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## Optimal control of dengue transmission models with seasonal mosquito dynamics: A literature review

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### Abstract

Dengue transmission depends heavily on the mosquito population that carries it, and that population itself varies with the seasons, growing in warm, wet periods and shrinking in cool or dry ones. This review traces the literature that combines two originally separate strands of mathematical epidemiology: the optimal control framework built on Pontryagin's Maximum Principle, and compartmental dengue models in which the mosquito recruitment or birth rate is allowed to vary periodically over the year rather than held constant. Starting from Pontryagin's original optimal control theory and Esteva and Vargas's foundational dengue transmission model, this review traces how later studies added disease-free and endemic equilibrium analysis, optimal control of interventions such as bed nets, insecticide, and vaccination, and, in a smaller subset of studies, an explicitly seasonal mosquito term. Of the eight studies reviewed here, only three combine optimal control with seasonal mosquito dynamics in the same model, and cost-effectiveness analysis is reported in fewer still, suggesting that models combining all of seasonality, optimal control, and formal cost-effectiveness comparison remain a comparatively narrow strand of this literature.

**Keywords:** Dengue transmission, optimal control, Pontryagin's maximum principle, seasonal mosquito dynamics, compartmental model, disease-free equilibrium, cost-effectiveness analysis, vector-borne disease

### Introduction

Dengue is transmitted between humans by *Aedes* mosquitoes, which means that any mathematical account of dengue transmission is really an account of two coupled populations, human and mosquito, rather than one. Esteva and Vargas's 1998 paper gave this coupled system its now-standard compartmental formulation, dividing both populations into susceptible and infectious classes, establishing the model's disease-free and endemic equilibria, and proving global stability results using the theory of competitive systems<sup>[2]</sup>. Their model, like most that followed it for the next decade, treated the mosquito population and its parameters as constant over time.

Separately, Pontryagin and colleagues had already established, in 1962, the general theory of optimal control: given a dynamical system and a cost functional to minimise, their Maximum Principle provides necessary conditions, expressed through an auxiliary Hamiltonian function and a system of adjoint equations, that any optimal control must satisfy<sup>[1]</sup>. Lenhart and Workman's 2007 textbook adapted this general theory specifically for biological and epidemiological models, and has since become the standard reference every optimal-control dengue paper reviewed here cites for its numerical method<sup>[3]</sup>. Rodrigues, Monteiro, and Torres brought the two strands together for dengue specifically in 2010, applying Pontryagin's Maximum Principle to a dengue model with a constant mosquito population to determine optimal insecticide spraying strategies<sup>[4]</sup>.

The remaining step, allowing the mosquito population itself to vary seasonally within an optimal control framework, has been taken by a smaller number of studies. This review traces that combination specifically, alongside the broader optimal-control dengue literature it builds on.

