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Novel entropy measures for q -rung orthopair fuzzy sets and their applications in multi-criteria decision making

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Abstract

The q -rung orthopair fuzzy set (q -ROFS) is a flexible generalisation of intuitionistic and Pythagorean fuzzy sets that widens the admissible space of membership and non-membership grades through an adjustable rung parameter q . Quantifying the fuzziness carried by such information is central to determining criterion importance in decision problems where weights are not known a priori. This paper proposes a novel trigonometric entropy measure for q -rung orthopair fuzzy sets that simultaneously incorporates the membership, non-membership and hesitancy (indeterminacy) grades, thereby aligning the measure with the three-dimensional structure of spherical fuzzy sets. The measure is shown, with complete proofs, to satisfy the four De Luca–Termini type axioms boundedness, sharpness, maximality and symmetry together with a monotonicity (resolution) property. An entropy-based objective weighting scheme and a full multi-criteria decision-making (MCDM) algorithm built on the q -rung orthopair fuzzy weighted averaging operator are then developed. The framework is illustrated on a green-supplier selection problem described by a 5×6 decision matrix (five suppliers evaluated on six criteria), for which the proposed method yields the ranking $A_1 > A_4 > A_2 > A_3 > A_5$. Sensitivity analysis across values of q and comparisons against equal weighting and the geometric aggregation operator confirm both the stability and the discriminating power of the measure.

Keywords: q -rung orthopair fuzzy set; entropy measure; spherical fuzzy set; hesitancy-aware; attribute weighting; multi-criteria decision making; green supplier selection

Introduction

Decision problems in engineering, management and the environmental sciences are routinely affected by vagueness and imprecision that ordinary crisp numbers cannot express. Since Zadeh's fuzzy sets [15], a hierarchy of generalisations has been advanced to capture increasingly nuanced expert judgements. Atanassov's intuitionistic fuzzy set (IFS) added a non-membership grade constrained by $\mu + \nu \leq 1$ [1], and Yager's Pythagorean fuzzy set (PFS) relaxed this to $\mu^2 + \nu^2 \leq 1$ [2]. To further enlarge the region of acceptable grades, Yager introduced the q -rung orthopair fuzzy set (q -ROFS), in which the constraint becomes $\mu^q + \nu^q \leq 1$ for $q \geq 1$ [3]. As q increases, the admissible orthopair region expands, so decision makers can freely assign grades that would violate the IFS or PFS constraints; IFS ($q=1$) and PFS ($q=2$) arise as special cases.

The expressive power of the q -ROFS has driven a rapid growth of aggregation operators [4, 5], distance and similarity measures [7, 11], and information measures [6, 12, 16], together with applications spanning supplier selection, fuel-technology assessment, intelligent transportation, and sustainable development [9, 10, 13, 14, 17]. A parallel line of research spherical fuzzy sets (SFS) [8] represents judgements through three *independent* degrees (membership, non-membership and hesitancy) restricted by $\mu^2 + \nu^2 + \pi^2 \leq 1$. Although the two models differ in whether hesitancy is derived (q -ROFS) or independent (SFS), both encode the same triad of information, which motivates entropy measures that treat all three grades on an equal footing. The determination of criterion weights is itself a delicate problem. Subjective weights, elicited directly from experts, are easy to obtain but vulnerable to bias, inconsistency and strategic manipulation, and they may bear little relation to how discriminating a criterion actually is on

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the alternatives at hand. Objective weighting schemes derive the weights from the assessment data, so that a criterion earns importance only to the extent that it separates the alternatives clearly. Entropy weighting is the most widely adopted objective scheme, but its behaviour is only as good as the underlying entropy measure.

Entropy quantifies the amount of fuzziness or uncertainty embodied in a fuzzy set and plays a decisive role when attribute weights must be inferred from the data itself. A criterion on which experts express decisive, low-fuzziness judgements is more informative and should carry a larger weight than a criterion on which the ratings are highly ambiguous. Despite the abundance of aggregation operators for q -rung orthopair information, entropy measures that (i) explicitly involve the hesitancy grade, (ii) admit closed-form axiomatic proofs, and (iii) reduce coherently to the spherical fuzzy setting remain comparatively scarce.

Contributions: This paper addresses that gap with the following contributions: (i) A novel trigonometric (sine-based) entropy measure for q -rung orthopair fuzzy sets is proposed that simultaneously incorporates membership, non-membership and hesitancy, thereby aligning the measure with the three-dimensional structure of spherical fuzzy sets. (ii) The five axiomatic properties boundedness, sharpness, maximality, symmetry and monotonicity are established with complete proofs, and the measure is shown to transfer to spherical fuzzy sets. (iii) An objective attribute-weighting model and a complete q -ROFS MCDM algorithm based on the q -rung orthopair fuzzy weighted averaging (q -ROFWA) operator are formulated. (iv) The framework is demonstrated on a green-supplier selection problem with a 5×6 decision matrix, followed by sensitivity and comparative analyses.

The need for a hesitancy-aware entropy is further underscored by the observation that, in many real-world evaluations, the panel's uncertainty about a supplier or alternative is itself a valuable piece of information. A criterion on which every expert is highly hesitant may signal a lack of available data or an inherently difficult-to-assess dimension, and an entropy that registers this hesitancy will automatically suppress such criteria in the weight vector. By contrast, a gap-based entropy that ignores hesitancy may inadvertently assign a non-trivial weight to a criterion on which experts have merely guessed, leading to a ranking that is driven partly by noise rather than by genuine evaluative content. The proposed trigonometric entropy addresses this shortcoming by embedding the hesitancy grade symmetrically alongside membership and non-membership in the ratio Φ , thereby ensuring that the measure is responsive to all three dimensions of a q -ROFS evaluation.

The remainder of the paper is organised as follows. Section 2 surveys related work. Section 3 reviews preliminaries. Section 4 introduces the entropy measure and its proofs. Sections 5 and 6 present the weighting model and the decision algorithm. Section 7 reports the application, and Section 8 the sensitivity and comparative studies. Section 9 concludes.

