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Fractional-order logistic growth models in the Caputo sense: A literature review

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Abstract

Replacing the ordinary derivative in the classical logistic growth equation with a Caputo fractional derivative gives a model that is better able to capture memory effects in population growth, but at a cost: unlike its integer-order counterpart, the resulting fractional logistic equation has no straightforward closed-form solution. This review traces the literature on this equation, from Caputo's original definition of the derivative and the predictor-corrector numerical method most commonly used to solve it, through El-Sayed, El-Mesiry, and El-Saka's foundational formulation of the fractional-order logistic equation, to a still-unresolved dispute over whether an exact solution exists at all. West proposed a closed-form function as an exact solution in 2015; Area, Losada, and Nieto showed within a year that the function does not actually satisfy the original equation; and subsequent authors have responded either by modifying the equation so that West's function becomes exact for a related model, or by setting the exactness question aside in favour of numerical and existence-uniqueness results. This review documents that dispute and the numerical literature that has grown up alongside it, and notes that no reviewed study yet supplies an exact, closed-form solution to the fractional logistic equation exactly as originally formulated.

Keywords: Fractional logistic equation, Caputo derivative, predictor-corrector method, existence and uniqueness, population growth model, exact solution, numerical methods, fractional calculus

Introduction

The classical logistic equation, in which the growth rate of a population slows as it approaches a carrying capacity, has a simple closed-form solution and has served as a basic population-growth model since Verhulst introduced it in the nineteenth century. Replacing its ordinary first derivative with a fractional derivative of order between zero and one gives a model capable of representing memory effects, where the current growth rate depends on the population's entire history rather than only its present value. The derivative most commonly used for this purpose is the one defined by Caputo in 1967, chosen largely because it allows initial conditions to be specified in ordinary, integer-order form rather than requiring fractional initial data ^[1].

El-Sayed, El-Mesiry, and El-Saka gave the resulting fractional-order logistic equation its now-standard formulation in 2007, establishing existence and uniqueness of a solution and studying its qualitative behaviour ^[3]. Solving the equation numerically generally relies on some variant of the predictor-corrector approach that Diethelm, Ford, and Freed developed in 2002 for fractional differential equations generally, which estimates the solution at each step and then refines that estimate using the fractional analogue of a trapezoidal correction ^[2].

Whether the fractional logistic equation has an exact, closed-form solution at all is a separate and, as this review shows, still partly unsettled question. West proposed a candidate closed-

form solution in 2015; the following year, Area, Losada, and Nieto showed that West's function, while numerically close to the true solution, does not exactly satisfy the equation as

posed [4, 5]. This review traces both that dispute and the broader numerical and existence-uniqueness literature that has developed around the fractional logistic equation since 2007.

