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## Preventive maintenance in two-component parallel repairable systems: A literature review

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### Abstract

A two-component parallel system continues to operate as long as at least one component is functioning, which already makes it more reliable than a single component on its own; adding preventive maintenance, scheduled servicing performed before a component fails, is a further way to raise availability and mean time to failure beyond what corrective repair alone can achieve. This review traces the literature on this combination, from Barlow and Hunter's 1960 foundational treatment of optimum preventive maintenance policy and Barlow, Proschan, and Hunter's general mathematical theory of reliability, through Markov and semi-Markov models of two-unit parallel systems with and without preventive maintenance, to a 2023 extension of the same Markov approach to four units. Of the seven studies reviewed here, five analyse a specific multi-unit parallel or component system, and of those five, three explicitly incorporate preventive maintenance alongside corrective repair while two consider corrective repair alone, indicating that preventive maintenance, while well established in the foundational theory, is not yet a universal feature of the applied two-unit reliability literature.

**Keywords:** Two-component parallel system, preventive maintenance, repairable system, Markov process, availability, mean time to failure, reliability, steady-state analysis

### Introduction

A parallel system of two components is, by definition, more reliable than either component alone: the system remains operational as long as at least one of the two components is functioning, and only fails when both have failed simultaneously. Barlow, Proschan, and Hunter's 1965 monograph established the general mathematical machinery for analysing systems of this kind, including the theory of increasing and decreasing failure rate distributions and the foundations of coherent system theory that later reliability models, parallel or otherwise, continue to build on [2].

A separate but closely related question is how maintenance should be scheduled once a system is in service. Barlow and Hunter's 1960 paper, published five years before the general reliability monograph, addressed this question directly for a single unit, deriving the optimum policy for preventive maintenance, servicing performed on a fixed schedule regardless of whether a failure has occurred, as opposed to purely corrective maintenance performed only after failure [1]. Extending this preventive-maintenance idea from a single unit to a two-component parallel system, and analysing the resulting Markov process for its steady-state availability and mean time to failure, is the specific combination this review traces.

This review follows how the two-unit parallel model has been analysed both with and without preventive maintenance since the 1980s, and how the same Markov approach has more recently been extended to systems with more than two units.

