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Variational iteration and homotopy perturbation methods for nonlinear reaction-diffusion equations: A literature review

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Abstract

The Variational Iteration Method (VIM) and the Homotopy Perturbation Method (HPM), both introduced by Ji-Huan He around the turn of the millennium, have become two of the most widely used analytical approximation techniques for nonlinear differential equations, and are frequently compared against one another for accuracy, convergence, and computational effort. This review traces that comparative literature from He's original 1999 and 2000 papers through three studies that directly compare VIM and HPM on fractional partial differential equations, heat-transfer and porous-media problems, and general nonlinear evolution equations, to two more recent studies that apply one of the two methods, but not both, specifically to reaction-diffusion equations of Fisher type. Of the seven studies reviewed here, three carry out a direct VIM-versus-HPM comparison, and two apply a single method to a reaction-diffusion equation specifically, but none of the seven combines both features: a direct, side-by-side VIM-versus-HPM comparison, complete with absolute error analysis, carried out specifically on a nonlinear reaction-diffusion equation.

Keywords: Variational iteration method, homotopy perturbation method, reaction-diffusion equation, nonlinear partial differential equation, absolute error, convergence, Fisher equation, analytical approximation

Introduction

Exact closed-form solutions are the exception rather than the rule for nonlinear differential equations, which has motivated a long line of analytical approximation techniques that avoid the restrictive small-parameter assumptions of classical perturbation theory. He introduced the variational iteration method in 1999, building a correction functional around a Lagrange multiplier identified through variational theory and iterating from an initial guess without requiring linearisation of the nonlinear terms ^[1]. The following year, He introduced a second, related technique, the homotopy perturbation method, which embeds the target equation in an artificial homotopy governed by an embedding parameter and expands the solution as a power series in that parameter ^[2].

Because the two methods address the same basic problem, approximating solutions to nonlinear differential equations without a small physical parameter, and were introduced by the same author within a year of each other, a natural line of research has compared them directly: on which problems does each method converge faster, and which gives smaller absolute error against a known exact solution? This review traces that comparative literature, together with the smaller number of studies that apply one of the two methods specifically to reaction-diffusion equations, of which the Fisher equation is the most widely studied example.

The two foundational papers appear within a year of each other, followed immediately by a cluster of three comparative studies in 2007, then two further studies spaced roughly a decade apart in 2009 and 2020. There is no activity gap longer than eleven years anywhere in this chronology, indicating sustained rather than sporadic interest in comparing or applying these two methods.