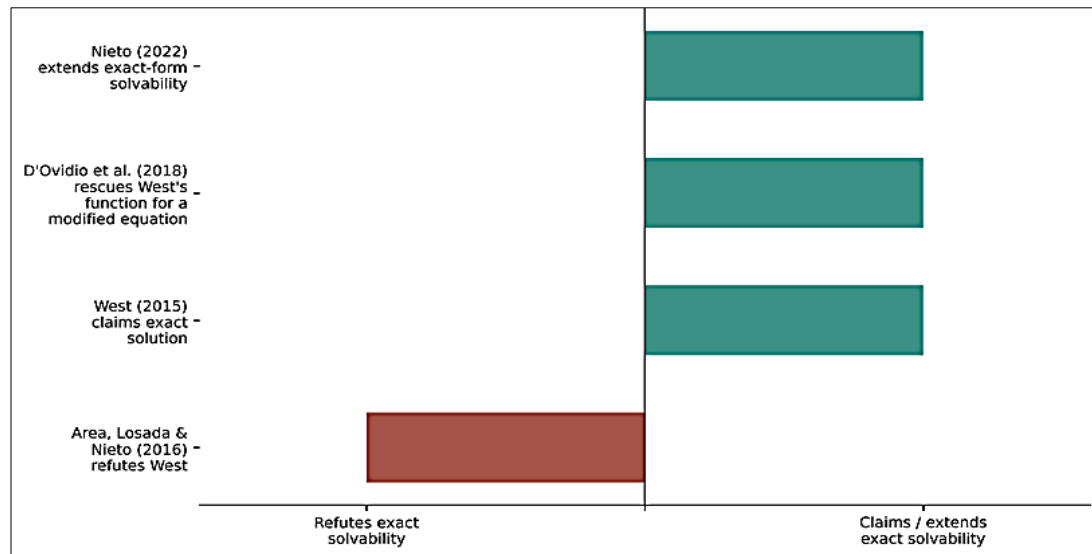


Fig 1: The exact-solution debate: four studies directly engaged with West's (2015) claim

Three of the four studies shown here ultimately side with the possibility of an exact, closed form, but only by working with a modified or generalized version of the equation¹⁵² rather than West's original target. Area, Losada, and Nieto's 2016 refutation remains the only study reviewed here that establishes a negative result about the original fractional logistic equation specifically.

Foundations: The Caputo derivative and the predictor-corrector method

Two technical results do most of the work in this literature. The first is the Caputo derivative itself, defined as a convolution of the function's ordinary derivative against a power-law kernel [1]. Its main practical advantage over the older Riemann-Liouville definition is that the Caputo

derivative of a constant is zero and its associated initial value problems take ordinary initial conditions, both of which make it easier to connect a fractional model to an integer-order one it is meant to generalize¹⁵², which is exactly the connection every fractional logistic study needs to make.

The second is the predictor-corrector method of Diethelm, Ford, and Freed, an adaptation of the classical Adams-Bashforth-Moulton scheme to the fractional setting [2]. Because the Caputo derivative depends on the function's entire history rather than only its most recent value, evaluating it numerically requires summing a weighted history of past values at every step, which is the main source of computational cost in this literature and the main technical challenge every numerical study reviewed here has to manage in some way.

Table 1: Chronological summary of reviewed studies

Study (Year)	Ref.	Relationship to Exact Solution	Method	Key Contribution
Caputo (1967)	[1]	Foundational (not logistic-specific)	N/A	Defined the Caputo fractional derivative
Diethelm, Ford & Freed (2002)	[2]	Foundational (not logistic-specific)	Predictor-corrector	General numerical method for fractional ODEs
El-Sayed, El-Mesiry & El-Saka (2007)	[3]	Numerical/approximate only	Existence-uniqueness, numerical	Introduced the fractional-order logistic equation
West (2015)	[4]	Claims exact solution	Analytic derivation	Proposed a closed-form function for the FLE
Area, Losada & Nieto (2016)	[5]	Refutes exact solvability	Analytic proof	Showed West's function does not solve the FLE
D'Ovidio, Loreti & Ahrabi (2018)	[6]	Claims/extends exact solution	Modified equation, analytic	Modified the FLE so West's function is exact
Yameni Noupoue <i>et al.</i> (2019)	[7]	Numerical/approximate only	Hadamard derivative, 3 numerical methods	Compared GEM, PSE, and CF numerical methods
Nieto (2022)	[8]	Claims/extends exact solution	Generalized proportional derivative	Extended exact-form solvability via Euler numbers
Singh (2026)	[9]	Numerical/approximate only	Variable-kernel Caputo, fixed-point	Existence-uniqueness and series solution for variable-kernel FLE

The search for an exact solution: West's function and its refutation

West's 2015 paper proposed a closed-form function, built from an exponential term modified to account for the fractional order, as an exact solution to the fractional logistic equation, and showed it to be in close numerical agreement with independently computed numerical solutions [4]. Area,

Losada, and Nieto's 2016 note, however, substituted West's function directly back into the original fractional differential equation and showed algebraically that it does not satisfy the equation exactly, despite the close numerical agreement West had reported [5]. Their result did not dispute that West's function is a good approximation; it disputed only the specific claim that the function is an exact solution.