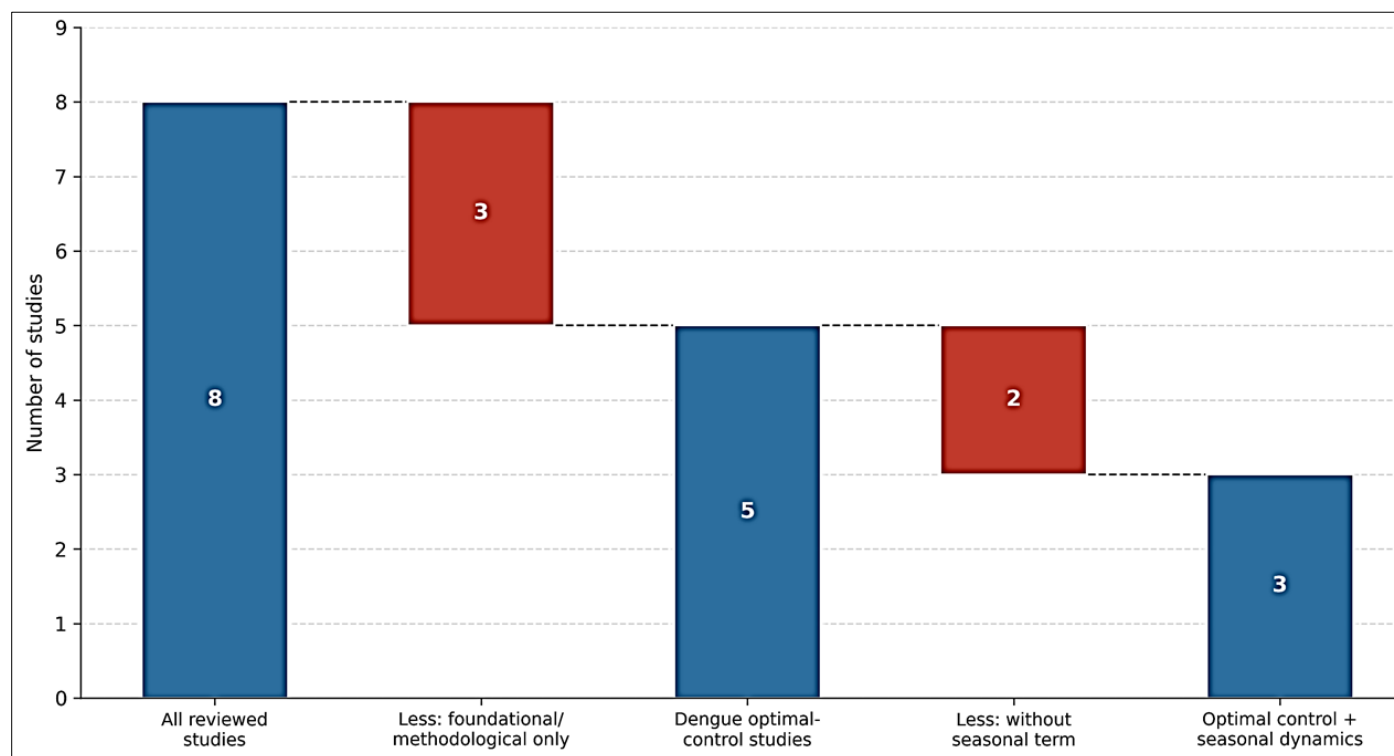


**Fig 1:** From all reviewed studies to the optimal-control-plus-seasonality subset ( $n = 8$ )

Of the eight studies reviewed here, three are foundational or purely methodological rather than dengue-specific optimal control studies. Of the five that do apply optimal control to a dengue model, two do not include a seasonal mosquito term, leaving three studies, just over a third of the total, that combine both features together.

#### Foundations: Optimal control theory and the dengue compartmental model

Pontryagin's Maximum Principle works by introducing an adjoint variable for each state variable in the dynamical system, combining them into a Hamiltonian function alongside the running cost, and showing that an optimal control must maximise (or minimise) this Hamiltonian at every instant, subject to the adjoint variables satisfying their own differential equations with a transversality condition at

the final time <sup>[1]</sup>. This turns an optimal control problem into a two-point boundary value problem, generally solved numerically using a forward-backward sweep method that alternates between solving the state equations forward in time and the adjoint equations backward in time until the two converge.

Esteva and Vargas's compartmental model, meanwhile, established the basic structure that later optimal-control studies extend: susceptible and infectious human compartments, susceptible and infectious mosquito compartments, and a basic reproduction number that determines whether the disease-free equilibrium is stable or an endemic equilibrium exists and is stable instead <sup>[2]</sup>. Every dengue-specific study reviewed here retains some version of this basic structure, adding control variables, a seasonal term, or both.

