

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452

NAAS Rating (2026): 4.49

Maths 2026; 11(7): 36-39

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<https://www.mathsjournal.com>

Received: 11-04-2026

Accepted: 15-05-2026

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On Liouville difference cordial labeling of some splitting graphs

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DOI: <https://www.doi.org/10.22271/math.2026.v11.i7a.2611>

Abstract

Graph labeling is an active area of research in graph theory with applications in coding theory, communication networks, and circuit design. Cordial labelings, introduced as a framework for balanced labeling schemes, have been extended in several directions, including those based on the Liouville lambda function $\lambda(n)$. In Liouville Difference Cordial Labeling, the label of each edge is determined by the absolute difference of the Liouville values assigned to its end vertices. A graph is said to admit such a labeling if the numbers of edges labeled -1 and 1 differ by at most one. In this paper, we investigate Liouville Difference Cordial Labeling for the splitting graphs of the Path, Comb, Ladder, Star, and Bistar. The results enlarge the class of graphs admitting this labeling and contribute to the study of Liouville-function-based cordial labelings.

Keywords: Graph labeling, Liouville lambda function, Liouville difference cordial labeling and splitting graph

Introduction

Graph labeling is a classical and well-studied branch of graph theory that has developed into a rich area of combinatorial research. Since Rosa's ^[1] pioneering work in 1967, the subject has attracted considerable attention because of its elegant structure and wide range of applications. Among the various labeling schemes, cordial labeling introduced by Cahit ^[2] occupies an important place. In a cordial labeling, the assigned labels are required to be balanced so that the corresponding edge labels differ by at most one in frequency. This notion has led to many variants, including prime cordial labeling, product cordial labeling, and difference cordial labeling and so on. Gallian's ^[3] survey provides a comprehensive account of the development of graph labeling.

A recent extension uses the Liouville lambda function ^[4, 5] $\lambda(n) = (-1)^{\Omega(n)}$, where $\Omega(n)$ denotes the number of prime factors of n , counted with multiplicity. Since $\lambda(n)$ is multiplicative and arithmetic in nature, it serves as a natural basis for defining new cordial-type labelings. Motivated by this idea, we previously introduced Liouville Difference Cordial Labeling (LDCL) and studied it for some acyclic graphs ^[6]. Formally, let $G = (V, E)$ be a simple graph with vertex set V and edge set E . A 1-1 function $f: V \rightarrow \{-1, 1\}$ is said to be a *Liouville Difference Cordial labeling* of the graph G if the induced edge function $f*: E \rightarrow \{-1, 1\}$ is defined by $f^*(uv) = \lambda(|f(u) - f(v)|)$, with the condition $|e_{f^*}(-1) - e_{f^*}(1)| \leq 1$ for all $uv \in E$, where $e_{f^*}(k)$ is the number of edges labeled with k . A graph admitting such a labeling is called a Liouville Difference Cordial graph.

In this paper, we study LDCL for the splitting graphs of several standard graph families. The splitting graph ^[7, 8, 9, 10] $Spl(G)$ is obtained from G by adding, for each vertex v , a new vertex v' adjacent to every neighbor of v . This operation preserves much of the adjacency structure of the original graph, making splitting graphs a natural class for investigating labeling properties. Using explicit constructions, we show that the splitting graphs of the Path, Comb, Ladder, Star, and Bistar admit Liouville Difference Cordial labeling.

Throughout this paper, let $G = G(V, E)$ be a simple, finite, connected, and undirected graph.

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Main Result

Theorem 2.1: For all $n > 1$, $Spl(P_n)$ is a Liouville Difference Cordial graph.

Proof: Let $V = \{u_i, u'_i; 1 \leq i \leq n\}$ and $E = \{u_i u_{i+1}; 1 \leq i < n\} \cup \{u_i u'_{i+1}; 1 \leq i < n\} \cup \{u_{i+1} u'_i; 1 \leq i < n\}$ denotes the vertex and edge set of the splitting graph of path P_n respectively.

$$\Rightarrow |V| = 2n \text{ and } |E| = 3n - 3.$$

Let us define the vertex labeling of the graph $Spl(P_n)$ by the $1 - 1$ function as follows: For all $1 \leq i \leq n$,

$$f(u_i) = 2^{3i-1} \text{ and}$$

$$f(u'_i) = 2^{3i}$$

Then for every edge in E , the induced edge function satisfies the following relations: For all $1 \leq i < n$,

$$f * (u_i u_{i+1}) = \begin{cases} -1 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

$$f * (u_i u'_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{2} \\ -1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

and

$$f * (u_{i+1} u'_i) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{2} \\ -1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

Case (i): When n is odd

The number of edges labelled with the labels -1 and 1 are equal and it is $\frac{3n-3}{2}$ in each case respectively.

$$\text{i.e., } e_{f*}(-1) = e_{f*}(1) = \frac{3n-3}{2}$$

Case (ii): When n is even

The number of edges with the labels -1 and 1 are $\left\lfloor \frac{3n-3}{2} \right\rfloor$ and $\left\lceil \frac{3n-3}{2} \right\rceil$, respectively.

$$\text{i.e., } e_{f*}(-1) = \left\lfloor \frac{3n-3}{2} \right\rfloor \text{ and } e_{f*}(1) = \left\lceil \frac{3n-3}{2} \right\rceil$$

In both the cases, the labeled edges always satisfy the condition $|e_{f*}(-1) - e_{f*}(1)| \leq 1$.