Related Work

The literature relevant to this study clusters around four themes: the theoretical foundations of orthopair fuzzy sets, aggregation operators, information measures, and decision-making applications.

Foundations and extensions: The orthopair family originates with Atanassov's intuitionistic fuzzy set ^[1], whose additive

constraint $\mu + \nu \leq 1$ Yager relaxed quadratically in the Pythagorean fuzzy set ^[2] and, subsequently, to arbitrary order in the q -rung orthopair fuzzy set ^[3]. The rung parameter grants the model a tunable expressiveness: any orthopair the decision maker wishes to state can be accommodated by choosing q large enough. In a complementary development, Kutlu Gündoğdu and Kahraman introduced spherical fuzzy sets, in which membership, non-membership and hesitancy are three independent coordinates on a unit sphere ^[8]. The present work exploits the shared three-degree structure of these two models.

Aggregation operators: Liu and Wang were the first to systematise information fusion in the q -rung setting, proposing the weighted averaging and geometric operators (q -ROFWA and q -ROFWG) that underpin most subsequent decision methods ^[4]. Peng, Dai and Garg later devised exponential operations together with a refined score function to overcome ranking ties ^[5]. These operators supply the aggregation machinery used in Section 6, but they presuppose a criterion weight vector, which is precisely what an entropy measure must furnish when weights are unknown.

Information measures: Peng and Liu established the inter-relations among distance, similarity, entropy and inclusion measures for q -rung orthopair fuzzy sets, providing systematic transformation formulae ^[6]. Liu, Chen and Peng developed cosine similarity and Euclidean distance measures ^[7], and more recently Ganie proposed several similarity and entropy measures with pattern-recognition and decision applications ^[12]. Khan *et al.* ^[16] introduced knowledge-based entropy measures for q -ROFS with applications in medical diagnosis, while Verma and Merigo ^[18] developed exponential entropy and distance measures under the Pythagorean fuzzy framework that extend naturally to higher-rung settings. A recurring shortcoming of purely membership-oriented constructions is that q -ROFNs differing only in hesitancy may receive identical scores, a deficiency the proposed measure explicitly targets (Section 4.4).

A closer examination of the existing q -ROFS entropy landscape reveals that the available measures can be broadly classified into three families according to the information they utilise. The first family, exemplified by the cosine-type entropy of Peng and Liu ^[6], measures fuzziness through the angular separation between the q -rung-powered membership and non-membership grades, effectively computing a normalised version of $|\mu^q - \nu^q|$. The second family, represented by the distance-based entropies of Ganie ^[12], quantifies fuzziness as the proximity of a q -ROFN to the nearest crisp set, again relying primarily on the membership–non-membership gap. A third, more recent category includes ratio-based constructions in which the minimum of the q -rung grades is compared with the maximum. All three families share a common structural limitation: two q -ROFNs that happen to exhibit the same gap $|\mu^q - \nu^q|$ but carry markedly different hesitancy levels will receive identical or nearly identical entropy scores, thereby concealing decision-relevant uncertainty that the q -ROFS model was designed to capture.

To illustrate this deficiency concretely, consider the three q -ROFNs $B_1 = (0.794, 0.585)$, $B_2 = (0.843, 0.669)$, and $B_3 = (0.737, 0.464)$, all of which share $|\mu^3 - \nu^3| = 0.30$ at $q = 3$ but possess π^q values of 0.30, 0.10, and 0.50, respectively. The cosine entropy of Peng and Liu ^[6] returns an identical value of 0.8910 for all three numbers, completely failing to differentiate a confident assessment (B_2 , low hesitancy) from a highly uncertain one (B_3 , high hesitancy). The proposed

measure, in contrast, assigns $E(B_2) = 0.7818 < E(B_1) = 0.8315 < E(B_3) = 0.8660$, correctly ranking the numbers in decreasing order of certainty. This monotone response to hesitancy is not merely a theoretical nicety; in the weighting step of the MCDM algorithm (Section 5), it ensures that criteria on which the panel expresses confident, low-hesitancy evaluations receive substantially larger weights than criteria assessed with comparable membership–non-membership gaps but pervasive indecision.

The comparative analysis presented later in this paper (Section 4.4 and Table 2) provides concrete numerical evidence that the proposed measure overcomes this shortcoming. In particular, the three-number experiment with B_1 , B_2 , and B_3 demonstrates that hesitancy-free measures assign identical entropy values to q -ROFNs that differ substantially in their confidence levels, whereas the proposed sine-based measure correctly discriminates among them. This discrimination capability has direct practical consequences: in the green supplier selection application (Section 7), the proposed entropy assigns a weight of 0.3807 to product quality versus 0.0037 to after-sales service, a 100-fold difference that reflects the genuine variation in evaluation decisiveness across criteria and that would be substantially attenuated under any gap-only entropy formulation.

Decision applications: q -rung orthopair fuzzy methods have been applied widely. Krishankumar *et al.* solved a green-supplier selection problem with unknown weight information [9]; Rani and Mishra combined the framework with a weighted aggregated sum product assessment for fuel-technology selection [10]; and Pinar and Boran built a group decision method for supplier selection on a novel distance measure [11]. Very recent contributions include Deveci *et al.*'s OPA-RAFSI model for autonomous-vehicle mobility [13], Qin *et al.*'s Choquet-integral VIKOR approach under a hesitant q -rung environment [14], and Wang *et al.*'s [19] multi-attribute group decision framework integrating power aggregation under q -ROFS. Against this backdrop, an entropy measure that is hesitancy-aware, axiomatically complete and spherically compatible, and that plugs directly into an operator-based ranking procedure, fills a clear methodological gap.

Preliminaries

This section collects the definitions used throughout. Let $X = \{x_1, x_2, \dots, x_n\}$ be a finite universe of discourse.

Definition 1 (q -rung orthopair fuzzy set [3]).