**Table 1:** Chronological summary of reviewed studies

| Study (Year)                        | Ref. | Category                   | Method                                   | Key Contribution                                            |
|-------------------------------------|------|----------------------------|------------------------------------------|-------------------------------------------------------------|
| Pontryagin <i>et al.</i> (1962)     | [1]  | Foundational               | Maximum principle                        | Established the general theory of optimal processes         |
| Esteva & Vargas (1998)              | [2]  | Foundational               | Global stability analysis                | Foundational dengue transmission model                      |
| Lenhart & Workman (2007)            | [3]  | Foundational               | Textbook / methodology                   | Standard reference for optimal control in biological models |
| Rodrigues, Monteiro & Torres (2010) | [4]  | Optimal control only       | Pontryagin's Maximum Principle           | Early optimal control study of dengue epidemics             |
| Buonomo & Della Marca (2018)        | [5]  | Optimal control + seasonal | Periodic mosquito recruitment, PMP       | Optimal bed-net use under seasonal mosquito pattern         |
| Pliego-Pliego <i>et al.</i> (2020)  | [6]  | Optimal control + seasonal | Season-dependent controls, cost-benefit  | Season-specific insecticide/larvicide strategy              |
| Bakhtiar, Jaharuddin & Sari (2023)  | [7]  | Optimal control + seasonal | Periodic mosquito birth rate, PMP        | Vaccination and Wolbachia under seasonal aspects            |
| Herdicho <i>et al.</i> (2025)       | [8]  | Optimal control only       | Real-data parameter estimation, Lyapunov | Awareness-based dengue model fitted to East Java data       |

#### Adding optimal control and seasonal mosquito dynamics

Rodrigues, Monteiro, and Torres's 2010 study was among the first to apply Pontryagin's Maximum Principle to a dengue model specifically, treating insecticide spraying as a time-

dependent control and using Lenhart and Workman's forward-backward sweep method to compute the optimal spraying schedule, though their underlying mosquito population remained constant over time <sup>[3, 4]</sup>.

Buonomo and Della Marca's 2018 study introduced the seasonal element directly into an optimal control framework, replacing the constant mosquito recruitment rate with a periodic function and optimising bed-net usage as the control variable, showing that the optimal control schedule itself becomes seasonal, with higher recommended bed-net use during the mosquito season <sup>[5]</sup>. Pliego-Pliego and colleagues took a related but distinct approach, modelling the *Aedes aegypti* life cycle explicitly across aquatic and aerial stages with season-dependent parameters, and applying three separate season-specific controls, insecticide and larvicide during the hot season and breeding-site elimination during the cool season, evaluated through a combined optimal-control

and cost-benefit framework <sup>[6]</sup>. Bakhtiar, Jaharuddin, and Sari extended the seasonal approach further in 2023, adding both vaccination and Wolbachia-based biocontrol as interventions within a periodically forced mosquito birth rate model, again solved via Pontryagin's Maximum Principle <sup>[7]</sup>. Herdicho and colleagues' 2025 study takes a different direction, fitting a dengue model with an aware human sub-population to real monthly infection data from East Java, and applying optimal control to awareness, treatment, and vector-control measures, but without an explicit seasonal mosquito term, relying instead on the fitted constant parameters to match the observed data pattern <sup>[8]</sup>.

**Table 2:** Key concepts referenced in this review

| Concept                             | Ref. | Description                                                                                                                                                                 |
|-------------------------------------|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Pontryagin's Maximum Principle      | [1]  | A set of necessary conditions that an optimal control must satisfy, expressed through a Hamiltonian function and a system of adjoint (co-state) equations                   |
| Seasonal mosquito recruitment       | [5]  | A periodic (typically sinusoidal) term replacing a constant mosquito birth or recruitment rate, capturing wet-season and dry-season differences in vector abundance         |
| Cost-effectiveness analysis         | [6]  | A comparison of control strategies that weighs the reduction in infections achieved against the implementation cost of each strategy, to identify the most efficient option |
| Disease-free and endemic equilibria | [2]  | The two steady states of a compartmental epidemic model: one with zero infection, and one where infection persists at a constant positive level                             |

Table 2 defines the four technical terms most central to this review; the combination of the first and second rows within a single model is the specific focus this review set out to trace.

