

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
NAAS Rating (2026): 4.49
Maths 2026; 11(2): 145-150
© 2026 Stats & Maths
<https://www.mathsjournal.com>
Received: 10-12-2025
Accepted: 09-02-2026

Dr. Lucas Meyer
Department of Applied
Mathematics, Faculty of
Computational Sciences, Rhine
Centre for Nonlinear Analysis,
Basel, Switzerland

Dr. Clara Baumann
Department of Applied
Mathematics, Faculty of
Computational Sciences, Rhine
Centre for Nonlinear Analysis,
Basel, Switzerland

Dr. Felix Hartmann
Department of Applied
Mathematics, Faculty of
Computational Sciences, Rhine
Centre for Nonlinear Analysis,
Basel, Switzerland

Corresponding Author:
Dr. Lucas Meyer
Department of Applied
Mathematics, Faculty of
Computational Sciences, Rhine
Centre for Nonlinear Analysis,
Basel, Switzerland

The homotopy analysis method for the Burgers-Fisher equation and its relation to other series-solution methods: A literature review

Lucas Meyer, Clara Baumann and Felix Hartmann

DOI: <https://www.doi.org/10.22271/math.2026.v11.i2b.2621>

Abstract

The Burgers-Fisher equation combines nonlinear convection, diffusion, and logistic-type reaction in a single partial differential equation, which makes it a natural test case for analytical approximation methods when exact closed-form solutions are unavailable for general parameter choices. This review traces how the Homotopy Analysis Method (HAM) and its main competitors, the Adomian Decomposition Method (ADM) and the Homotopy Perturbation Method (HPM), have been applied to the Burgers-Fisher equation and its close relatives, the Fisher, Burgers, and Burgers-Huxley equations. Particular attention is given to how each method establishes convergence of the resulting series solution and how the small number of studies that compare methods directly report relative accuracy. The review finds that most studies apply a single method to a single equation, that formal convergence arguments are more often asserted through numerical demonstration than proved analytically, and that only one study in this literature carries out a direct multi-method comparison, including HAM and ADM together, with an accompanying convergence proof and error analysis, on the Burgers-Fisher equation specifically.

Keywords: Burgers-Fisher equation, homotopy analysis method, Adomian decomposition method, homotopy perturbation method, convergence theorem, error analysis, nonlinear partial differential equation, series solution

Introduction

The Burgers-Fisher equation is built from two older and separately well-studied equations. Fisher introduced his equation in 1937 to model the spread of an advantageous gene through a population, combining ordinary diffusion with a logistic reaction term ^[1]. Burgers introduced his equation in 1948 as a simplified model of turbulence, combining diffusion with a nonlinear convection term proportional to the product of the solution and its own spatial derivative ^[2]. The Burgers-Fisher equation places both nonlinear terms, convection and logistic reaction, alongside diffusion in a single equation, which gives it travelling-wave solutions that are of interest in fields ranging from flame propagation to population dynamics, but which also makes exact closed-form solutions available only for special parameter choices.

Where an exact solution is not available, researchers have generally turned to series-based analytical approximation methods rather than purely numerical discretization schemes, since a series solution can be evaluated on the whole domain at once and often reveals the qualitative structure of the solution more directly. Three such methods dominate the literature reviewed here. The Adomian decomposition method, developed earlier and applied to the Burgers-Huxley and Burgers-Fisher equations by Ismail, Raslan, and Abd Rabboh in 2004, builds the solution as a series whose terms are generated by a recursive formula involving so-called Adomian polynomials for the nonlinear term ^[5]. The homotopy perturbation method, applied to the Burgers and Burgers-Fisher equations by Rashidi, Ganji, and Dinarvand in 2009, embeds the equation in a homotopy parameterised by a single artificial parameter and expands the solution as a power series in that parameter ^[7]. The homotopy analysis method, formalised by Liao and applied to the Burgers-Fisher equation by Babolian and Saeidian in 2009, generalises this idea by introducing an auxiliary convergence-control parameter that is not tied

To any small physical quantity in the original equation, giving the user explicit control over the convergence region of the resulting series ^[3, 8]. This review traces how these three methods, and the smaller number of studies that compare them directly, have been applied to the Burgers-Fisher

equation and its close relatives, with particular attention to how convergence of the series solution is established and how accuracy is assessed against exact or high-precision numerical solutions.