A q -rung orthopair fuzzy set A on X is $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where $\mu_A: X \rightarrow [0, 1]$ and $\nu_A: X \rightarrow [0, 1]$ denote the membership and non-membership grades, subject to

$$0 \leq \mu_A(x)^q + \nu_A(x)^q \leq 1, q \geq 1. \quad (1)$$

The hesitancy (indeterminacy) grade is

$$\pi_A(x) = (1 - \mu_A(x)^q - \nu_A(x)^q)^{1/q}. \quad (2)$$

For brevity a pair $a = (\mu, \nu)$ satisfying the constraint is called a q -rung orthopair fuzzy number (q -ROFN). When $q = 1$ the model reduces to an IFS and when $q = 2$ to a PFS. The complement of A is $A^c = \{(x, \nu_A(x), \mu_A(x))\}$.

Definition 2 (Operations on q -ROFNs [4])

For two q -ROFNs $a_1 = (\mu_1, \nu_1)$ and $a_2 = (\mu_2, \nu_2)$ and a scalar $\lambda > 0$:

$$a_1 \oplus a_2 = ((\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q)^{1/q}, \nu_1 \nu_2) \quad (3)$$

$$a_1 \otimes a_2 = (\mu_1 \mu_2, (\nu_1^q + \nu_2^q - \nu_1^q \nu_2^q)^{1/q}) \quad (4)$$

$$\lambda a_1 = ((1 - (1 - \mu^q)^m)^{1/q}, \nu^m) \quad (5)$$

Definition 3 (Score and accuracy [4]).

For a q -ROFN $a = (\mu, \nu)$ the score and accuracy functions are

$$S(a) = \mu^q - \nu^q \in [-1, 1], H(a) = \mu^q + \nu^q \in [0, 1]. \quad (6)$$

For two q -ROFNs, $a_1 > a_2$ if $S(a_1) > S(a_2)$; ties in the score are broken by the accuracy.

Definition 4 (q -ROFWA operator [4]).

Let $a_j = (\mu_j, \nu_j)$, $j = 1, \dots, n$, be a collection of q -ROFNs with weight vector $\mathbf{w} = (w_1, \dots, w_n)^T$, $w_j \geq 0$, $\sum w_j = 1$. The q -rung orthopair fuzzy weighted averaging operator is

$$q\text{-ROFWA}(a_1, \dots, a_n) = ((1 - \prod_j (1 - \mu_j^q)^{w_j})^{1/q}, \prod_j \nu_j^{w_j}) \quad (7)$$

Definition 5 (Spherical fuzzy set [8]).

A spherical fuzzy set A on X assigns three independent grades $\mu_A, \nu_A, \pi_A \in [0, 1]$ with $\mu_A(x)^2 + \nu_A(x)^2 + \pi_A(x)^2 \leq 1$. Unlike the q -ROFS, in which π is a derived quantity, the spherical fuzzy set treats hesitancy as a free parameter, yet both share the same three-degree description of an expert opinion.

Remark 1 (q -ROFS versus SFS: the status of hesitancy).

Although q -ROFS and SFS both associate three grades (μ, ν, π) with each element, the two models differ fundamentally in how the third grade arises. In a q -ROFS, hesitancy is a derived quantity: given the orthopair (μ, ν) and the rung q , the hesitancy is fully determined as $\pi = (1 - \mu^q - \nu^q)^{1/q}$. The decision maker specifies only μ and ν ; π carries no additional degree of freedom. In a spherical fuzzy set, by contrast, μ, ν , and π are three independent parameters whose squares must satisfy $\mu^2 + \nu^2 + \pi^2 \leq 1$. A q -ROFS entropy that depends on μ^q, ν^q , and $\pi^q = 1 - \mu^q - \nu^q$ is not introducing a redundant variable; rather, it ensures that the entropy reflects the full information content of the orthopair constraint. When the same form is transplanted to the spherical setting (Corollary 1), the derived grade becomes a genuinely independent input, and the spherical entropy inherits all axiomatic properties except monotonicity, which requires the side condition that π be held fixed.

Definition 6 (Axioms of a q -ROFS entropy).

A mapping $E: q\text{-ROFS}(X) \rightarrow [0, 1]$ is a q -rung orthopair fuzzy entropy if it satisfies: (E1) Boundedness: $0 \leq E(A) \leq 1$; (E2) Sharpness: $E(A) = 0$ iff A is crisp, i.e. $(\mu_i, \nu_i) \in \{(1, 0), (0, 1)\}$ for every i ; (E3) Maximality: $E(A) = 1$ iff $\mu_i = \nu_i$ for every i ; (E4) Symmetry: $E(A) = E(A^c)$; (E5) Monotonicity (resolution): $E(A) \leq E(B)$ whenever A is less fuzzy than B .

A Novel Entropy Measure

Definition and Rationale

An informative entropy for orthopair fuzzy data should attain its maximum when membership and non-membership are indistinguishable ($\mu = \nu$), vanish for crisp opinions, and grow with the hesitancy carried by an element.

The central innovation of the proposed construction lies in the treatment of the hesitancy grade as a first-class component of the entropy rather than a residual by-product. Traditional gap-

based entropy measures for q -ROFS operate on the single statistic $|\mu^q - v^q|$. While this statistic captures how decisively an expert leans towards membership or non-membership, it is entirely silent on how much information remains uncommitted that is, how large the hesitation margin π^q is. Two evaluations can exhibit the same gap yet embody fundamentally different levels of confidence: a pair $(\mu, v) = (0.843, 0.669)$ at $q = 3$ yields $\pi^q \approx 0.10$ (high confidence), whereas $(0.737, 0.464)$ yields the same gap but $\pi^q \approx 0.50$ (high indecision). An entropy that ignores π conflates these two qualitatively distinct assessments and therefore misinforms the weight-determination step.

The proposed ratio Φ_i resolves this conflation by embedding the hesitancy alongside both the dominant and subordinate q -rung grades. Algebraically (Lemma 1), Φ_i simplifies to $(1 - M_i)/(1 - m_i)$, which increases whenever the dominant grade M_i decreases or the subordinate grade m_i increases. Crucially, for a fixed gap $M_i - m_i$, an increase in π^q pushes Φ_i upward and hence raises the entropy, producing exactly the discrimination that gap-only measures lack. The sine transformation preserves this ordering while compressing the scale into $[0, 1]$ in a manner that enhances sensitivity near the balanced point $\mu = v$.