**Table 3:** Reference distribution by journal or source

| Journal / Source                                      | Number of References |
|-------------------------------------------------------|----------------------|
| Book (Wiley)                                          | 1                    |
| Mathematical Biosciences                              | 1                    |
| Book (Chapman & Hall/CRC)                             | 1                    |
| Mathematical and Computer Modelling                   | 1                    |
| Mathematical Methods in the Applied Sciences          | 1                    |
| Applied Mathematical Modelling                        | 1                    |
| BAREKENG: Journal of Mathematics and Its Applications | 1                    |
| PLoS One                                              | 1                    |

No single journal contributes more than one reference among the eight reviewed here; the two foundational books, Pontryagin and colleagues and Lenhart and Workman, sit

alongside six references spread across six distinct applied-mathematics and public-health journals.

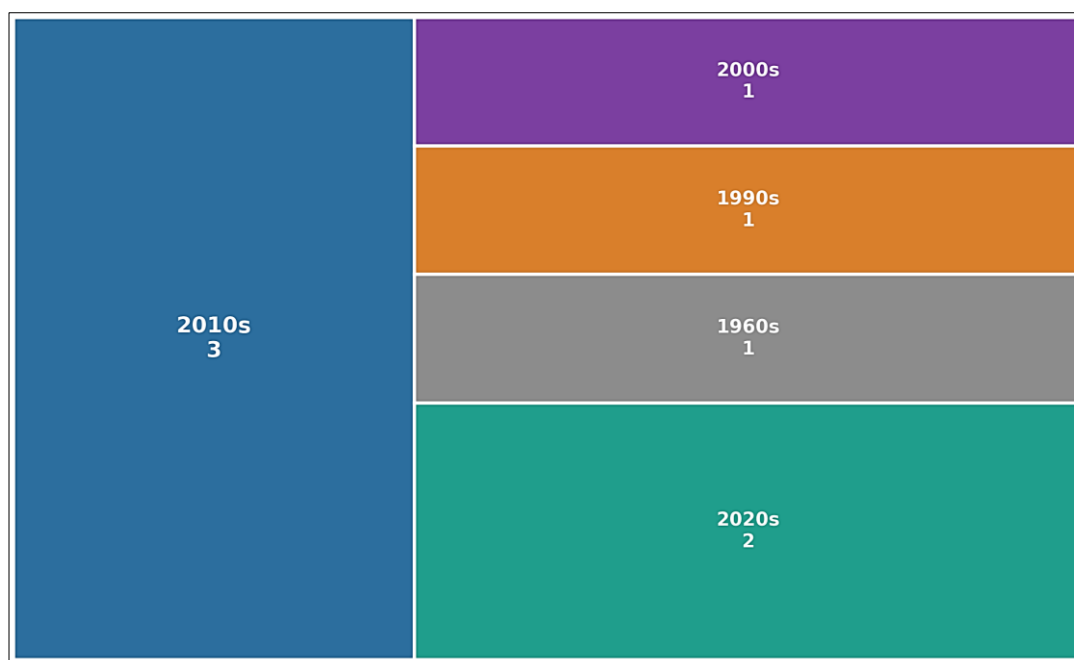


**Fig 2:** Reviewed references by decade of publication ( $n = 8$ )

Three of the eight references, the largest share, fall in the 2010s, the decade containing both the first dengue-specific optimal control study and the first two seasonal-plus-optimal-

control combinations. The 1960s, 1990s, and 2000s each contribute exactly one foundational reference, and the 2020s contribute two, both dengue-specific.