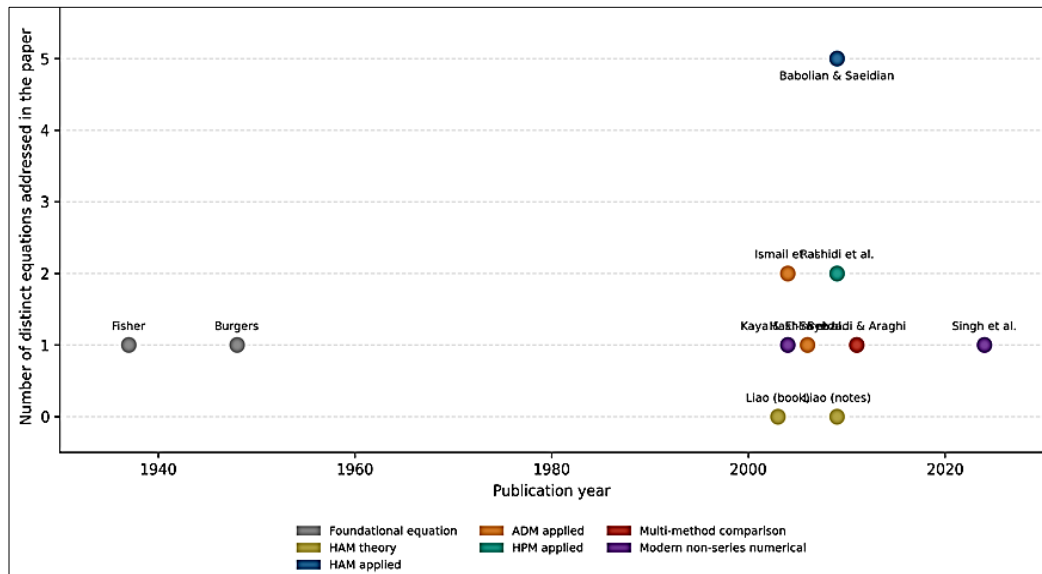


Fig 1: Publication year against the number of distinct equations addressed in the same paper, coloured by method category ($n = 11$)

Most points sit at or near the bottom of the plot, one or two equations per paper, and cluster tightly between 2003 and 2011. Babolian and Saeidian stand apart both in height, five equations in one paper, and in method, HAM, while the two points furthest to the right, Behzadi and Araghi in 2011 and Singh and colleagues in 2024, mark the most recent analytical and numerical treatments respectively, thirteen years apart.

Foundations: The two component equations and the homotopy analysis method

Fisher's equation and Burgers' equation were developed independently and for different purposes, but both belong to the broader family of nonlinear reaction-diffusion and convection-diffusion equations that resist exact solution outside special cases ^[1, 2]. The Burgers-Fisher equation combines the convection term from the first with the logistic reaction term from the second, and its generalized form raises

the reaction term to an arbitrary power, which is the version most of the studies reviewed here actually solve.

The homotopy analysis method itself rests on a single technical device: rather than expanding the unknown solution directly in a small physical parameter, as classical perturbation methods do, HAM embeds the equation in an artificial homotopy governed by a convergence-control parameter that has no physical meaning and can be chosen freely ^[3]. Liao's later work formalised the definitions and proved general theorems establishing when the resulting homotopy-Maclaurin series converges, giving later authors a template that can be applied to a new equation largely by checking that a small number of technical conditions hold, rather than by constructing a new convergence proof from first principles each time ^[9]. This is the theoretical machinery that Babolian and Saeidian later invoke, without substantial modification, when applying HAM to the Burgers-Fisher equation.

