

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452

NAAS Rating (2026): 4.49

Maths 2026; 11(7):171-175

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<https://www.mathsjournal.com>

Received: 30-04-2026

Accepted: 30-05-2026

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Solving the bessel functions using the sadik transform

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Abstract

In this paper, we find that the Sadik transform of Bessel functions is essential for solving a variety of differential equations and mathematical problems. This study main objective is to solve linked Bessel relations by evaluating Bessel functions using the Sadik integral transform. Bessel functions can be used to solve a variety of equations in cylindrical or spherical dimensions, including heat equations, wave equations, Laplace equations, and Helmholtz equations. Additionally, this study derives the Sadik transform of first-kind Bessel functions and shows how to use it to solve fractional differential equations and evaluate complex integrals. The results demonstrate the Sadik transforms effectiveness grace, its simplicity as a powerful alternative to conventional integral transforms, increasing its applicability in the physical sciences and sophisticated engineering.

Keywords: Sadik transform, bessel function, inverse sadik transform

Introduction

Bessel function are now essential for solving problems in the fields of engineering, Heat transfer, vibrations, hydrodynamics, fluid mechanics, nuclear physics, mathematical physics, atomic physics, acoustics, radio physics, and stress analysis, and nuclear reactor flux distribution. The nonnegative integer is represented by the Bessel function of order [2,5,7,8]. It is also clear how crucial integral transformations are for precisely solving differential equations. Numerous integral transformations have been proposed by researchers, each of which addresses a problem in life sometimes social and economic, sometimes engineering, physical, medical, or astronomical with the goal of finding solutions using less complicated, more technical, and simpler approaches. It is thought that the authentic transformation is easier to use and more universal [9-13].

$$J_n(t) = \frac{t^n}{2^n n!} \left\{ 1 - \frac{t^2}{2(2n+2)} + \frac{t^4}{2.4(2n+2)(2n+4)} - \frac{t^6}{2.4.6(2n+2)(2n+4)(2n+6)} + \dots \right\} \quad (1)$$

For $n = 0$, The zero order Bessel function is represented by. $J_0(t)$ The following infinite power series is used to define.

$$J_0(t) = \left\{ 1 - \frac{t^2}{2^2} + \frac{t^4}{2^2 4^2} - \frac{t^6}{2^2 4^2 6^2} + \dots \right\} \quad (2)$$

For $n = 1$, The following infinite power series defines Bessel function of order one, symbolizes by $J_1(t)$.

$$J_1(t) = \left\{ \frac{t}{2} - \frac{t^3}{2^3 . 4} + \frac{t^5}{2^2 4^2 . 6} - \dots \right\} \quad (3)$$

For $n = 2$, The following infinite power series defines Bessel function of order one, symbolizes by $J_2(t)$.

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$$J_2(t) = \left\{ \frac{t^2}{2.4} - \frac{t^4}{2^2.4.6} + \frac{t^6}{2^2.4^2.6.8} - \dots \right\} \quad (4)$$

Some useful properties of sadik transform:

Sadik Transform (S – Transform)

Integral transforms methods (*L* - Transform, *Mo* - Transform, *M* - Transform, *S* - Transform ,..... etc) are practical mathematical tools that can be used to solve complex science and engineering problems that can be expressed in terms of integral and the differential equations.

Definition: [3,8] The Sadik Transform of the function $f(x)$, $x \geq 0$ can be defined

$$S\{f(x)\} = \frac{1}{v^\beta} \int_0^\infty f(x) e^{-xv^\alpha} dx,$$

when v is a complex variable and $\alpha \neq 0, \beta$ are any actual numbers. The Sadik transform operator is denoted by the letter S .

It should be noted that his Sadik transform of then functions $f(x)$ since $x \geq 0$ exists if $f(x)$ is The only prerequisites for the existence of Sadik transforms the function are piecewise continuity and exponential order $f(x)$.

Convolution Theorem the Sadik Transform

Table 1: (Sadik –Transform) for some elementary functions

S.N	$f(x)$	$Sf(x)$
1	1	$\frac{1}{v^{\alpha+\beta}}$
2	x	$\frac{1}{v^{2\alpha+\beta}}$
3	x^2	$\frac{2!}{v^{3\alpha+\beta}}$
4	$x^n, n \in N$	$\frac{n!}{v^{(n+1)\alpha+\beta}}$
5	e^{ax}	$\frac{1}{v^\beta(v^\alpha - a)}$
6	$\sin ax$	$\frac{a}{v^\beta(v^{2\alpha} + a^2)}$
7	$\cos ax$	$\frac{v^\alpha}{v^\beta(v^{2\alpha} + a^2)}$
8	$\sinh ax$	$\frac{a}{v^\beta(v^{2\alpha} - a^2)}$
9	$\cosh ax$	$\frac{v^\alpha}{v^\beta(v^{2\alpha} - a^2)}$

Dualities between transforms

To illustrate the significance of these linkages for the integral transforms mentioned, we give examples using φ as a symbol for the other transform result. The dualities between the *L*-Transform and a number of popular transforms, including the *M*-Transform, *S*-Transform, *Mo*-Transform, and γ -Transform, will be covered in this section.