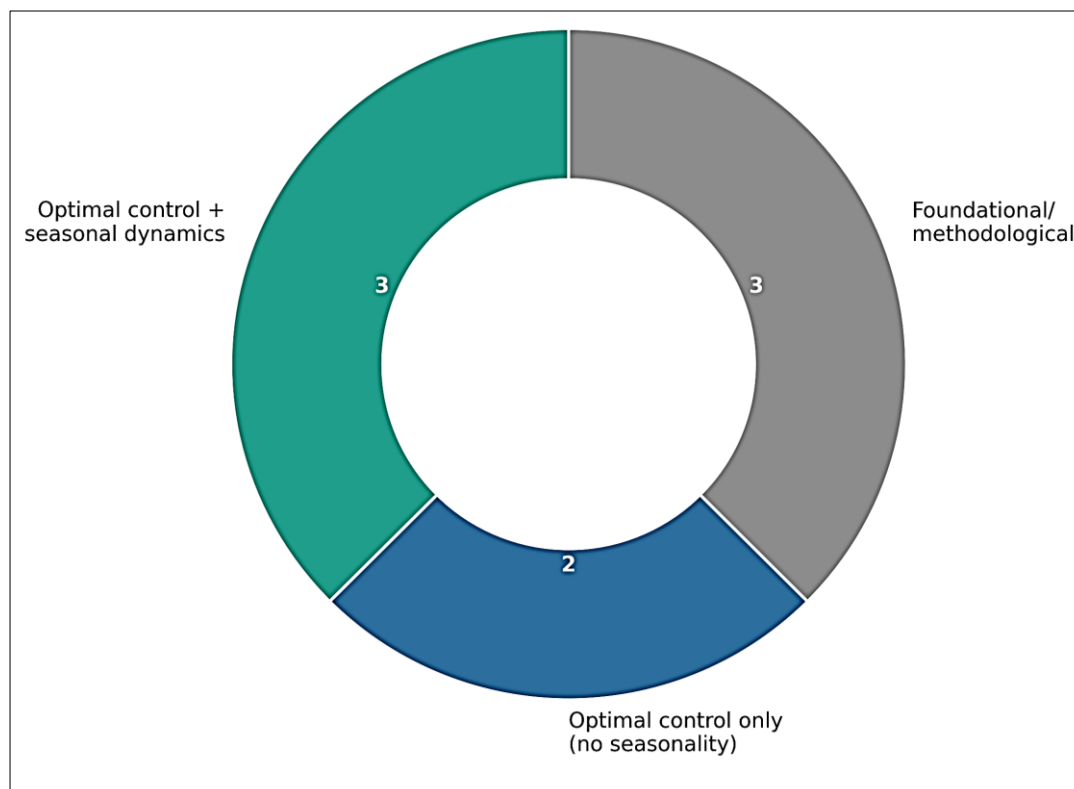


**Fig 3:** Reviewed studies by modelling category ( $n = 8$ )

The eight reviewed studies split into three foundational or methodological references, two dengue-specific optimal control studies without seasonality, and three that combine

optimal control with seasonal mosquito dynamics, the specific combination this review set out to trace.