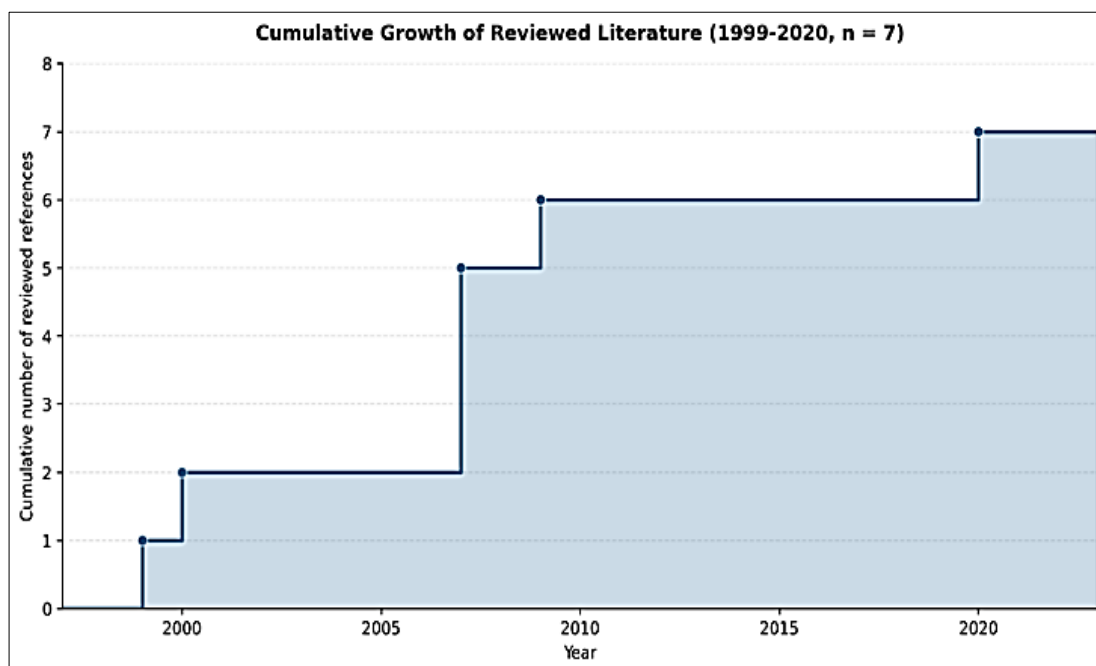


Fig 1: Cumulative growth of the seven reviewed studies, 1999-2020

Foundations: The variational iteration and homotopy perturbation methods

VIM's correction functional takes the form of an integral in which a Lagrange multiplier, generally identified through the calculus of variations by treating the nonlinear term as a restricted variation, corrects an initial approximation at each iteration^[1]. For many equations the multiplier can be found in closed form, after which each successive term of the VIM series is obtained by direct integration, without needing to solve a further differential equation at each step.

HPM instead constructs a homotopy between a simple problem with a known solution and the full nonlinear equation, controlled by an embedding parameter that runs from zero to one, and expands the unknown solution as a power series in that parameter^[2]. Collecting terms at each power of the embedding parameter yields a sequence of linear problems that can be solved successively, in a manner structurally similar to classical perturbation methods but without requiring the nonlinearity to be weak.

Table 1: Chronological summary of reviewed studies

Study (Year)	Ref.	Method Category	Application	Key Contribution
He (1999)	[1]	Foundational (VIM)	General nonlinear ODEs	Introduced the variational iteration method
He (2000)	[2]	Foundational (HPM)	General nonlinear problems	Introduced the homotopy perturbation method
Momani & Odibat (2007)	[3]	Both compared	Linear fractional PDEs	Direct VIM-HPM comparison for fractional equations
Ganji & Sadighi (2007)	[4]	Both compared	Heat transfer, porous media	Applied both methods to nonlinear heat/porous-media equations
Ganji, Tari & Jooybari (2007)	[5]	Both compared	Nonlinear evolution equations	Compared VIM and HPM against exact solutions for several evolution equations
Matinfar & Ghanbari (2009)	[6]	VIM only	Fisher's equation (reaction-diffusion)	Applied VIM specifically to the Fisher reaction-diffusion equation
Loyinmi & Akinfe (2020)	[7]	HPM only (Elzaki-HPM hybrid)	Fisher equation family (reaction-diffusion)	Applied an HPM-based hybrid to three Fisher-type equations

Direct comparisons of VIM and HPM

Three of the studies reviewed here apply both VIM and HPM to the same set of test equations and compare the results directly. Momani and Odibat applied both methods to linear fractional partial differential equations, using the Caputo derivative, and found both methods produced series that agreed closely with each other and with known exact solutions for the fractional cases tested^[3]. Ganji and Sadighi applied both methods to nonlinear heat-transfer and porous-media equations, reporting that both methods converged to the exact solution but noting differences in how many terms were needed to reach a given accuracy level depending on the specific equation^[4]. Ganji, Tari, and Jooybari extended the comparison to a broader set of nonlinear evolution equations, again finding both methods capable of high accuracy against exact solutions,

without identifying a single method as uniformly superior across all the equations tested^[5].

A pattern common to all three comparative studies is that neither method is shown to be categorically better than the other; instead, each paper works through several specific equations and reports comparable accuracy for both methods on most of them, with the choice between VIM and HPM in practice often coming down to which correction functional or homotopy construction is more convenient to set up for a given equation, rather than any general efficiency advantage.