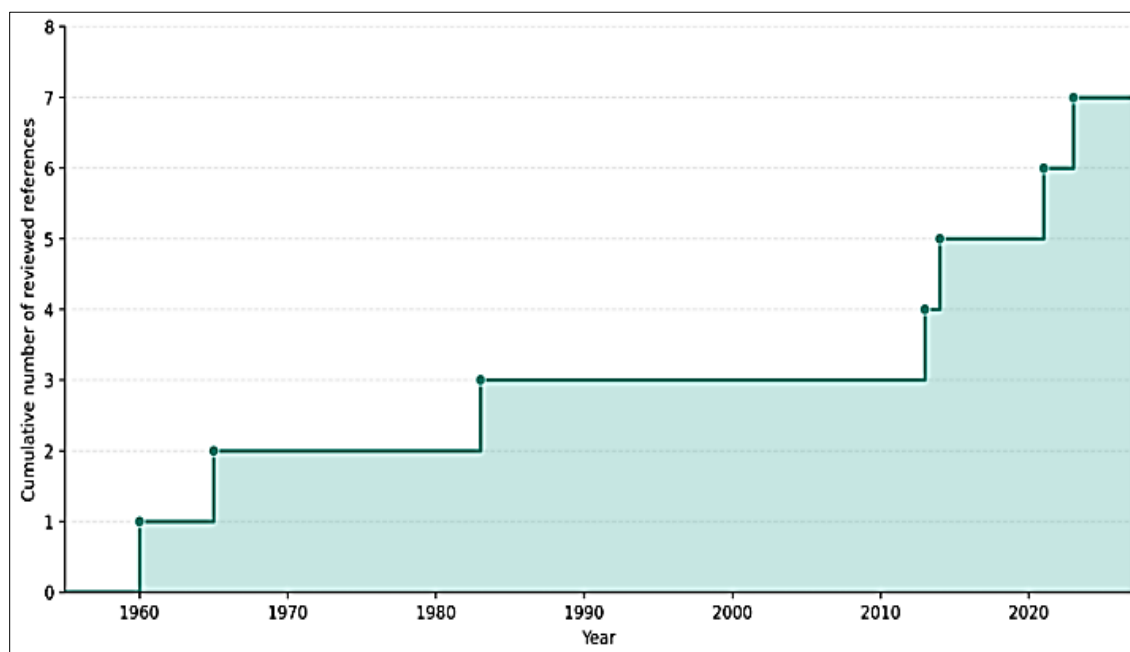


**Fig 1:** Cumulative growth of the seven reviewed studies, 1960-2023

The curve rises to two by 1965 and then stays flat for eighteen years before Chakravarthy's 1983 paper, after which growth resumes in a roughly steady trickle rather than a sharp cluster, reaching seven by 2023. Unlike some of the other topics reviewed in this series, this literature shows no single decade of concentrated activity.

#### **Foundations: Preventive maintenance theory and the general reliability framework**

Barlow and Hunter's optimum preventive maintenance policy rests on a simple trade-off: preventive maintenance carries a cost even when performed on a unit that has not yet failed, but waiting for failure risks a more expensive or more disruptive

corrective repair, so an optimum schedule balances the two <sup>[1]</sup>. Their original treatment was for a single unit; every later study reviewed here that includes preventive maintenance adapts this same cost-and-timing logic to a multi-unit setting, generally by allowing one unit to undergo preventive maintenance while the parallel unit continues to operate. Barlow, Proschan, and Hunter's monograph, published five years later, supplies the broader distributional and structural theory, increasing and decreasing failure rate classes, coherent system structure functions, and redundancy comparisons, that the more specific two-unit models discussed below draw on when specifying failure-time distributions and justifying which system configurations are worth analysing in detail <sup>[2]</sup>.

**Table 1:** Chronological summary of reviewed studies

| Study (Year)                     | Ref. | Units                  | PM Included?           | Key Contribution                                            |
|----------------------------------|------|------------------------|------------------------|-------------------------------------------------------------|
| Barlow & Hunter (1960)           | [1]  | General theory         | Yes (PM policy itself) | Foundational optimum preventive maintenance policy          |
| Barlow, Proschan & Hunter (1965) | [2]  | General theory         | No                     | Foundational mathematical theory of reliability             |
| Chakravarthy (1983)              | [3]  | Two-unit parallel      | No                     | Phase-type repair distribution for a parallel system        |
| EL-Sherbeny (2013)               | [4]  | Two-unit parallel      | Yes                    | Two non-identical units with PM and multiple failure types  |
| Rathee & Malik (2014)            | [5]  | Two-unit parallel      | Yes                    | Priority to PM of one unit over replacement of the other    |
| Bahou <i>et al.</i> (2021)       | [6]  | Two-component parallel | No                     | Availability under stochastic dependence between components |
| Yadav <i>et al.</i> (2023)       | [7]  | Four-unit              | Yes                    | Markov approach extending PM analysis to four units         |

#### **Two-unit parallel systems with and without preventive maintenance**

Chakravarthy's 1983 paper analysed a two-unit parallel system without preventive maintenance, focusing instead on generalising the repair-time distribution itself: rather than assuming exponential repair times, as most Markov reliability models do for tractability, Chakravarthy used a phase-type distribution, built from a sequence of exponential phases, general enough to approximate a wide range of realistic repair-time shapes while keeping the overall process analytically tractable <sup>[3]</sup>. Bahou and colleagues took a different route to relaxing the standard assumptions, keeping repair times relatively simple but instead modelling stochastic dependence between the two components directly through a Cox proportional hazards approach, showing that the degree

of dependence has a considerable effect on system availability even without any preventive maintenance in the model <sup>[6]</sup>. El-Sherbeny's 2013 paper brought preventive maintenance into a two-unit parallel setting explicitly, analysing two non-identical units subject to several distinct failure types alongside scheduled preventive maintenance, and deriving availability and related measures via the same steady-state Markov balance-equation approach used throughout this literature <sup>[4]</sup>. Rathee and Malik's 2014 paper introduced a further refinement, priority: rather than treating preventive maintenance and repair symmetrically, their model gives priority to preventive maintenance of one unit over the replacement of the other, reflecting a common real-world practice of scheduling planned servicing ahead of reactive replacement work whenever both are due at the same time <sup>[5]</sup>.