Table 1: Chronological summary of reviewed studies

Study (Year)	Ref.	Method	Equation(s) Solved	Key Contribution
Fisher (1937)	[1]	N/A (foundational)	Fisher's equation (introduced)	Population-genetics reaction-diffusion model
Burgers (1948)	[2]	N/A (foundational)	Burgers' equation (introduced)	Nonlinear convection-diffusion turbulence model
Liao (2003)	[3]	HAM theory	General nonlinear equations	Convergence-control parameter framework
Kaya & El-Sayed (2004)	[4]	Decomposition-based simulation	Generalized Burgers-Fisher	Explicit and numerical solutions, no discretization
Ismail, Raslan & Abd Rabboh (2004)	[5]	ADM	Burgers-Huxley & Burgers-Fisher	Early ADM application to both equations
Hashim, Noorani & Al-Hadidi (2006)	[6]	ADM	Generalized Burgers-Huxley	ADM extended to generalized form
Rashidi, Ganji & Dinarvand (2009)	[7]	HPM	Burgers & Burgers-Fisher	Explicit analytical solutions via HPM
Babolian & Saeidian (2009)	[8]	HAM	Burgers, Fisher, Huxley, Burgers-Fisher, Burgers-Huxley	Unified initial guess/operator across five equations
Liao (2009)	[9]	HAM theory	General	Formal convergence definitions and theorems
Behzadi & Araghi (2011)	[10]	ADM, MADM, VIM, MVIM, MHPM, HAM	Burgers-Fisher	Direct six-method comparison with convergence proofs
Singh, Balyan, Mittal & Saini (2024)	[11]	Time-space pseudospectral	Generalized Burgers-Fisher (multidimensional)	High-accuracy modern numerical benchmark

Applying series methods to the Burgers-Fisher family

Ismail, Raslan, and Abd Rabboh's 2004 application of the Adomian decomposition method to the Burgers-Huxley and Burgers-Fisher equations illustrated the basic ADM recipe for this equation family: split the nonlinear reaction and convection terms using Adomian polynomials, generate successive terms of the solution series recursively, and sum a finite number of terms as the approximate solution ^[5]. Hashim, Noorani, and Al-Hadidi extended the same approach to the generalized Burgers-Huxley equation, where the reaction term carries an arbitrary exponent rather than the quadratic or cubic forms treated in the earlier paper ^[6].

Rashidi, Ganji, and Dinarvand's 2009 application of the homotopy perturbation method to the Burgers and Burgers-Fisher equations followed a structurally similar recipe to ADM, expanding the solution in powers of an embedding parameter and solving a sequence of linear problems, but starting from a homotopy between the full nonlinear equation and a simpler linear one rather than decomposing the nonlinear term directly ^[7]. Babolian and Saeidian's HAM treatment of the same equation family went a step further methodologically: rather than treating each of the Burgers, Fisher, Huxley, Burgers-Fisher, and Burgers-Huxley equations as a separate problem requiring its own initial guess and auxiliary operator, they showed that a single choice of initial guess, linear operator, and auxiliary function could be reused across all five equations, differing only in how the nonlinear term entered the governing homotopy ^[8]. This reuse is possible precisely because of the convergence-control parameter's flexibility, and it is one of the more distinctive methodological points to come out of the HAM literature on this equation family. Kaya and El-Sayed's 2004 treatment of the generalized Burgers-Fisher equation sits slightly apart from the other studies reviewed here: rather than a homotopy

or decomposition-based series, they used a decomposition scheme to obtain explicit solutions directly from the initial condition and then constructed numerical solutions without any discretization of the domain, comparing the results against known analytical solutions for several values of the reaction exponent ^[4].