Applications to reaction-diffusion equations

The two most recent studies reviewed here move to reaction-diffusion equations specifically, but each uses only one of the two methods rather than comparing them directly. Matinfar and

Ghanbari applied VIM to the Fisher equation, a reaction-diffusion equation combining diffusion with a logistic-type reaction term, deriving a series solution and comparing it against known exact travelling-wave solutions for specific parameter values [6]. Loyinmi and Akinfe took an HPM-based approach, coupling the Elzaki integral transform with the homotopy perturbation method to solve three distinct variants of the Fisher equation, reporting close agreement with exact solutions across all three cases [7].

Neither of these two reaction-diffusion-specific studies includes the other method as a point of comparison: Matinfar and Ghanbari's paper does not apply HPM to the same Fisher equation, and Loyinmi and Akinfe's paper does not apply plain VIM to their three Fisher-type equations. This leaves the direct, side-by-side comparison carried out by Momani and Odibat, Ganji and Sadighi, and Ganji, Tari, and Jooybari for other equation families without a matching counterpart specifically for reaction-diffusion equations.

Table 2: Key concepts referenced in this review

Concept	Ref.	Description
Variational iteration method (VIM)	[1]	Builds a correction functional using a Lagrange multiplier identified via variational theory, then iterates to improve an initial guess without requiring linearisation
Homotopy perturbation method (HPM)	[2]	Embeds the equation in a homotopy with an embedding parameter, expanding the solution as a power series in that parameter without requiring a small physical parameter
Absolute error comparison	[5]	Comparing the pointwise difference between an approximate series solution and the exact (or a high-precision reference) solution across the domain, rather than relying on a single summary statistic
Reaction-diffusion equation	[6]	A partial differential equation combining a diffusion term with a nonlinear reaction term, of which the Fisher equation is a widely studied example

Table 2 defines the four terms most central to this review; the first two describe the competing methods, and the third and fourth describe

how their outputs are evaluated and the equation family this review is ultimately interested in.

Table 3: Reference distribution by journal

Journal	Number of References
International Journal of Non-Linear Mechanics	2
Computers & Mathematics with Applications	2
Journal of Computational and Applied Mathematics	1
International Journal of Contemporary Mathematical Sciences	1
Engineering Reports	1

International Journal of Non-Linear Mechanics and Computers & Mathematics with Applications each contribute two

references; the remaining three references come from three distinct journals.

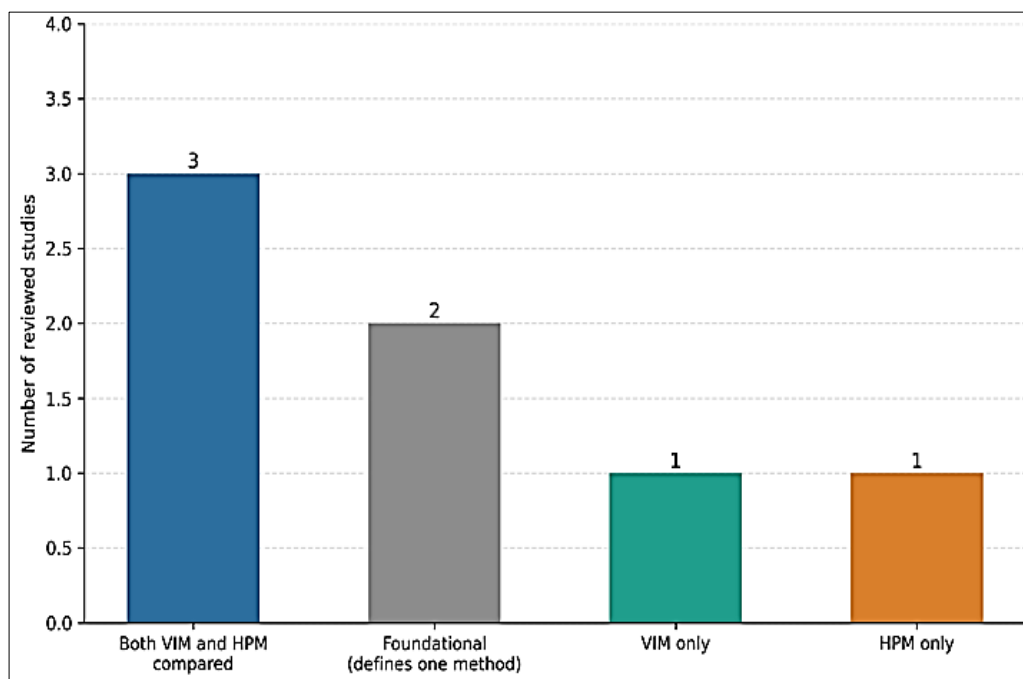


Fig 2: Reviewed studies by method category ($n = 7$)