**Table 2:** Key concepts referenced in this review

| Concept                        | Ref. | Description                                                                                                                                                                                         |
|--------------------------------|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Preventive maintenance         | [1]  | Scheduled maintenance performed before failure occurs, intended to reduce the rate of unexpected breakdowns relative to relying on corrective repair alone                                          |
| Mean time to failure (MTTF)    | [2]  | The expected value of the time until a non-repairable failure occurs, or, for a repairable system, the expected duration of an up period                                                            |
| Steady-state availability      | [3]  | The long-run proportion of time a repairable system is in an operating (up) state, obtained from the stationary distribution of the underlying Markov process                                       |
| Phase-type repair distribution | [3]  | A repair-time distribution built from a sequence or mixture of exponential phases, general enough to approximate many non-exponential repair-time distributions while keeping the process Markovian |

Table 2 defines the four terms most central to this review; the third and fourth rows describe the two main analytical tools the studies

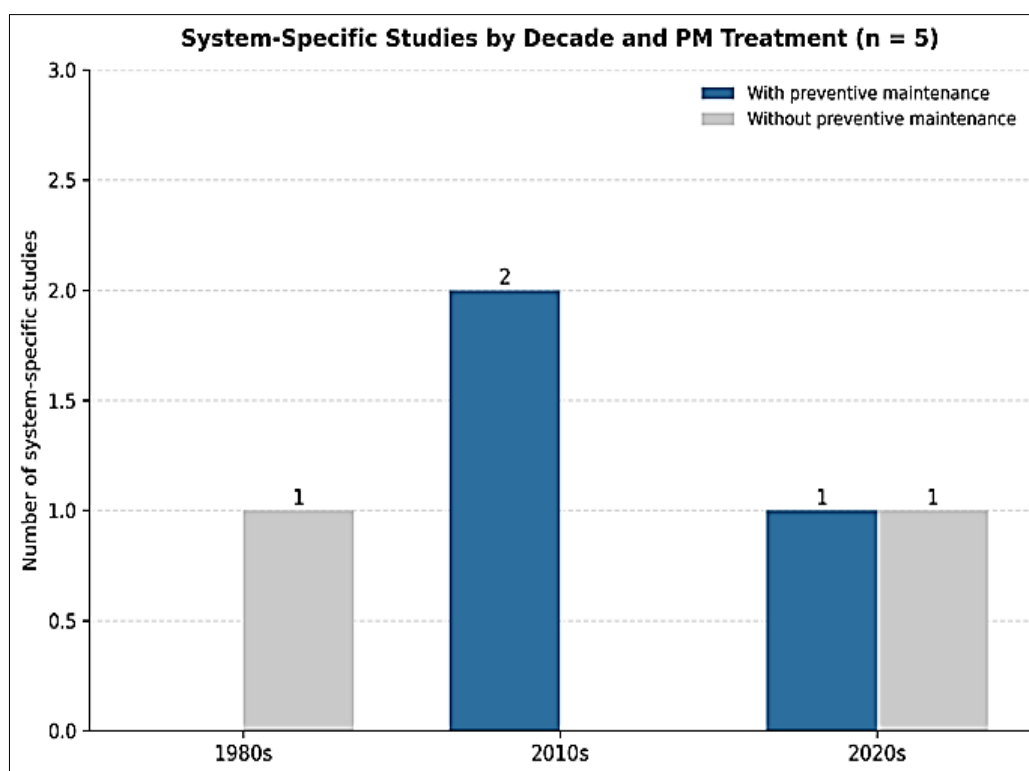
above use to turn a system description into a solvable stochastic model.

