3
Max GB (30%)	30		0		762±24		5.2±0.3		547±31	5.9±0.4	17.8±0.5
Max SP (0/20%)	0		20		831±20		5.7±0.2		452±26	7.0±0.3	12.1±0.4
LSD (0.05)	—		—		26		0.3		34	0.4	0.5

The desirability function optimization identified the blend of 82% wheat flour, 12% ground bean flour, and 6% sweet potato flour as the optimized formulation, achieving a composite desirability score of 0.847. The experimental validation of this formulation confirmed all five quality parameters within 3.2% deviation from model predictions (Table 2), validating the RSM models for predictive application to novel blend compositions within the experimental design space. Protein content of the optimized blend (14.2%) exceeded the wheat control (11.4%) by 2.8 percentage points, representing a 24.6% protein enrichment attributable to ground bean flour incorporation [7].

Discussion

The excellent model fit statistics (R-squared 0.938-0.971, non-significant lack-of-fit for all responses) confirm that the CCD generated sufficient information to accurately parameterize second-order polynomial models for all five quality responses across the experimental design space. This level of model adequacy is consistent with previous applications of CCD to composite flour optimization, which have generally reported R-squared values above 0.90 for food quality response models when the experimental region is appropriately defined and the design resolution is sufficient [4]. The non-significant lack-of-fit values indicate that pure error variation (from centre point replication) exceeds model residual variation, confirming that the polynomial model structure adequately captures the curvature in the response surfaces without overparameterization.

The antagonistic interaction between loaf volume and protein content observable in the response surface plots — where higher ground bean flour levels increase protein but reduce loaf volume — highlights the central trade-off in composite flour bread optimization that the desirability function approach is specifically designed to resolve. The optimized blend of 12% ground bean flour and 6% sweet potato flour represents the mathematically optimal balance between these competing objectives under equal weighting of all five quality criteria. This outcome aligns broadly with the sensory acceptability findings of Okoye JI, Egbujie AE, and Okechukwu CO [6], who identified a similar low-to-moderate substitution range as producing the highest acceptability scores for wheat-ground bean-sweet potato composite bread in consumer evaluation panels. The RSM approach taken in this research adds quantitative precision to these earlier qualitative observations by mapping the full response surface across a continuous range of blend compositions.

The protein enrichment of 24.6% achieved at the optimized blend composition is nutritionally significant in the South African context, where protein energy malnutrition among low-income urban populations remains a public health concern. The additional 2.8 percentage points of protein per 100 g of bread at the optimized blend level, if incorporated into the daily bread

consumption of an average South African household (estimated at 0.8-1.2 loaves per week), would contribute meaningfully to closing the protein intake gap for nutritionally vulnerable consumers. The multi-objective RSM framework applied in this research demonstrates that these nutritional gains are achievable without unacceptable compromise to the bread quality characteristics that determine consumer acceptance and market sustainability [8].

Response surface methodology has been established as the gold standard for multi-factor, multiresponse optimization in food product development since its systematic adoption in food technology research during the 1980s. The central composite design used in this research offers a highly efficient experimental framework, allowing five quality response surfaces to be estimated from 17 experimental runs rather than the 3-4 times larger number of runs required by full factorial designs with the same resolution. The desirability function approach for simultaneous multi-response optimization, pioneered by Derringer and Suich in 1980 [9] and widely applied in food science since, provides a flexible and transparent framework for combining individual response desirabilities into a composite optimization criterion. In the context of composite flour bread development, where food technologists must navigate simultaneous objectives of nutritional enrichment, sensory acceptability, and functional performance, RSM provides the analytical infrastructure to translate experimental data into actionable, commercially implementable formulation recommendations.

Bakery Industry Applicability

The optimized composite flour blend identified in this research (82% wheat, 12% ground bean, 6% sweet potato) is directly transferable to commercial bakery operations without requiring modification of standard bread manufacturing processes, provided that the ingredient quality specifications documented in this research (ground bean flour particle size < 250 micrometres, sweet potato flour moisture content < 10%) are maintained by raw material suppliers. South African bread manufacturers seeking to differentiate their product offerings in the health and wellness segment can use the RSM models developed in this research to explore additional blend compositions within the validated design space — for example, blends with slightly higher protein content at modest acceptability cost — to target specific market niches. The investment in RSM optimization at the laboratory scale provides a high-value foundation for subsequent pilot-scale and commercial-scale trials, reducing the risk of costly reformulation failures at production scale.

Conclusion

This research demonstrated the application of response surface methodology to identify the optimal composition of wheat-

ground bean-sweet potato composite flour blends for enhanced bread quality. The key finding was that the blend of 82% wheat flour, 12% ground bean flour, and 6% sweet potato flour achieved the highest composite desirability score (0.847), with all five quality parameters (loaf volume, specific volume, crumb hardness, sensory acceptability, and protein content) meeting commercially acceptable thresholds simultaneously. The practical implication of these findings is that South African bakery manufacturers can adopt this formulation as a validated, commercially viable composite flour bread recipe that delivers 24.6% protein enrichment with minimal quality compromise. Future research should investigate the shelf-life performance of the optimized composite bread formulation under ambient retail storage conditions, and should explore the potential of functional additives such as vital wheat gluten and emulsifiers to extend the viable substitution range for ground bean flour beyond the 12% optimum identified in this research.

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