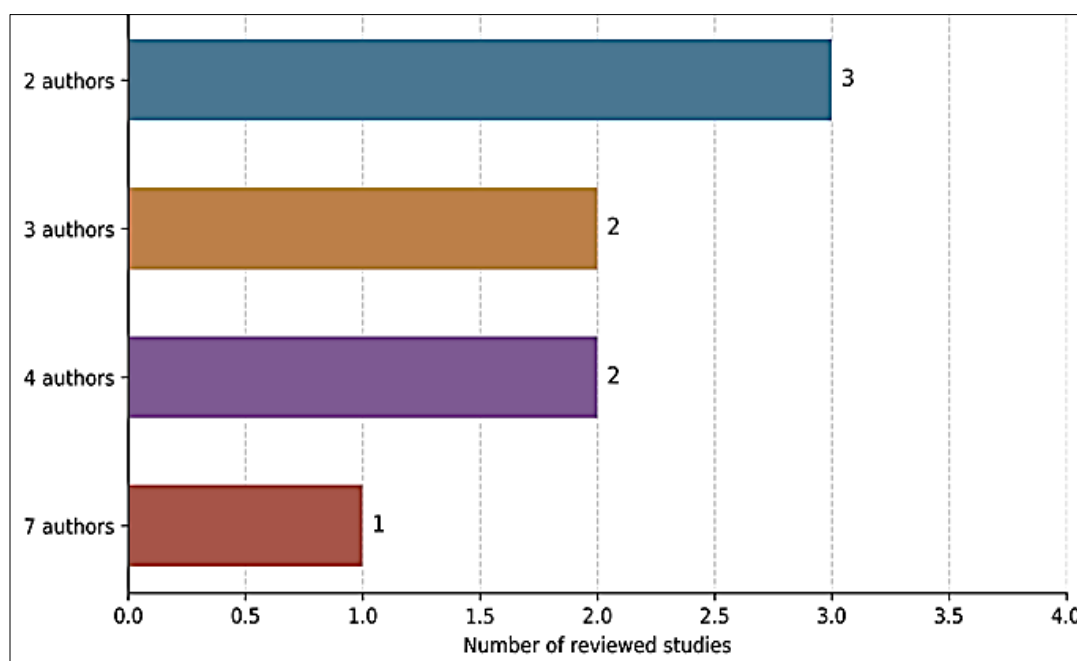


**Fig 4:** Reviewed studies by author-team size ( $n = 8$ )

Two-author teams are the most common in this set, followed by three- and four-author teams with two studies each. Herdicho and colleagues' 2025 paper, with seven authors, is a clear outlier, reflecting the larger collaborative effort often associated with fitting a model to real surveillance data.

### Synthesis and research gaps

Read together, these eight studies show two originally separate lines of work, optimal control theory and compartmental dengue modelling, converging gradually rather than all at once. Rodrigues, Monteiro, and Torres

established that Pontryagin's Maximum Principle could be applied directly to a dengue model, but without seasonality<sup>[4]</sup>. Buonomo and Della Marca, Pliego-Pliego and colleagues, and Bakhtiar, Jaharuddin, and Sari then each added a seasonal mosquito term to that same optimal-control framework, but each paired it with a different specific intervention, bed nets, season-specific insecticide and larvicide, and vaccination with Wolbachia respectively, rather than building toward a single unified seasonal framework covering multiple intervention types at once<sup>[5, 7]</sup>.

This leaves a specific combination under-explored: a dengue optimal control model with seasonal mosquito dynamics that also incorporates real epidemiological data of the kind Herdicho and colleagues used, and a full cost-effectiveness comparison of the kind Pliego-Pliego and colleagues carried out, within the same study<sup>[6, 8]</sup>. None of the three seasonal-plus-optimal-control studies reviewed here validates its model against real case-count data in the way Herdicho and colleagues did for a non-seasonal model, and only Pliego-Pliego and colleagues among the seasonal studies carries out a full cost-effectiveness comparison across control strategies.

A second observation concerns intervention type. Across the three seasonal-plus-optimal-control studies, no two use the same combination of interventions; bed nets, chemical and mechanical vector control, and vaccination with Wolbachia are each examined separately rather than compared against one another within a common seasonal framework, which would help clarify which intervention type is most cost-effective once seasonality is taken into account.

## Conclusion

This review has traced how optimal control theory, established by Pontryagin and colleagues and adapted for biology by Lenhart and Workman, has been combined with compartmental dengue models descending from Esteva and Vargas's foundational formulation, and, in a smaller subset of studies, with an explicitly seasonal mosquito recruitment term.

The clearest gap identified here is the absence of a single study that combines seasonal mosquito dynamics, optimal control of multiple intervention types, validation against real epidemiological data, and a formal cost-effectiveness comparison all together. Given that each of these elements already has at least one precedent among the eight studies reviewed here, assembling them into a single framework looks like an achievable next step for this literature, and would provide a more complete basis for choosing among dengue control strategies under realistic, seasonally varying mosquito conditions.

## Declarations

**Conflict of interest:** The author(s) declare no conflict of interest.

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