Definition 7 (Proposed entropy).

Let $A = \{\{x_i, \mu_i, v_i\}\}^n$ be a q -ROFS, and write $\pi^q = 1 - \mu^q - v^q$. Define

$$E(A) = (1/n) \sum_i \sin(\pi/2 \cdot \Phi_i) \quad (8)$$

Where;

$$\Phi_i = [\min(\mu_i^q, v_i^q) + \pi_i^q] / [\max(\mu_i^q, v_i^q) + \pi_i^q] \quad (9)$$

All three grades μ_i, v_i, π_i appear explicitly in (9), which is what makes the measure compatible with the spherical fuzzy representation (Corollary 1).

Lemma 1

For each i , writing $M_i = \max(\mu_i^q, v_i^q)$ and $m_i = \min(\mu_i^q, v_i^q)$,

$$\Phi_i = (1 - M_i) / (1 - m_i), \Phi_i \in [0, 1]. \quad (10)$$

Proof: Because $m_i + M_i = \mu_i^q + v_i^q$ and $\pi_i^q = 1 - \mu_i^q - v_i^q$, we have $m_i + \pi_i^q = 1 - M_i$ and $M_i + \pi_i^q = 1 - m_i$, which gives the stated form. Since $0 \leq m_i \leq M_i$ and $M_i \leq 1$, we have $0 \leq 1 - M_i \leq 1 - m_i$ and $1 - m_i > 0$, so $\Phi_i \in [0, 1]$. $\square$

The identity established in Lemma 1 is the algebraic cornerstone of the subsequent analysis: it expresses Φ_i solely in terms of the two order statistics M_i and m_i of the pair $\{\mu_i^q, v_i^q\}$, which makes the dependence on each axiomatic input transparent and tractable. With this identity in hand, the proof of the five axiomatic properties reduces to standard monotonicity arguments on the increasing sine map composed with the ratio $(1 - M_i)/(1 - m_i)$.

4.2 Axiomatic Properties

Theorem 1.

The mapping E in Definition 7 is a q -rung orthopair fuzzy entropy; i.e. it satisfies (E1)–(E5) of Definition 6.

Proof: Throughout, $g(t) = \sin(\pi t/2)$ is continuous and strictly increasing on $[0, 1]$ with $g(0) = 0$ and $g(1) = 1$.

- (E1) Boundedness. By Lemma 1, $\Phi_i \in [0, 1]$, hence $g(\Phi_i)$

$\in [0, 1]$. The average of terms in $[0, 1]$ lies in $[0, 1]$, so $0 \leq E(A) \leq 1$.

- (E2) Sharpness. As g is nonnegative on $[0, 1]$ and vanishes only at 0, $E(A) = 0$ iff $\Phi_i = 0$ for all i , i.e. $1 - M_i = 0$, i.e. $\max(\mu_i^q, v_i^q) = 1$. Given $\mu_i^q + v_i^q \leq 1$, one grade equalling 1 forces the other to be 0; thus $(\mu_i, v_i) \in \{(1, 0), (0, 1)\}$, which is precisely the crisp case.
- (E3) Maximality. Since g attains 1 on $[0, 1]$ only at 1, $E(A) = 1$ iff $\Phi_i = 1$ for all i , i.e. $M_i = m_i$. This holds iff $\mu_i^q = v_i^q$, equivalently $\mu_i = v_i$ (both nonnegative, $q \geq 1$), for every i .
- (E4) Symmetry. In A^c the pair (μ_i, v_i) is replaced by (v_i, μ_i) . The quantities $\min$, $\max$ and π^q are all invariant under this swap, hence each Φ_i and therefore E is unchanged.
- (E5) Monotonicity. Assume A is less fuzzy than B and fix i . Consider the case $\mu_B \geq \mu_A$, so that $M_B = \mu_B^q$, $m_B = v_B^q$, and by hypothesis $\mu_A \geq \mu_B$, $v_A \leq v_B$. Then $M_A \geq M_B$ and $m_A \leq m_B$. Consequently $\Phi_A = (1 - M_A)/(1 - m_A) \leq (1 - M_B)/(1 - m_B) = \Phi_B$. The case $\mu_B \leq \mu_A$ is identical after the membership–non-membership interchange. Because g is increasing, $g(\Phi_A) \leq g(\Phi_B)$ for every i ; summing and dividing by n gives $E(A) \leq E(B)$.

The proof above demonstrates that the axiomatic validity of the measure rests on only two structural ingredients: the orthopair constraint $\mu^q + v^q \leq 1$ with $q \geq 1$, and the strict monotonicity of the sine function on $[0, \pi/2]$. Neither ingredient is specific to any particular value of q , which immediately yields the following specialisation results.

Remark 2.

Setting $q = 1$ or $q = 2$ specialises E to a valid entropy for intuitionistic and Pythagorean fuzzy sets, respectively, since the proofs of Theorem 1 use only $q \geq 1$ and the orthopair constraint.

Remark 3 (Equivalent closed form).

By Lemma 1, the measure admits the compact form $E(A) = (1/n) \sum_i \sin(\pi/2 \cdot (1 - \max(\mu_i^q, v_i^q))/(1 - \min(\mu_i^q, v_i^q)))$, which avoids explicit evaluation of π^q .

Corollary 1 (Spherical fuzzy extension).

Let A be a spherical fuzzy set with independent grades (μ_i, v_i, π_i) and $\mu^2 + v^2 + \pi^2 \leq 1$. Define $E_S(A) = (1/n) \sum \sin(\pi/2 \cdot [\min(\mu_i^2, v_i^2) + \pi_i^2] / [\max(\mu_i^2, v_i^2) + \pi_i^2])$. Then E_S satisfies boundedness, symmetry and maximality, and is monotone under refinements that hold the indeterminacy fixed.

Corollary 2 (Associated knowledge measure).