Mahgoub – Sadik Duality

MahgoubTransform of then is function $f(x)$, since $x \geq 0$ such that define as:

$$M\{f(x)\} = v \int_0^\infty f(x) e^{-vx} dx,$$

and Sadik Transforms of then function $f(x)$, since $x \geq 0$ symbolizes by $S\{f(x)\}$ and can be defined as:

$$S\{f(x)\} = \frac{1}{v^\beta} \int_0^\infty f(x) e^{-xv^\alpha} dx,$$

To solve integral equation in any transform we should use the convolution theorem it as we have observed in solving the integral equations by previous transform. Let the sadik Transform of the functions $f_1(x)$ and $f_2(x)$ is $S\{f_1(x)\}$ and $S\{f_2(x)\}$ respectively, then: Inverse of *S* – Transform [3].

$$S\{f_1(x) * f_2(x)\} = v^\beta S\{f_1(x)\} \cdot S\{f_2(x)\} [6].$$

If $S\{f(x)\}$ is the Sadik Transforms of the function $f(x)$, the $f(x)$ is known as the inverse of the Sadik Transform and has the following mathematical definition.

$$f(x) = S^{-1}\{S\{f(x)\}\},$$

Such that operator S^{-1} is called the inverse *S* - Transform operator and for sure it have that linearity property:

$$\begin{aligned} & S^{-1}\{a S\{f_1(x)\} + b S\{f_2(x)\}\} \\ &= a S^{-1}\{S\{f_1(x)\}\} + b S^{-1}\{S\{f_2(x)\}\} \\ &= a f_1(x) + b f_2(x). \end{aligned}$$

Now, let $v = u^\alpha$ then

$$\begin{aligned} M\{f(x)\} &= v \cdot \int_0^\infty f(x) \cdot e^{-u^\alpha x} dx = v^{\beta+1} \left[\frac{1}{v^\beta} \int_0^\infty f(x) x dx \right] \\ &= v^{\beta+1} \cdot \varphi(u^\alpha, \beta). \end{aligned}$$

On the other hand: Let $r = v^\alpha$, then

$$\begin{aligned} S\{f(x)\} &= \frac{1}{v^\beta} \int_0^\infty f(x) e^{-xv^\alpha} dx = \frac{1}{v^\beta v} \left\{ v \int_0^\infty f(x) e^{-xr} dx \right\} \\ &= \frac{1}{v^\beta v} \{ \varphi(r) \} \\ \text{Thus } S\{f(x)\} &= \frac{1}{v^\beta v} \{ \varphi(r) \}. \end{aligned}$$

Sadik – mohand duality

Sadik Transforms of then function $f(\gamma)$, since $x \geq 0$ denoted by $S\{f(\gamma)\}$ and can be they know:

$$S\{f(\gamma)\} = \frac{1}{v^\beta} \int_0^\infty f(\gamma) e^{-\gamma v^\alpha} d\gamma,$$

With mohand Transform:

$$Mo\{f(\gamma)\} = v^2 \int_0^\infty f(\gamma) e^{-v\gamma} d\gamma = \varphi(v).$$

Now let $v^\alpha = u$, then we have:

$$\begin{aligned} S\{f(\gamma)\} &= \frac{1}{v^\beta} \int_0^\infty f(\gamma) e^{-\gamma v^\alpha} d\gamma = \frac{1}{v^\beta v^2} \left\{ v^2 \int_0^\infty f(\gamma) \cdot e^{-u\gamma} d\gamma \right\} \\ &= \frac{1}{v^\beta v^2} \cdot \varphi(u) \end{aligned}$$

Thus

$$S\{f(x)\} = \frac{1}{v^\beta v^2} \cdot \varphi(u). \text{ On the other hand:}$$

Let $v = r^\alpha$, then:

$$\begin{aligned} Mo\{f(\gamma)\} &= v^2 \int_0^\infty f(\gamma) e^{-v\gamma} d\gamma = v^2 v^\beta \left\{ \frac{1}{v^\beta} \int_0^\infty f(\gamma) e^{-\gamma r^\alpha} d\gamma \right\} \\ &= v^2 v^\beta \varphi(r^\alpha, \beta) \end{aligned}$$