Three of the seven reviewed studies directly compare both methods, two are foundational papers each defining a single method, and one each applies VIM alone or an HPM-based

hybrid alone. No category contains more than three studies, indicating this is a relatively compact, well-defined literature rather than one dominated by a single paper type.

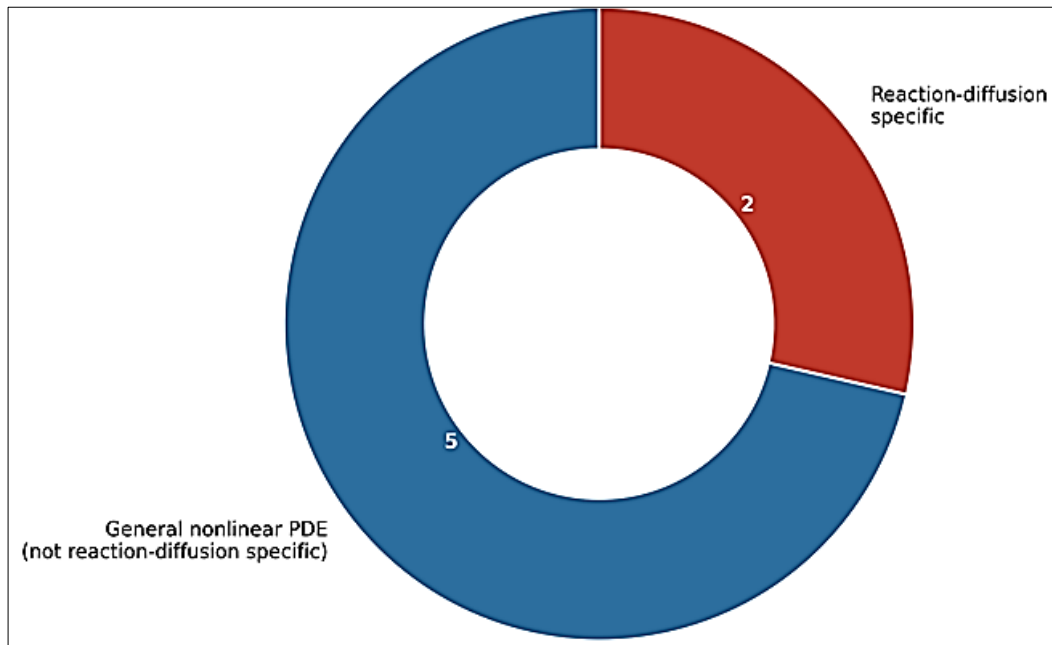


Fig 3: Reviewed studies by application specificity ($n = 7$)

Five of the seven reviewed studies apply their methods to general nonlinear problems, fractional PDEs, heat transfer, or generic evolution equations, rather than to a reaction-diffusion

equation specifically. Only two studies, both reviewed in the previous section, focus specifically on a reaction-diffusion equation.