Comparing methods and establishing convergence

Most of the studies reviewed so far apply one method to one equation and demonstrate accuracy by comparing the resulting series against an exact or reference solution at a handful of points, which shows that the method works for the case tested but does not by itself establish convergence in general or indicate how the method would perform against a genuinely different technique on the same problem. Behzadi and Araghi's 2011 study is the clearest exception in this literature. They applied six methods, the Adomian decomposition method, a modified Adomian decomposition method, the variational iteration method, a modified variational iteration method, a modified homotopy perturbation method, and the homotopy analysis method, to the same Burgers-Fisher equation, proved existence and uniqueness of the solution and convergence of each method's series, and compared the resulting approximations on a single numerical example ^[10]. This makes Behzadi and Araghi's paper the closest existing precedent for a study that combines a HAM formulation, a direct comparison with the Adomian decomposition method, a convergence theorem, and an error analysis, on the Burgers-Fisher equation specifically, all in one place. Every other study reviewed here supplies at most two of these four elements: a method applied to the equation and a numerical comparison against an exact solution, without a general convergence theorem covering the method as applied to this particular equation.

Table 2: Key equation and method forms referenced in this review

Item	Ref.	Form (described)
Fisher's equation	^[1]	A diffusion term plus a logistic reaction term equal to u times the quantity one minus u
Burgers' equation	^[2]	A diffusion term minus a convection term proportional to u times its own spatial derivative
Generalized Burgers-Fisher equation	^[4]	Burgers' convection term combined with a logistic-type reaction term raised to a general power
HAM zeroth-order deformation equation	^[3]	A convex homotopy between a linear auxiliary operator applied to an initial guess and the full nonlinear operator, controlled by an embedding parameter and a convergence-control parameter

Table 2 summarises, in words rather than full notation, the equations and the core HAM construction discussed above; every method-application study reviewed here works with some variant of the third

row, applied to one or more of the equations named in the first two rows.

Table 3: Reference distribution by journal or source

Journal / Source	Number of References
Annals of Eugenics	1
Advances in Applied Mechanics	1
Book (Chapman & Hall/CRC)	1
Applied Mathematics and Computation	2
Mathematical and Computer Modelling	1
Numerical Methods for Partial Differential Equations	1
Communications in Nonlinear Science and Numerical Simulation	2
Mathematical and Computational Applications	1
Mathematics and Computers in Simulation	1

Applied Mathematics and Computation and Communications in Nonlinear Science and Numerical Simulation each account for two of the eleven references; every remaining reference comes from a

distinct journal or book, indicating this literature is spread across general applied-mathematics venues rather than concentrated in a single specialist journal.

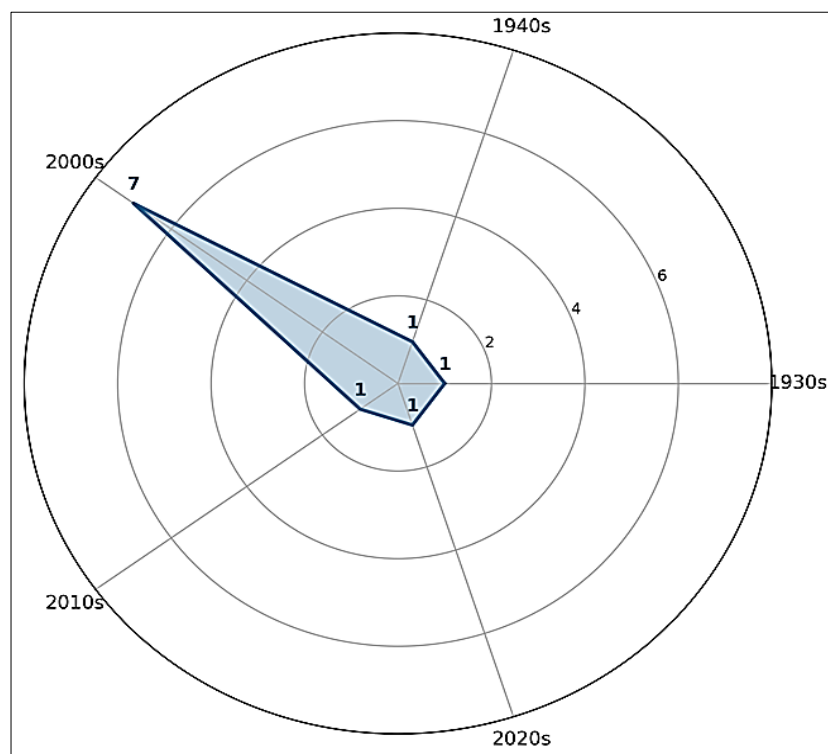


Fig 3: Reviewed references by decade of publication ($n = 11$)