D'Ovidio, Loreti, and Ahrabi responded to this gap in 2018 not by abandoning West's function but by changing the target: they introduced a modified fractional logistic equation, adding a nonlinear term related to the classical (integer-order) logistic solution, for which West's function is an exact solution^[6]. This makes their contribution a genuine rescue of West's function, but for a different equation than the one West had originally set out to solve. Nieto's 2022 paper extended this line of work further, using a generalized proportional fractional derivative and a fractional generalization of the Euler numbers¹⁵³¹⁵³¹⁵³ to construct exact-form solutions for a broader family of logistic-type equations, again working with a generalized rather than the original equation¹⁵³^[8].

Numerical and existence-uniqueness studies

Alongside the exact-solution dispute, a separate strand of the

literature has set the exactness question aside and focused on existence, uniqueness, and numerical approximation. Yameni Noupoue, Tandoğdu, and Awadalla used a Hadamard rather than Caputo derivative and integral to establish existence and uniqueness, then compared three numerical approaches, a generalized Euler method, a power series expansion method¹⁵³, and a Caputo-Fabrizio method, against a real dataset, finding that the generalized Euler and power series methods¹⁵³ produced the lowest error at a fractional order close to one^[7]. Singh's 2026 study returns to the Caputo setting but allows the kernel of the fractional derivative to vary, proving existence and uniqueness via a fixed-point argument and constructing an infinite-series analytic solution alongside a numerical treatment, making it the most recent attempt reviewed here to extend the equation's analytical tractability without engaging directly in the West dispute^[9].

Table 2: Key concepts referenced in this review

Concept	Ref.	Description
Caputo fractional derivative	[1]	A fractional-order derivative defined via a convolution integral of the integer-order derivative, chosen because it allows initial conditions expressed in ordinary, integer-order form
Fractional logistic equation (FLE)	[3]	The classical logistic growth equation with its first-order derivative replaced by a Caputo derivative of order between zero and one
Predictor-corrector method	[2]	A two-stage numerical scheme that first estimates (predicts) the solution at the next step, then refines (corrects) that estimate using the predicted value
West function	[4,5]	A closed-form expression proposed as a solution to the FLE, later shown to satisfy a related but not identical equation

Table 2 defines the four terms most central to this review; the dispute traced in the next section is fundamentally about whether the second

and fourth rows can be connected by an exact, rather than only approximate, relationship.

Table 3: Reference distribution by journal

Journal	Number of References
Geophysical Journal of the Royal Astronomical Society	1
Nonlinear Dynamics	1
Applied Mathematics Letters	1
Physica A: Statistical Mechanics and its Applications	3
Advances in Continuous and Discrete Models	1
Fractional Calculus and Applied Analysis	1
Mathematical Methods in the Applied Sciences	1

Physica A: Statistical Mechanics and its Applications carries three of the nine references, including West's original claim and both direct

responses to it, making it the de facto venue for the exact-solution dispute; every other journal contributes exactly one reference.