Thus

$$Mo\{f(\gamma)\} = v^2 v^\beta \varphi(r^\alpha, \beta).$$

Example 1 Solve the following integral equation using sadik integral transforms

$$u((x)) = \cos x + \sin x - \int_0^x u(t) dt$$

Solution

Since

$$S\{u(x)\} = \frac{1}{v^\beta} \cdot \{\varphi(v)\}$$

$$S\{u(x)\} = \frac{1}{v^\beta} \cdot \frac{v}{1+v^2}$$

now let $v = r^\alpha$

$$S\{u(x)\} = \frac{1}{v^\beta} \cdot \frac{r^\alpha}{1+r^{2\alpha}}$$

$$S\{u(x)\} = \frac{1}{(r^\alpha)^\beta} \cdot \frac{r^\alpha}{1+r^{2\alpha}}$$

$$\therefore u(x) = S^{-1} \left\{ \frac{1}{(r^\alpha)^\beta} \cdot \frac{r^\alpha}{1+r^{2\alpha}} \right\}$$

$$u(x) = \cos x$$

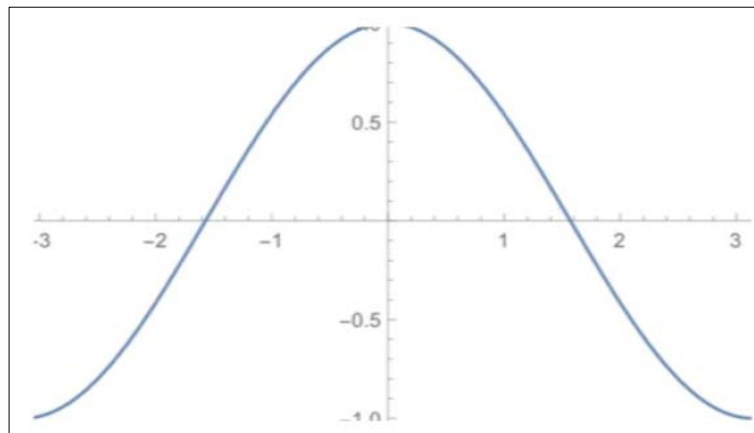


Fig 1: $u(x) = \cos x$

Relation between $(J_{0(t)})$ and $(J_{1(t)})$ [4]:

$$\frac{d}{dt} J_{0(t)} = -J_{1(t)} \quad (5)$$

Relation between $(J_{0(t)})$ and $(J_{2(t)})$ [4]:

$$J_{2(t)} = J_{0(t)} + 2J_{0(t)}'' \quad (6)$$

Sadik transform to solve Bessel function

Sadik transform to solve Bessel function of zero

order: Applying the sadik transform to both sides of equation (2) yields the result.

$$J_{0(t)} = \left\{ 1 - \frac{t^2}{2^2} + \frac{t^4}{2^2 4^2} - \dots \right\}$$

$$= \left\{ \frac{1}{v^{\alpha+\beta}} - \frac{2!}{2^2 v^{3\alpha+\beta}} + \frac{4!}{2^2 4^2 v^{5\alpha+\beta}} - \dots \right\}$$

$$= \left\{ \frac{1}{v^{\alpha+\beta}} - \frac{1}{2 v^{3\alpha+\beta}} + \frac{24}{4.16 v^{5\alpha+\beta}} - \dots \right\}$$

$$= \frac{1}{v^{\alpha+\beta}} \left\{ 1 - \frac{1}{2 v^{2\alpha+\beta}} + \frac{3}{8 v^{4\alpha+\beta}} \right\} \quad (7)$$

Sadik transform to solve Bessel function of order one

Equation (3) can be solved by applying the Sadik transform to both sides:

$$J_{1(t)} = \left\{ \frac{t}{2} - \frac{t^3}{2^3 4} + \frac{t^5}{2^2 4^2 6} - \dots \right\}$$

$$= \left\{ \frac{2}{2 v^{\alpha+\beta}} - \frac{3!}{2^3 4 v^{4\alpha+\beta}} + \frac{5!}{2^2 4^2 6 v^{6\alpha+\beta}} - \dots \right\}$$

$$= \left\{ \frac{1}{v^{\alpha+\beta}} - \frac{3}{16v^{4\alpha+\beta}} + \frac{5}{16v^{6\alpha+\beta}} - \dots \right\} \quad (8)$$

Sadik transform to solve Bessel's function of order two

Equation (4) can be solved by applying the Sadik transform to both sides

$$\begin{aligned} J_2(t) &= \left\{ \frac{t^2}{2.4} - \frac{t^4}{2^2.4.6} + \frac{t^6}{2^2.4^2.6.8} - \dots \right\} \\ &= \left\{ \frac{2!}{8v^{\alpha+\beta}} - \frac{4!}{96v^{4\alpha+\beta}} + \frac{6!}{3072v^{6\alpha+\beta}} - \dots \right\} \\ &= \left\{ \frac{v^3}{4v^{3\alpha+\beta}} - \frac{v^5}{4v^{5\alpha+\beta}} + \frac{120.v^7}{5v^{7\alpha+\beta}} - \dots \right\} \end{aligned} \quad (9)$$