**Table 3:** Reference distribution by journal

| Journal / Source                                              | Number of References |
|---------------------------------------------------------------|----------------------|
| Operations Research                                           | 1                    |
| Book (Wiley)                                                  | 1                    |
| OR Spectrum                                                   | 1                    |
| Mathematical Problems in Engineering                          | 1                    |
| International Journal of Engineering Development and Research | 1                    |
| IEOM Conference Proceedings                                   | 1                    |
| Reliability: Theory & Applications                            | 1                    |

No journal in this list contributes more than one reference among the seven studies reviewed here, reflecting how this

literature is spread across operations research, general engineering, and specialist reliability venues.

**Fig 2:** System-specific studies by decade and preventive-maintenance treatment ( $n = 5$ )

Excluding the two foundational, non-system-specific references, the five remaining studies split three with preventive maintenance and two without. Both preventive-

maintenance studies from the 2010s appear alongside no non-preventive-maintenance study in that decade, while the 2020s contain one study of each type.

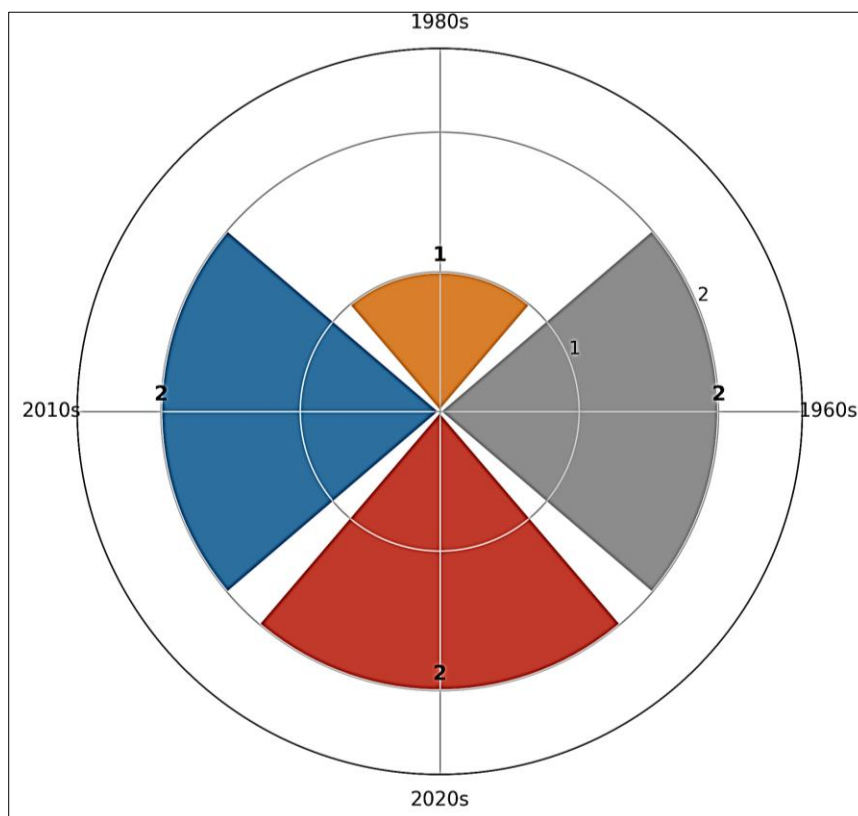


**Fig 3:** Reviewed references by decade of publication ( $n = 7$ )

References are spread fairly evenly across four decades, with two each in the 1960s, 2010s, and 2020s and one in the 1980s,

in contrast to the sharper decade-level clustering seen in some of the other topics reviewed in this series.