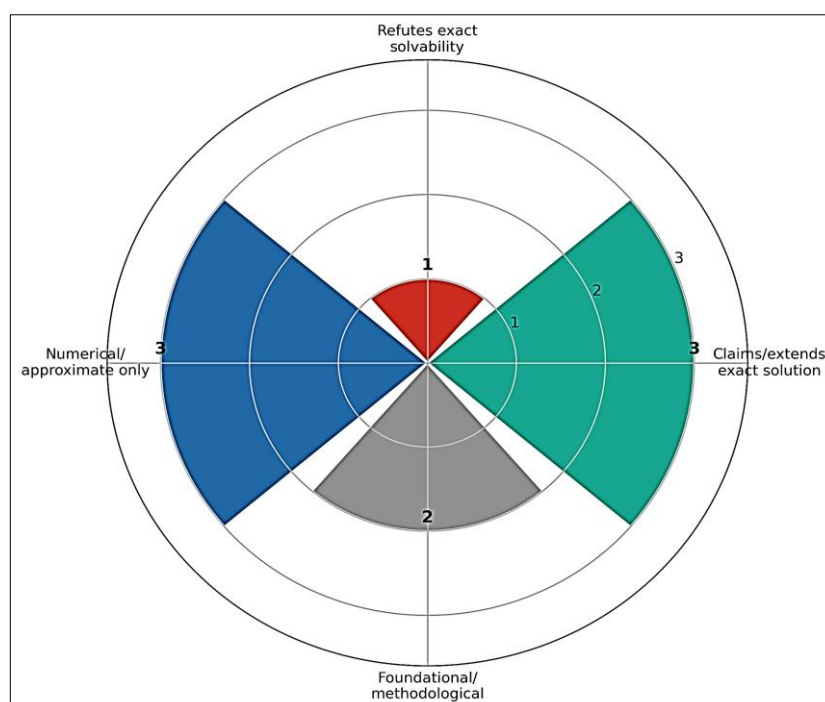


Fig 2: Reviewed studies by relationship to the exact-solution question ($n = 9$)

Three of the nine reviewed studies claim or extend some form of exact solvability, but, as Figure 1 already showed, all three do so for a modified or generalised equation rather than the original. Three more studies are purely numerical or

approximate, one directly refutes exact solvability for the original equation, and two are foundational papers not specific to the logistic setting at all.

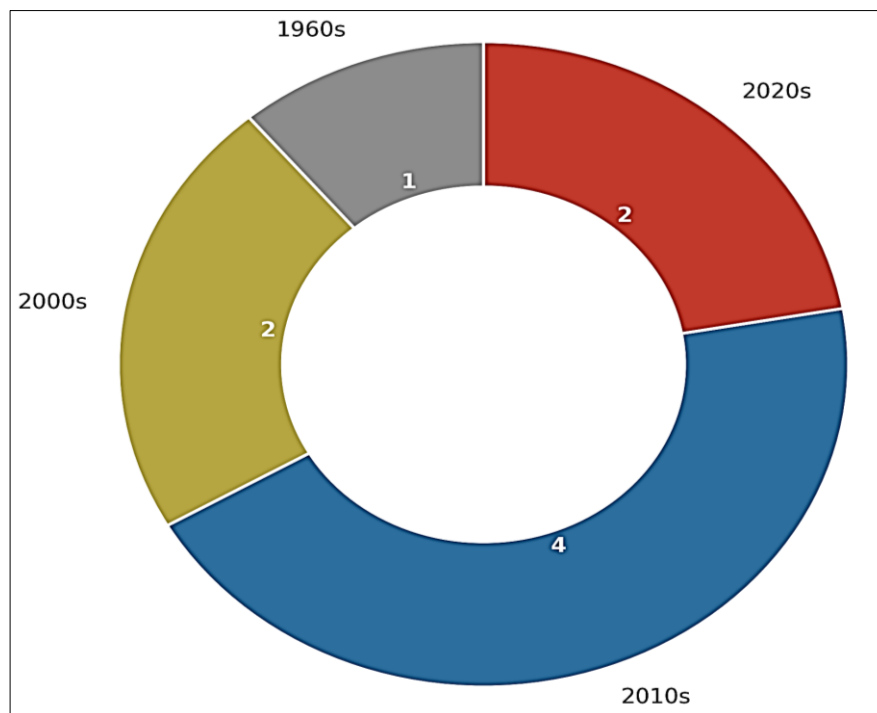


Fig 3: Reviewed references by decade of publication ($n = 9$)

Four of the nine references, the largest share, come from the 2010s, the decade containing both West's claim and its refutation. The 2020s already contribute two references

despite covering only a few years so far, suggesting continued rather than declining interest in this equation.