The mapping $K(A) = 1 - E(A)$ is a knowledge (certainty) measure: it lies in $[0, 1]$, attains its maximum 1 exactly for crisp sets, vanishes exactly when $\mu_i = v_i$ for all i , satisfies $K(A) = K(A^c)$, and reverses the monotonicity of E .

Numerical Illustration of the Measure

Table 1: Element-wise entropy for representative q -ROFNs ($q = 3$)

(μ, v)	μ^3	v^3	π^3	Φ_i	$e_i = \sin(\pi\Phi/2)$
(0.88, 0.20)	0.6815	0.0080	0.3105	0.3228	0.4833
(0.57, 0.57)	0.1852	0.1852	0.6296	1.0000	1.0000
(0.55, 0.58)	0.1664	0.1951	0.6385	0.9641	0.9984
(0.90, 0.30)	0.7290	0.0270	0.2440	0.2786	0.4236
(0.40, 0.60)	0.0640	0.2160	0.7200	0.8364	0.9666

Table 1 evaluates the element entropy for representative q-ROFNs at $q = 3$. The decisive opinion (0.88, 0.20) produces a low entropy ($e = 0.4833$): its large membership grade ($\mu^3 = 0.6815$) and tiny non-membership grade ($\nu^3 = 0.0080$) yield a ratio $\Phi = 0.3228$, which the sine map compresses into the lower half of the $[0, 1]$ scale. The balanced opinion (0.57, 0.57) achieves $\Phi = 1.0000$ and hence the maximum entropy $e = 1.0000$, confirming property (E3). The nearly balanced pair (0.55, 0.58) returns $\Phi = 0.9641$ and $e = 0.9984$, demonstrating the enhanced sensitivity of the sine function near $\Phi = 1$: even a slight departure from perfect balance reduces the entropy noticeably. The asymmetric pair (0.40, 0.60) has a moderate gap but very high hesitancy ($\pi^3 = 0.7200$), resulting in $\Phi = 0.8364$ and $e = 0.9666$ confirming that the measure correctly assigns elevated entropy to hesitant evaluations. These numerical examples validate all five axiomatic properties at the element level and illustrate the practical discrimination capability of the proposed formulation.

It is worth noting that the equivalent closed form of Remark 3 is particularly useful for computational implementation, as it avoids the explicit calculation of π^q and requires only the two order statistics $M_i = \max(\mu_i^q, \nu_i^q)$ and $m_i = \min(\mu_i^q, \nu_i^q)$. Each element entropy is computed in constant time using one comparison, one subtraction, one division and one sine evaluation, making the overall entropy computation $O(n)$ for a q-ROFS defined on n elements.

Comparison with Hesitancy-Free Entropies

To highlight the practical benefit of embedding the hesitancy grade, we contrast the proposed measure E with a representative hesitancy-free cosine entropy E^c [6], which depends solely on the gap $|\mu^q - \nu^q|$, and with the untransformed ratio $E^r(A) = (1/n)\sum \Phi_i$. Table 2 evaluates the three measures on q-ROFNs that share $|\mu^3 - \nu^3| = 0.30$ at $q = 3$ but carry markedly different hesitancy.

Table 2: Three q-ROFNs with identical gap but different hesitancy ($q = 3$)

q-ROFN	π^q	$ \mu^q - \nu^q $	E (prop.)	E^c (cos.)	E^r (ratio)
$B_1 = (0.794, 0.585)$	0.30	0.30	0.8315	0.8910	0.6250
$B_2 = (0.843, 0.669)$	0.10	0.30	0.7818	0.8910	0.5714
$B_3 = (0.737, 0.464)$	0.50	0.30	0.8660	0.8910	0.6667

Because E^c ignores the hesitancy, it returns the identical value 0.8910 for all three numbers and cannot discriminate between a confident and an undecided assessment. The proposed measure, in contrast, orders them exactly as intuition demands: the most hesitant number B_3 receives the highest entropy (0.8660) and the most certain number B_2 the lowest (0.7818). This monotone response to hesitancy is the property that makes the measure suitable for weighting criteria on which experts differ not only in their leaning but also in their confidence.

Entropy-Based Attribute Weighting

In many practical settings the relative importance of criteria is unknown and must be extracted from the assessment data. The proposed entropy provides an objective weighting rule: criteria that receive decisive, low-entropy evaluations are more discriminating and should be weighted more heavily.

Let $D = (d_{ij})_{m \times n}$ be a normalised q -rung orthopair fuzzy decision matrix, where $d_{ij} = (\mu_{ij}, \nu_{ij})$ is the assessment of alternative A_i on criterion C_j . Treating column j as a q -ROFS, its entropy is

$$E_j = (1/m) \sum_i \sin(\pi/2 \cdot \Phi_{ij}), \quad \Phi_{ij} = [\min(\mu_{ij}^q, \nu_{ij}^q) + \pi_{ij}^q] / [\max(\mu_{ij}^q, \nu_{ij}^q) + \pi_{ij}^q] \quad (11)$$

The divergence $1 - E_j$ measures the information content of C_j , and the criterion weights are obtained by normalisation:

$$w_j = (1 - E_j) / \sum_k (1 - E_k), \quad w_j \geq 0, \quad \sum w_j = 1. \quad (12)$$

The Proposed MCDM Method

The complete decision procedure combines the entropy weights of Section 5 with the q-ROFWA aggregation operator.

- **Step 1.** Construct the q -rung orthopair fuzzy decision matrix $D = (d_{ij})_{m \times n}$ from the experts' assessments.
- **Step 2.** Normalise D . For a benefit criterion the rating is retained; for a cost criterion it is complemented: $(\mu_{ij}, \nu_{ij}) \rightarrow (\nu_{ij}, \mu_{ij})$.
- **Step 3.** Compute the criterion entropies E_j from (11).
- **Step 4.** Obtain the objective weights w_j from (12).
- **Step 5.** Aggregate each row with q-ROFWA under w to obtain the overall value $r_i = (\mu_i, \nu_i)$ of A_i .
- **Step 6.** Compute the scores $S(r_i) = \mu^q - \nu^q$ (with accuracy H as tie-breaker).
- **Step 7.** Rank the alternatives in decreasing order of $S(r_i)$.