Seven of the eleven references, nearly two-thirds, fall within the 2000s, when ADM, HPM, and HAM were each first applied to this equation family. Activity outside that decade is

thin: one reference each in the 1930s, 1940s, 2010s, and 2020s.

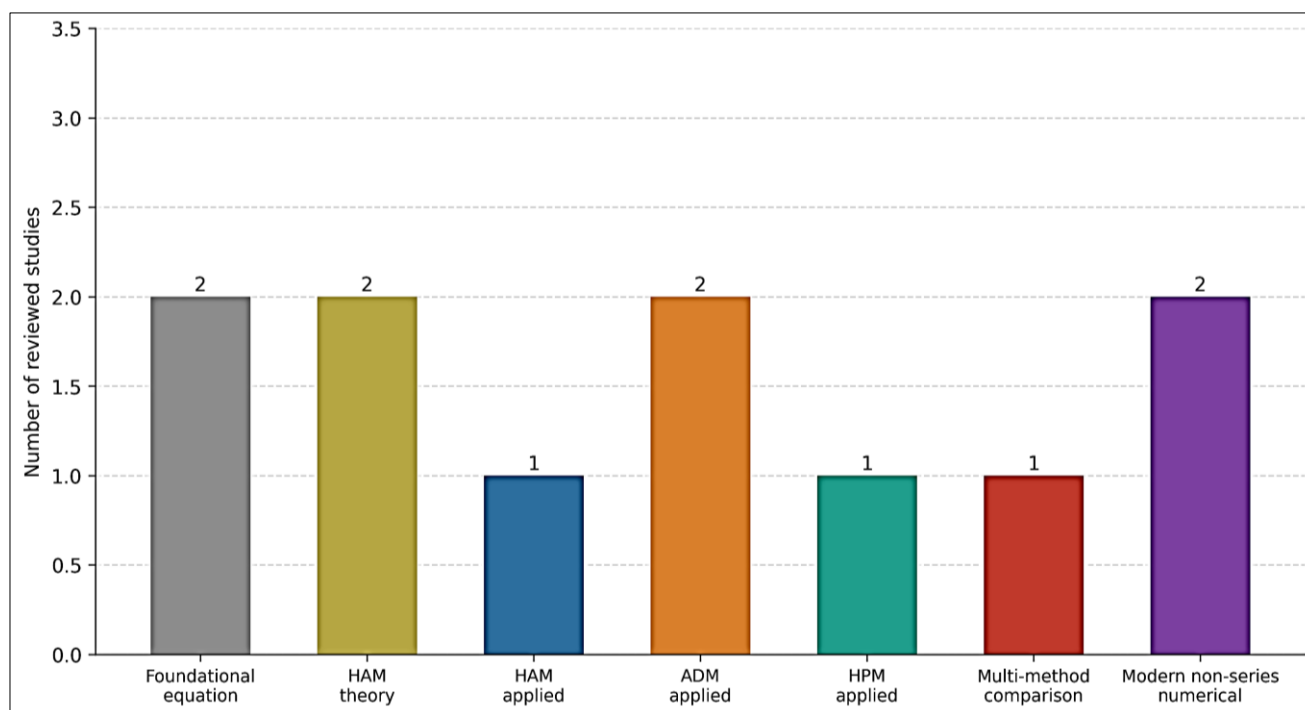


Fig 2: Number of reviewed studies by method category ($n = 11$)

Figure 2 groups the same eleven studies by method category. ADM and the modern non-series numerical methods each account for two studies, HAM theory accounts for two more, and HAM applied, HPM applied, and multi-method comparison each account for exactly one study. The single

multi-method comparison, Behzadi and Araghi, is the only entry that combines HAM with ADM in one paper, which is the empirical basis for treating it as the closest precedent for the kind of comparative, convergence-and-error-analysis study this review set out to locate.