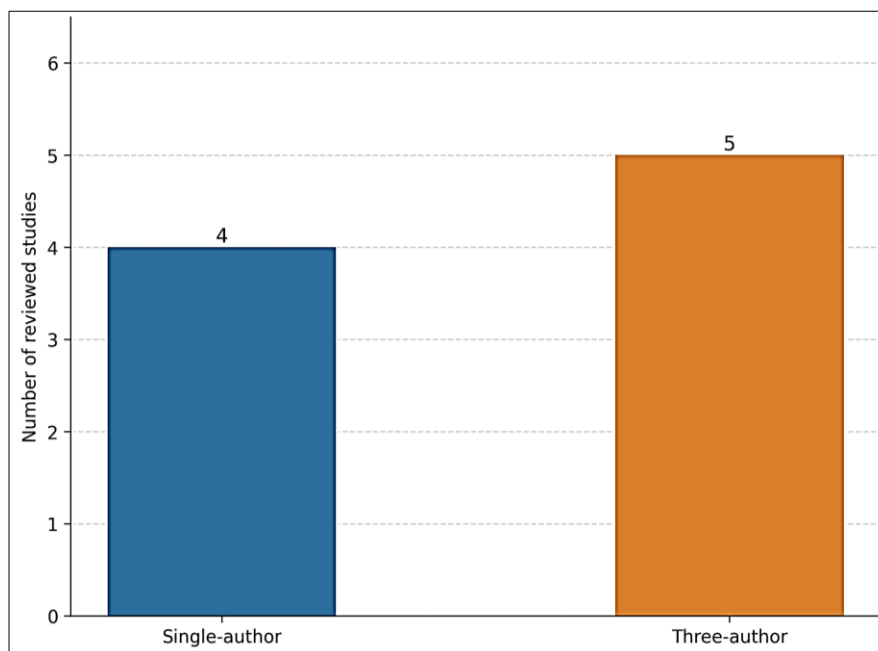


Fig 4: Reviewed studies by author-team size ($n = 9$)

The nine reviewed studies split cleanly between single-author and three-author papers, with no two-author studies at all in this particular set. Both of the papers most central to the exact-solution dispute, West's original claim and Area, Losada, and Nieto's refutation, sit on opposite sides of this split: West's paper is single-authored, while the refutation has three authors.

Synthesis and research gaps

Read together, these nine studies trace a field organised around a single unresolved question rather than a steadily accumulating consensus. Every study that claims some form of exact solvability, West, D'Ovidio and colleagues, and Nieto, either targets or ends up needing a modified version of the equation El-Sayed, El-Mesiry, and El-Saka originally

formulated in 2007 [3, 4, 6, 8]. The one study that engages the original equation directly on the exactness question, Area, Losada, and Nieto, reaches a negative result [5].

This leaves the original fractional logistic equation, exactly as formulated in 2007, without either a confirmed exact solution or a proof that no such solution exists in closed form; the literature reviewed here has, in effect, answered the question for several nearby equations while leaving the original one open. The numerical and existence-uniqueness studies reviewed here, Yameni Noupoue and colleagues and Singh, sidestep this question productively but do not resolve it, since establishing that a unique solution exists is a different claim from exhibiting that solution in closed form [7, 9].

A second observation concerns method reuse. The predictor-corrector approach of Diethelm, Ford, and Freed, though not developed specifically for the logistic equation, underlies or is directly comparable to the numerical methods used in most of the studies reviewed here, which suggests that any future attempt to resolve the exact-solution question for the original equation will still need a numerical benchmark of this kind to check candidate solutions against, exactly as Area, Losada, and Nieto did in 2016 [2, 5].

Conclusion

This review has traced the literature on the fractional-order logistic equation from Caputo's 1967 definition of the derivative and El-Sayed, El-Mesiry, and El-Saka's 2007 formulation of the equation itself, through a still-unresolved dispute over whether the equation admits an exact, closed-form solution.

The clearest gap identified here is definitional rather than computational: no reviewed study either exhibits a confirmed exact solution to the original fractional logistic equation or proves that none exists. Every claimed exact solution in this literature applies to a modified or generalised version of the equation instead. Settling the original question, one way or the other, using the same analytic and numerical tools already developed across the nine studies reviewed here, would resolve a dispute that has now run for roughly a decade without a direct answer.

Declarations

Conflict of interest: The author(s) declare no conflict of interest.

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