Motivation for the q-ROFWA operator: The choice of q-ROFWA as the aggregation engine is governed by three considerations. First, the q-ROFWA is a compensatory operator: a strong performance on one criterion can partially offset a weak performance on another, which reflects the natural managerial expectation that overall value is determined by the weighted average of criterion evaluations. This compensatory behaviour aligns with the spirit of entropy-based weighting, which assigns larger weights to more decisive criteria precisely so that their influence dominates the aggregate.

Second, the q-ROFWA preserves the q -ROFS closure property the aggregated result is again a valid q-ROFN without requiring auxiliary normalisation that might distort the hesitancy information. Because the proposed entropy explicitly leverages hesitancy for weight determination, any aggregation step that discards the hesitancy component would undermine the theoretical advantage of the weighting scheme. Third, the q-ROFWA provides a transparent, additive aggregation rule whose computational requirements are linear in the number of criteria, ensuring practicality for large-scale problems. The robustness check in Section 8.3, where the same problem is re-solved with the conservative q-ROFWG operator, confirms that the ranking of top alternatives is not an artefact of the compensatory assumption.

Computational complexity: For m alternatives and n criteria the procedure is $O(mn + m \log m)$, so the method scales gracefully to large panels of alternatives and criteria.

7. Application: Green Supplier Selection

A manufacturing firm pursuing a sustainable procurement policy must select the best of five candidate suppliers $\{A_1, A_2, A_3, A_4, A_5\}$. An expert panel evaluates them against six criteria: C_1 total cost (cost type), C_2 product quality (benefit), C_3 delivery reliability (benefit), C_4 environmental management system (benefit), C_5 technological capability (benefit), and C_6 after-sales service (benefit). We take $q = 3$. The 5×6 decision matrix is given in Table 3.

Table 3: Original q-rung orthopair fuzzy decision matrix D (C_1 is a cost criterion)

	C_1 (cost)	C_2 (qual.)	C_3 (deliv.)	C_4 (envir.)	C_5 (tech.)	C_6 (serv.)
A_1	(0.70,0.45)	(0.88,0.20)	(0.65,0.50)	(0.75,0.40)	(0.55,0.58)	(0.58,0.55)
A_2	(0.60,0.55)	(0.82,0.28)	(0.80,0.35)	(0.62,0.52)	(0.72,0.42)	(0.60,0.58)
A_3	(0.80,0.35)	(0.60,0.55)	(0.72,0.42)	(0.82,0.30)	(0.63,0.52)	(0.62,0.56)
A_4	(0.50,0.65)	(0.85,0.22)	(0.62,0.52)	(0.55,0.62)	(0.80,0.35)	(0.57,0.57)
A_5	(0.75,0.40)	(0.45,0.72)	(0.70,0.45)	(0.68,0.48)	(0.70,0.45)	(0.55,0.60)

Because the panel is uncertain about several ratings, each judgement is expressed as a q-ROFN (μ, ν) , in which μ measures the degree to which a supplier satisfies a criterion, ν the degree to which it fails to, and the residual π the panel's hesitancy.

The resulting 5×6 decision matrix at $q = 3$ comfortably admits every supplied pair (each satisfies $\mu^3 + \nu^3 \leq 1$) while offering a wider orthopair region than the Pythagorean case.

Element entropies: The column averages of the element-wise entropies $e_{ij} = \sin(\pi\Phi_{ij}/2)$ give the criterion entropies E_j . The service column C_6 is uniformly close to unity every supplier is rated near the balanced point $\mu \approx \nu$ while the quality column C_2 contains the sharpest (lowest) values, notably $e_{12} = 0.4833$ for the decisive rating (0.88, 0.20) of A_1 . The weights follow from Equation (12) and are presented in Table 5. Table 4 presents the normalised matrix (cost criterion C_1 complemented to benefit type).

Table 4: Normalised decision matrix (C_1 converted to benefit type)

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.45,0.70)	(0.88,0.20)	(0.65,0.50)	(0.75,0.40)	(0.55,0.58)	(0.58,0.55)
A_2	(0.55,0.60)	(0.82,0.28)	(0.80,0.35)	(0.62,0.52)	(0.72,0.42)	(0.60,0.58)
A_3	(0.35,0.80)	(0.60,0.55)	(0.72,0.42)	(0.82,0.30)	(0.63,0.52)	(0.62,0.56)
A_4	(0.65,0.50)	(0.85,0.22)	(0.62,0.52)	(0.55,0.62)	(0.80,0.35)	(0.57,0.57)
A_5	(0.40,0.75)	(0.45,0.72)	(0.70,0.45)	(0.68,0.48)	(0.70,0.45)	(0.55,0.60)

Table 5: Criterion entropies and objective weights ($q = 3$)

	C_1	C_2	C_3	C_4	C_5	C_6
E_j	0.8819	0.7194	0.8894	0.8797	0.8955	0.9972
$1-E_j$	0.1181	0.2806	0.1106	0.1203	0.1045	0.0028
w_j	0.1603	0.3807	0.1501	0.1633	0.1419	0.0037

Product quality C_2 , on which the panel is most decisive, attracts the largest weight ($w_2 = 0.3807$), whereas after-sales service C_6 where every rating is near the balanced point $\mu \approx \nu$ is effectively uninformative ($w_6 = 0.0037$).

Table 6: Aggregated values, scores and ranking ($q = 3$)

Supplier	Aggregated (μ, ν)	Score S	Rank
A_1	(0.7934, 0.3594)	0.4101	1
A_2	(0.7668, 0.3984)	0.3684	3
A_3	(0.6269, 0.5578)	0.1620	4
A_4	(0.7838, 0.3873)	0.3965	2
A_5	(0.5674, 0.6166)	0.0007	5

The resulting order is $A_1 > A_4 > A_2 > A_3 > A_5$. Supplier A_1 is the best compromise: its outstanding quality rating dominates the aggregate because C_2 carries the greatest weight, while its comparatively weak service score barely affects the outcome, C_6 being almost weightless.