By applying the second derivative Sadik transform property, we arrive at:

$$S\{h''(t)\} = v^2 H_{(v)}$$

As a result, the Sadik transform of from $J_{0(t)}$ equation (7) is:

$$(h)''t = J_{0(t)} \text{ and } h(0)=0 \text{ and } h'(0)=0$$

Consequently, by applying the transform relation linked to the Bessel function of order one

$$h(t) = t^2 J_{1(t)}$$

and taking the inverse Sadik transform, we ultimately obtain:

$$v^2 H_{(v)} + h_{(t)} = S\{J_{0(t)}\}$$

Equation (7) thus gives the transform of $\{J_{0(t)}\}$, and the inverse transform step that links the answer to $\{J_{0(t)}\}$ yields equation (8).

$$S\{J_{0(t)}\} = \left\{ \frac{1}{v^2} - \frac{1}{v^4} + \frac{3}{v^6} - \dots \right\}$$

$$H_{(v)} = \frac{1}{v^2} S\{J_{0(t)}\} \quad (10)$$

Practical example: Transient response of a cylindrical system

The transient response of a cylindrical system in physics and engineering frequently requires the solution of differential equations with boundary conditions determined by the object's geometry. system, like a thin circular plate or a cylindrical heat conductor, responds to a time-dependent stimulus by using the Sadik Transform. the Problem since by equation (7,8,10) we option:

Examine a mechanical system in which the following second-order differential equation controls the displacement $h(t)$:

$$(h)''t + h_{(t)} = J_{0(t)}$$

With Initial conditions: $h(0)=0$ and $h'(0)=0$

$$v^2 h_{(t)} + h_{(t)} = S\{J_{0(t)}\}$$

We use the following property for second-order derivatives by applying the Sadik Transform $S\{h(t)\} = h(v)$ to both sides of the equation:

$$S\{J_{0(t)}\} = \frac{1}{\sqrt{v^2+1}}$$

Substituting the known transform for the zero-order Bessel function,

$$(v^2 + 1) H_{(v)} = \frac{1}{\sqrt{v^2+1}} \text{ we option}$$

Solving for $h(t)$, we get:

$$H_{(v)} = \frac{1}{(v^2+1)^{3/2}}$$

Using the inverse Sadik transform property for Bessel functions, the solution in the time domain is:

$$h_{(t)} = \frac{t}{2} J_{1(t)}$$

Conclusion

In this work, we have effectively illustrated how the Sadik integral transform can be used to solve differential equations involving Bessel functions. We have demonstrated that the Sadik transform offers a simple and effective substitute for traditional techniques like Laplace or Hankel transforms by moving the analysis from the time domain to the algebraic v -domain. The power of the Sadik transform is demonstrated by modeling the transient response of cylindrical systems and the vibration of a system under Bessel-type excitation. It produces results like $h_{(t)} = \frac{t}{2} J_{1(t)}$ with little computational overhead by breaking down complicated differential equations into manageable algebraic forms. In the end, this study demonstrates that the Sadik transform is a very useful solution for engineers and physicists working with cylindrical or spherical coordinates in addition to being a lovely mathematical tool. Its capacity to solve non-homogeneous and fractional equations indicates that it will become a more valuable tool in the field of advanced computational engineering.

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حل دوال بيسل باستخدام تحويل صادق

المستخلص:

(لدوال بيسل يُعد *Sadik Transform* في هذا البحث، نُبَيِّن أن تحويل صادق) أداة أساسية في حل مجموعة متنوعة من المعادلات التفاضلية والمسائل الرياضية. يهدف هذا البحث بشكل رئيسي إلى حل العلاقات المرتبطة بدوال بيسل من خلال تقييم هذه الدوال باستخدام تحويل صادق التكامل. تُستخدم دوال بيسل في حل العديد من المعادلات في الإحداثيات الأسطوانية أو الكروية، مثل معادلات الحرارة، والموجات، ولابلاس، هلمهولتز . علاوة على ذلك، تستعرض هذه الدراسة اشتقاق تحويل صادق لدوال بيسل من النوع الأول، وتوضح كيفية توظيفه في حل المعادلات التفاضلية الكسرية وتقييم التكاملات المعقدة. وتُظهر النتائج فعالية تحويل صادق وبساطته كبديل قوي للتحويلات التكاملية التقليدية، مما يعزز من قابليته للتطبيق في العلوم الفيزيائية والهندسة المتقدمة.