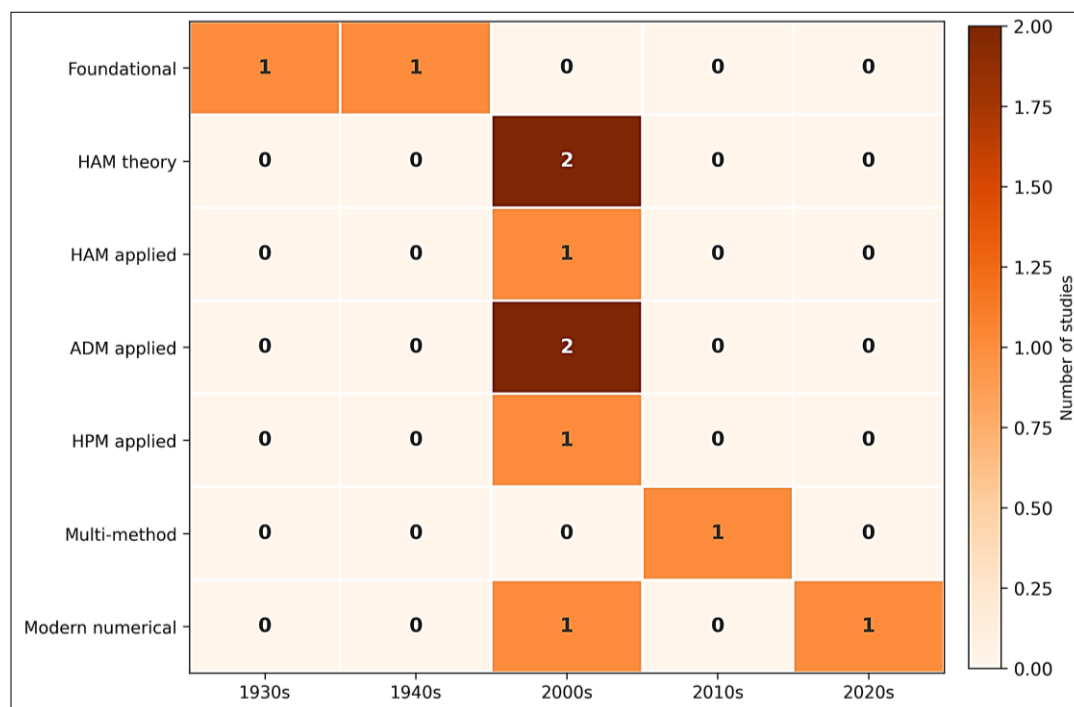


Fig 4: Method category by decade of publication ($n = 11$)

Figure 4 recasts the same eleven studies as a method-by-decade matrix. Every method-application cell with a count above one, HAM theory and ADM applied, each with two studies, falls within the single 2000s column, which is also the only decade containing more than two method categories at once. The modern non-series numerical row is the only one with entries in two different decades, 2004 and 2024, twenty years apart, which is consistent with the comparatively quiet analytical-methods period noted in the Synthesis below.

Synthesis and research gaps

Taken together, this literature shows a field where the individual methods, ADM, HPM, and HAM, are each well established for the Burgers-Fisher equation and its close relatives, and where the underlying convergence theory for HAM in general is well developed thanks to Liao's foundational work [3, 9]. What is comparatively rare is a study that brings a convergence theorem, a direct comparison against the Adomian decomposition method, and a quantitative error analysis together specifically for the Burgers-Fisher equation using HAM as the primary method. Behzadi and Araghi come closest, but their comparison spans six methods rather than focusing specifically on HAM's comparison against ADM, and their convergence arguments, while rigorous, are presented for each method somewhat separately rather than within a single unified HAM-centred convergence theorem of the kind Liao's general theory would support [3, 9, 10]. A study that applied Liao's general convergence theorems directly to the Burgers-Fisher equation, benchmarked the resulting HAM series specifically against Ismail, Raslan, and Abd Rabboh's or Kaya and El-Sayed's ADM-family results, and reported a quantitative error analysis at multiple orders of approximation would fill a gap that the chronology and method counts in Figures 1 and 2 both point to [4, 5, 9].