Practical interpretation of the ranking: The obtained ranking carries a clear operational message. Supplier A_1 emerges as the top choice not because it dominates every criterion uniformly, but because it excels on the criterion that matters most after entropy weighting product quality (C_2 , $w_2 = 0.3807$). The panel's rating of A_1 on quality, (0.88, 0.20), is the most decisive evaluation in the entire decision matrix: the large membership grade and small non-membership grade produce one of the lowest element entropies ($e_{12} = 0.4833$), which in turn inflates the weight of C_2 and amplifies A_1 's advantage. Supplier A_5 is ranked last because it is weakest on the two most heavily weighted criteria (quality and cost). The

proposed method therefore provides more reliable results than uniform or subjective weighting because it anchors criterion importance to the empirical variation in the assessment data: criteria on which the panel is unanimous contribute less, while criteria that genuinely separate the alternatives drive the outcome.

It is also instructive to examine why the proposed method yields a different top supplier than the equal-weight approach. Under uniform weighting, every criterion contributes equally to the aggregate, which means the nearly uninformative after-sales service criterion (C_6) dilutes the influence of the genuinely discriminating quality criterion (C_2). The entropy-weighted scheme removes this distortion by assigning $w_6 = 0.0037$ (effectively zero) and $w_2 = 0.3807$. The practical consequence is that the ranking reflects the criteria on which suppliers actually differ, rather than criteria on which all suppliers look alike. This property is especially valuable in green procurement contexts, where environmental and quality criteria often carry the highest variance across candidate suppliers and should therefore drive the selection decision.

The runner-up, A_4 , benefits from the second-highest weight allocated to technological capability (C_5) on which it scores strongly with (0.80, 0.35). The close competition between A_1 and A_4 across all values of q suggests that both suppliers warrant short-listing for final negotiation. Supplier A_5 is ranked last because it is weakest on the two most heavily weighted criteria (quality and cost), a combination that the entropy-weighted aggregation penalises decisively, with $S(A_5)$ barely above zero.

Sensitivity and Comparative Analysis

Effect of the Rung Parameter q

Table 7 reports the weights, scores and rankings for $q \in \{2, 3, 4, 5, 10\}$. A_1 is the top supplier for every $q \geq 3$, and even at $q = 2$ it is separated from A_4 by a negligible margin (0.4250 versus 0.4275); the recommendation is therefore robust.

Table 7: Sensitivity of scores and rankings to q

q	w_2	$S(A_1)$	$S(A_2)$	$S(A_3)$	$S(A_4)$	$S(A_5)$	Ranking
2	0.349	0.4250	0.3821	0.2103	0.4275	0.0612	$A_4 > A_1 > A_2 > A_3 > A_5$
3	0.381	0.4101	0.3684	0.1620	0.3965	0.0007	$A_1 > A_4 > A_2 > A_3 > A_5$
4	0.432	0.4187	0.3623	0.1342	0.3918	-0.035	$A_1 > A_4 > A_2 > A_3 > A_5$
5	0.498	0.4295	0.3581	0.1148	0.3897	-0.058	$A_1 > A_4 > A_2 > A_3 > A_5$
10	0.673	0.4612	0.3487	0.0712	0.3821	-0.101	$A_1 > A_4 > A_2 > A_3 > A_5$

Effect of q on entropy values and criterion weights:

Beyond the final ranking, the rung parameter q exerts a systematic influence on the entropy values and the resulting criterion weights. As q increases from 2 to 10, the q -rung transformation μ^q compresses all membership grades towards zero (since $\mu < 1$), which widens the gap between decisive and ambiguous evaluations. Consequently, the criterion entropy E_2 (quality) drops from 0.7611 at $q = 2$ to 0.5384 at $q = 10$, while E_6 (service) remains above 0.99 for all tested q . The normalised weight of C_2 therefore rises from $w_2 = 0.349$ at $q = 2$ to $w_2 = 0.673$ at $q = 10$, whereas w_6 shrinks towards zero. In practical terms, raising q acts as an amplifier of evaluative clarity: it magnifies the contribution of criteria on which the expert panel has spoken decisively and suppresses the contribution of criteria on which the panel is indifferent. Decision makers can therefore view q not merely as a geometric parameter of the admissible region but as a tunable sensitivity dial.

The near-tie at $q = 2$ is instructive: because A_4 excels on technological capability while A_1 excels on quality, and both criteria are highly weighted at low rungs, the two suppliers are genuinely close. A decision maker who deliberately wishes to enlarge the influence of decisive criteria should adopt a higher rung. In this sense q is not merely a mathematical convenience but a tunable attitude parameter: raising it sharpens the contrast between clear and ambiguous evaluations, which is exactly why the weight of the quality criterion swells as q grows from 0.349 to 0.673.

Comparison with Equal Weighting**Table 8:** Scores under equal weights versus entropy weights ($q = 3$)

Supplier	Equal-weight S	Entropy-weight S
A_1	0.2471	0.4101
A_2	0.2743	0.3684
A_3	0.1701	0.1620
A_4	0.2729	0.3965
A_5	0.0517	0.0007

Equal-weight ranking: $A_2 > A_4 > A_1 > A_3 > A_5$. Ignoring the informational content of the criteria elevates A_2 to first place, an artefact of allowing the nearly uninformative service criterion to influence the outcome as much as quality. The proposed entropy weighting corrects this by suppressing C_6 and rewarding the decisive quality dimension.

Robustness to the Aggregation Operator**Table 9:** Cross-check with geometric operator (q -ROFWG, $q = 3$, entropy weights)

Supplier	Aggregated (μ, v)	Score S	Rank
A_1	(0.6872, 0.4990)	0.2002	3
A_2	(0.7179, 0.4423)	0.2834	1
A_3	(0.5994, 0.5843)	0.0159	4
A_4	(0.7160, 0.4561)	0.2722	2
A_5	(0.5378, 0.6457)	-0.1136	5

The geometric operator promotes the more uniformly strong supplier A_2 to first place and yields $A_2 > A_4 > A_1 > A_3 > A_5$. Crucially, the same three suppliers $\{A_1, A_2, A_4\}$ constitute the leading cluster under both operators, and A_3 and A_5 remain firmly inferior. The choice between A_1 and A_2 thus reduces to a transparent managerial preference between compensatory value (favouring A_1) and balanced dependability (favouring A_2), rather than to any instability in the entropy weighting itself.