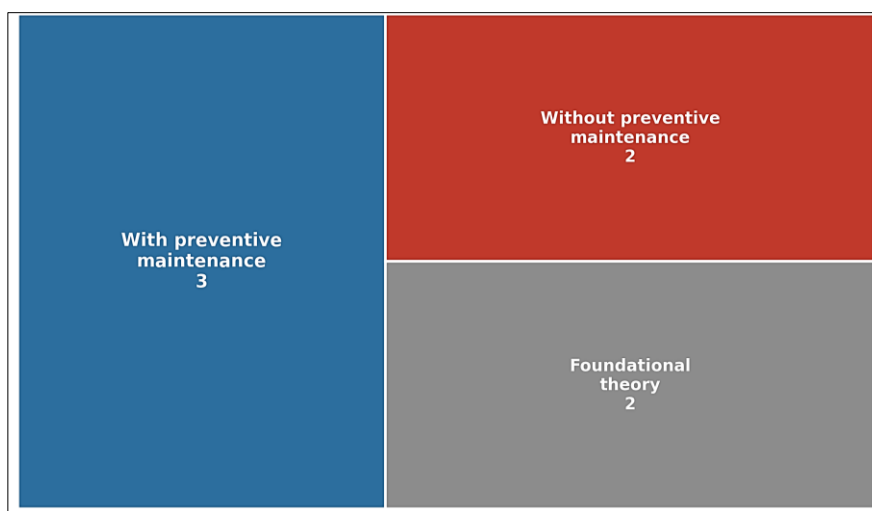


**Fig 4:** Reviewed studies by preventive-maintenance treatment ( $n = 7$ )

Three of the seven reviewed studies incorporate preventive maintenance directly into a multi-unit system model, two analyse a multi-unit system with corrective repair only, and two are foundational theoretical references that underpin both groups without analysing a specific multi-unit system themselves.

#### Synthesis and research gaps

Read together, these seven studies show a two-unit parallel reliability literature that has developed the preventive-maintenance and non-preventive-maintenance strands somewhat independently. Chakravarthy's phase-type repair extension and Bahou and colleagues' stochastic-dependence extension both relax a standard modelling assumption,

exponential repair times and component independence respectively, but neither incorporates preventive maintenance into its more general model [3, 6]. Conversely, El-Sherbeny's and Rathee and Malik's preventive-maintenance models retain the simpler exponential, independent-component assumptions that the non-preventive-maintenance studies were specifically trying to relax [4, 5].

This suggests a specific, well-defined gap: a two-component parallel system that combines preventive maintenance with either a phase-type repair distribution or an explicit model of stochastic dependence between components has not yet appeared in the literature reviewed here. Given that Chakravarthy's phase-type approach and the standard preventive-maintenance formulation both operate within the

same Markov or semi-Markov framework, combining them looks feasible using tools already established in this literature. A second observation follows from Yadav and colleagues' extension of the Markov approach from two units to four <sup>[7]</sup>. Their paper demonstrates that the same steady-state balance-equation method scales to larger systems, at the cost of a rapidly growing number of states to track. Whether the priority-based preventive-maintenance scheme that Rathee and Malik developed for two units extends cleanly to four, or whether the added complexity requires a qualitatively different approach, is not addressed by any study reviewed here.

## Conclusion

This review has traced how preventive maintenance, established in general terms by Barlow and Hunter and situated within the broader reliability theory of Barlow, Proschan, and Hunter, has been incorporated into two-unit parallel system models over more than four decades, alongside a separate strand of two-unit and multi-unit studies that instead relax other modelling assumptions without preventive maintenance.

The clearest gap identified here is the absence of a two-component parallel system model that combines preventive maintenance with either a phase-type repair distribution or explicit inter-component dependence, both of which have been studied on their own but not together. Extending Rathee and Malik's priority-based preventive-maintenance scheme using the phase-type or dependence tools already developed by Chakravarthy and by Bahou and colleagues, and testing whether Yadav and colleagues' four-unit scaling approach carries over to a preventive-maintenance setting, would address two related gaps using methods already present across the studies reviewed here.

## Declarations

**Conflict of interest:** The author(s) declare no conflict of interest.

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