A second observation is that the modern numerical literature on this equation, represented here by Singh, Balyan, Mittal, and Saini's 2024 pseudospectral study, has moved toward high-accuracy grid-based methods and multidimensional generalizations, while the analytical series-method literature

on this specific equation has been comparatively quiet since 2011 [11]. Whether HAM's convergence-control parameter offers any advantage over these modern numerical benchmarks, in terms of either accuracy or computational cost, does not appear to have been tested directly in the literature reviewed here.

Conclusion

This review has traced how the Adomian decomposition method, the homotopy perturbation method, and the homotopy analysis method have each been applied to the Burgers-Fisher equation and its close relatives, generally one method and one equation at a time, against a backdrop of well-developed general convergence theory for HAM and a single direct multi-method comparison from 2011.

The clearest gap identified here is the absence of a study that applies HAM to the Burgers-Fisher equation specifically, invokes Liao's general convergence theorems in a formal proof tailored to this equation, compares the result quantitatively against the Adomian decomposition method, and reports an error analysis across multiple orders of approximation, all within a single self-contained treatment. Given how well developed each individual piece already is in the literature surveyed here, assembling them into one study looks like a tractable next step, and comparing the result against the modern pseudospectral benchmarks that have emerged since 2011 would help clarify where series methods still offer a practical advantage.

Declarations

Conflict of interest: The author(s) declare no conflict of interest.

Funding: This review received no specific funding.

References

1. Fisher RA. The wave of advance of advantageous genes. *Ann Eugen.* 1937;7(4):355-369.
2. Burgers JM. A mathematical model illustrating the theory of turbulence. *Adv Appl Mech.* 1948;1:171-199.

3. Liao SJ. Beyond Perturbation: Introduction to the Homotopy Analysis Method. Boca Raton (FL): Chapman & Hall/CRC; 2003.
4. Kaya D, El-Sayed SM. A numerical simulation and explicit solutions of the generalized Burgers-Fisher equation. Appl Math Comput. 2004;152:403-413.
5. Ismail HNA, Raslan K, Abd Rabboh AA. Adomian decomposition method for Burger's-Huxley and Burger's-Fisher equations. Appl Math Comput. 2004;159(1):291-301.
6. Hashim I, Noorani MSM, Al-Hadidi MRS. Solving the generalized Burgers-Huxley equation using the Adomian decomposition method. Math Comput Model. 2006;43(11-12):1404-1411.
7. Rashidi MM, Ganji DD, Dinarvand S. Explicit analytical solutions of the generalized Burger and Burger-Fisher equations by homotopy perturbation method. Numer Methods Partial Differ Equ. 2009;25:409-417.
8. Babolian E, Saeidian J. Analytic approximate solutions to Burgers, Fisher, Huxley equations and two combined forms of these equations. Commun Nonlinear Sci Numer Simul. 2009;14(5):1984-1992.
9. Liao SJ. Notes on the homotopy analysis method: Some definitions and theorems. Commun Nonlinear Sci Numer Simul. 2009;14(4):983-997.
10. Behzadi SS, Araghi MAF. Numerical solution for solving Burger's-Fisher equation by using iterative methods. Math Comput Appl. 2011;16(2):443-455.
11. Singh H, Balyan LK, Mittal AK, Saini P. A numerically robust and stable time-space pseudospectral approach for multidimensional generalized Burgers-Fisher equation. Math Comput Simul. 2024.