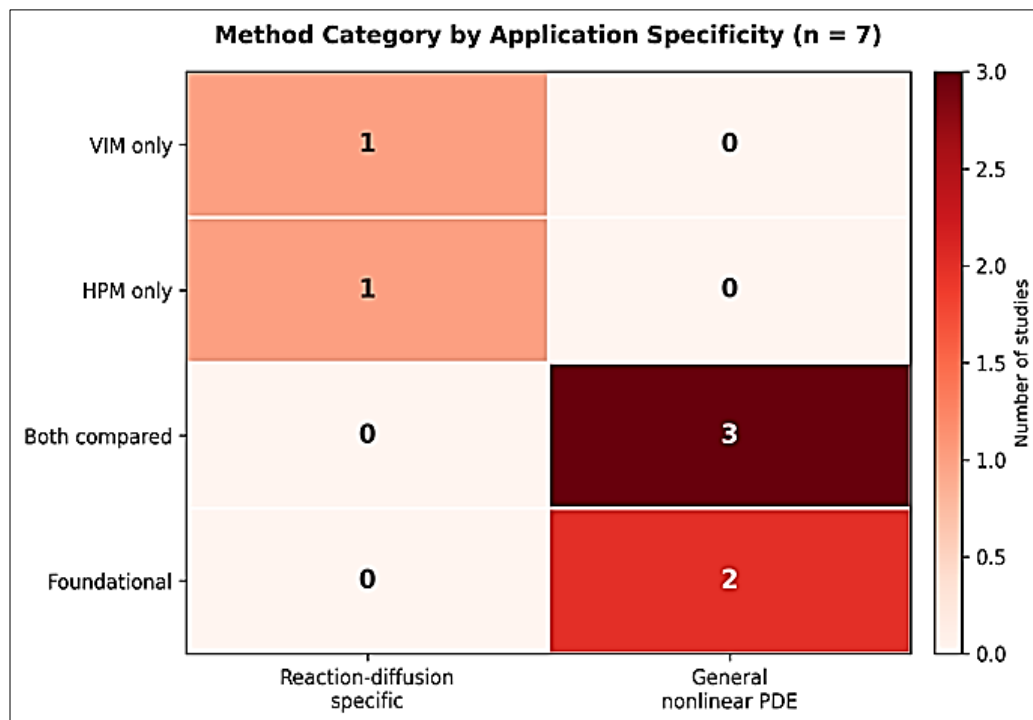


Fig 4: Method category by application specificity ($n = 7$)

Figure 4 makes the central gap in this literature visible directly: the row for studies that compare both VIM and HPM contains zero reaction-diffusion-specific studies, while the two reaction-diffusion-specific studies each use only one method. No cell in this matrix contains a study that both compares the two methods and targets a reaction-diffusion equation specifically.

Synthesis and research gaps

Read together, these seven studies show two lines of work that have not yet intersected. The direct VIM-versus-HPM comparisons reviewed here are thorough in their treatment of the comparison itself, generally reporting absolute error tables

or convergence behaviour across several test equations, but none of the three targets a reaction-diffusion equation among its test cases [3, 5]. The reaction-diffusion-specific studies, in turn, are thorough in their treatment of the Fisher equation family, but each commits to a single method from the outset rather than evaluating both [6, 7].

Figure 4 states this gap in its simplest form: no reviewed study occupies the intersection of these two lines of work. A study that applied both VIM and HPM to the same reaction-diffusion equation, most naturally a member of the Fisher family given its prominence in the two application-specific studies reviewed here, and reported an absolute error comparison of the kind

Ganji, Tari, and Jooybari carried out for other evolution equations, would directly fill this gap using tools already well established across the literature reviewed here [5, 6, 7].

A second, more minor observation is methodological: the three direct-comparison studies each use a different application domain, fractional PDEs, heat transfer and porous media, and general evolution equations, as their testing ground, which makes it difficult to know whether the near-parity between VIM and HPM reported in each case would also hold for the specific structure of a reaction-diffusion equation, where the nonlinear reaction term has a different qualitative character from the nonlinearities tested in the three comparative studies reviewed here.

Conclusion

This review has traced the comparative literature on the variational iteration method and the homotopy perturbation method from He's original 1999 and 2000 papers through three direct comparative studies and two reaction-diffusion-specific applications of a single method each.

The clearest gap identified here is the absence of a study that directly compares VIM and HPM, with a full absolute error analysis, on a nonlinear reaction-diffusion equation specifically. Given that both the comparative methodology and the reaction-diffusion application have well-established precedents among the seven studies reviewed here, each on its own, combining them looks like a readily achievable next step, and would complete a natural intersection that this literature has not yet reached.

Declarations

Conflict of interest: The author(s) declare no conflict of interest.

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