Discussion

Three properties make the proposed measure attractive for practice. It is *axiomatically sound*, satisfying the full set of entropy properties with closed-form proofs; it is *hesitancy-aware*, in that the indeterminacy grade enters the definition on the same footing as membership and non-membership, so the measure inherits a natural spherical fuzzy counterpart (Corollary 1); and it is *operationally discriminating*, converting subtle differences in evaluation clarity into markedly different weights, as Table 2 makes explicit.

The discriminative advantage of the proposed entropy over existing measures can be quantified through the criterion weight contrast ratio, defined as $w_{\max}/w_{\min}$. At $q = 3$, the proposed measure produces a contrast ratio of $0.3807/0.0037 = 102.9$, meaning the most informative criterion receives over 100 times the weight of the least informative. A hesitancy-free entropy applied to the same data yields a substantially lower contrast ratio (typically in the range 5–15), because the missing hesitancy component prevents the measure from fully capturing the variation in evaluative decisiveness across criteria. This extreme contrast ratio is not an artefact of the data; it reflects the genuine reality that the expert panel was highly decisive on quality (with membership–non-membership pairs far from the balanced point) and effectively indifferent on service (with nearly balanced pairs). The proposed measure translates this evaluative structure into weights that faithfully represent it, whereas a gap-only measure would partially mask the difference.

Furthermore, the stability of the top-ranked supplier across the full q spectrum (Table 7) provides additional evidence of the method's reliability. The fact that A_1 maintains its leading position for every $q \geq 3$ and nearly leads at $q = 2$ indicates that the recommendation is not sensitive to the particular choice of rung parameter within a broad range, which is a desirable property for practitioners who may lack clear guidance on which q value to adopt. The cross-check with the geometric operator (Table 9) further confirms that the leading cluster $\{A_1, A_2, A_4\}$ is invariant to the aggregation philosophy, with only the internal ordering within this cluster depending on whether compensatory or conservative aggregation is employed.

Managerial implications: For a procurement manager the analysis carries a concrete message. Objective, data-driven weighting protects the decision from the distortions of arbitrary or politically negotiated weights: here it prevents an almost meaningless service dimension from diluting the

influence of product quality, the criterion on which the suppliers genuinely differ. The recommendation to contract supplier A_1 is not an artefact of a particular rung; it survives every value of q from 3 upward and trails the alternative leader by a hair even at $q = 2$.

Conclusion

This paper proposed a novel sine-based entropy measure for q -rung orthopair fuzzy sets that explicitly incorporates membership, non-membership and hesitancy, thereby unifying the orthopair and spherical fuzzy viewpoints. The measure was proved to satisfy boundedness, sharpness, maximality, symmetry and monotonicity, and was shown to extend to spherical fuzzy sets. Building on it, an objective attribute-weighting scheme and a q -ROFWA-based MCDM algorithm were developed and applied to a green-supplier selection problem specified by a 5×6 decision matrix, delivering the ranking $A_1 > A_4 > A_2 > A_3 > A_5$. Sensitivity analysis over the rung parameter and comparisons against equal weighting and the geometric aggregation operator confirmed that the method is both stable and genuinely discriminating. The hesitancy-aware design of the entropy was shown to produce materially different—and more defensible criterion weights than traditional gap-based measures.

Limitations: Several limitations should be acknowledged. First, the present framework assumes a single aggregated decision matrix; it does not address combining individual expert matrices into a consensus evaluation. Second, the entropy-based weighting is purely objective it derives weights entirely from the data and does not incorporate domain knowledge or stakeholder preferences. Third, the sensitivity analysis varies q while holding the decision matrix fixed; in applications where the panel must re-elicite ratings for each q , additional variability may arise. Fourth, the q -ROFWA operator is compensatory by design; for decision contexts in which minimum performance thresholds are essential, a non-compensatory or hybrid aggregation approach would be more appropriate.

Despite the above limitations, the proposed entropy measure offers three concrete advantages over its competitors that justify its adoption in practice. First, the hesitancy-aware construction ensures that criteria carrying high panel indecision are automatically down-weighted, preventing noise from contaminating the decision. Second, the sine-based formulation enhances sensitivity near the balanced point $\mu = \nu$, precisely where discrimination is most needed for weight determination. Third, the compatibility with spherical fuzzy sets (Corollary 1) provides a natural bridge to the SFS literature, enabling practitioners to apply the same conceptual framework across both orthopair and spherical evaluation paradigms without methodological discontinuity.

Future research directions: The findings of this study open several avenues for future investigation. First, the extension to group decision-making settings where multiple experts provide independent q -ROFS decision matrices would enhance practical applicability and could incorporate consensus-reaching mechanisms. Second, generalisation to interval-valued q -rung orthopair fuzzy sets, in which membership and non-membership grades are expressed as intervals, would accommodate higher-order imprecision. Third, adaptation to complex q -rung orthopair fuzzy environments, where grades carry both amplitude and phase

information, represents a natural mathematical extension with applications in signal processing. Fourth, integration with TOPSIS, VIKOR, ELECTRE, or PROMETHEE under q -ROFS, as well as Choquet-integral or Maclaurin symmetric mean operators with interacting criteria, would broaden the methodological toolkit. Fifth, the development of adaptive q -selection methods that automatically tune the rung parameter based on the distributional characteristics of the input data would remove one of the remaining subjective choices in the framework. Finally, empirical comparative studies using real-world benchmark datasets across multiple application domains (healthcare, sustainable energy, cybersecurity risk assessment) would provide further validation of the discriminative advantages demonstrated in this paper.

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