Hence, for all $n > 1$, $Spl(P_n)$ is a Liouville Difference Cordial graph.

Theorem 2.2: For all $n \geq 1$, $Spl(P_n \bullet K_1)$ is a Liouville Difference Cordial graph.

Proof: Let $V = \{u_i, v_i, u'_i, v'_i; 1 \leq i \leq n\}$ and $E = \bigcup_{i=1}^n E_i$, denotes the vertex set and edge set of the splitting graph of the comb graph $P_n \bullet K_1$ respectively, where $E_1 = \{u_i u_{i+1}; 1 \leq i < n\}$; $E_2 = \{u_i v_i; 1 \leq i \leq n\}$; $E_3 = \{u_i u'_{i+1}; 1 \leq i < n\}$; $E_4 = \{u_{i+1} u'_i; 1 \leq i < n\}$; $E_5 = \{u_i v'_i; 1 \leq i \leq n\}$ and $E_6 = \{u'_i v_i; 1 \leq i \leq n\}$

$$\Rightarrow |V| = 4n \text{ and } |E| = 6n - 3.$$

Let the vertex labeling of the graph $Spl(P_n \bullet K_1)$ by the $1 - 1$ function as follows: For all $1 \leq i \leq n$,

$$f(u_i) = 2^{3i}$$

$$f(v_i) = 2^{3i-1}$$

$$f(u'_i) = 2^{3i+1} \text{ and}$$

$$f(v'_i) = 2^{3i} + 3^i$$

Then by the definition of Liouville Difference cordial labeling, for every edge in E , the following results holds good. For all $1 \leq i < n$,

$$f * (u_i u_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{2} \\ -1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

$$f * (u_i u'_{i+1}) = f * (u_{i+1} u'_i) = \begin{cases} -1 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

and for all $1 \leq i \leq n$,

$$f * (u_i v_i) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{2} \\ -1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

$$f * (u_i v'_i) = f * (u'_i v_i) = \begin{cases} -1 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

The number of edges labelled with the labels -1 and 1 $\left\lfloor \frac{6n-1}{2} \right\rfloor$ and $\left\lceil \frac{6n-1}{2} \right\rceil$, respectively.

$$\text{i.e., } e_{f*}(-1) = \left\lfloor \frac{6n-1}{2} \right\rfloor \text{ and } e_{f*}(1) = \left\lceil \frac{6n-1}{2} \right\rceil$$

Thus, the labeled edges satisfy the relation $|e_{f*}(-1) - e_{f*}(1)| \leq 1$.

Hence, for all $n \geq 1$, $Spl(P_n \bullet K_1)$ is a Liouville Difference Cordial graph.

Theorem 2.3: For all $n \geq 1$, there exists a Liouville Difference Cordial labeling in $Spl(L_n)$.

Proof: Let $V = \{u_i, v_i, u'_i, v'_i; 1 \leq i \leq n\}$ and $E = \bigcup_{j=1}^9 E_j$, denotes the vertex set and edge set of the splitting graph of the ladder L_n respectively, where

$$E_1 = \{u_i u_{i+1}; 1 \leq i < n\};$$

$$E_2 = \{v_i v_{i+1}; 1 \leq i < n\};$$

$$E_3 = \{u_i v_i; 1 \leq i \leq n\};$$

$$E_4 = \{u_i u'_{i+1}; 1 \leq i < n\};$$

$$E_5 = \{u_{i+1} u'_i; 1 \leq i < n\};$$

$$E_6 = \{v_i v'_{i+1}; 1 \leq i < n\};$$

$$E_7 = \{v_{i+1} v'_i; 1 \leq i < n\};$$

$$E_8 = \{u_i v'_i; 1 \leq i \leq n\};$$

$$E_9 = \{u'_i v_i; 1 \leq i \leq n\}$$

$$\Rightarrow |V| = 4n \text{ and } |E| = 9n - 6.$$

Let us define the vertex labeling of the graph $Spl(L_n)$ by the 1-1 function as follows: For all $1 \leq i \leq n$,

$$f(u_i) = 2^{5i-3}$$

$$f(v_i) = 2^{5i-2}$$

$$f(u'_i) = 2^{5i-1} \text{ and}$$

$$f(v'_i) = 2^{5i}$$

Then by the definition of the induced edge function, every edge in E , the following results: For all $1 \leq i < n$,

$$f * (u_i u_{i+1}) = f * (u_i u'_{i+1}) = f * (u_{i+1} u'_i) = \begin{cases} -1 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

$$f * (v_i v_{i+1}) = f * (v_i v'_{i+1}) = f * (v_{i+1} v'_i) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{2} \\ -1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

and for all $1 \leq i \leq n$,

$$f * (u_i v_i) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{2} \\ -1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

$$f * (u_i v'_i) = f * (u'_i v_i) = \begin{cases} -1 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

Case (i): When n is odd

The number of edges with the labels -1 and 1 are $\left\lfloor \frac{9n-4}{2} \right\rfloor$ and $\left\lfloor \frac{9n-6}{2} \right\rfloor$, respectively.

$$\text{i.e., } e_{f*}(-1) = \left\lfloor \frac{9n-4}{2} \right\rfloor \text{ and } e_{f*}(1) = \left\lfloor \frac{9n-6}{2} \right\rfloor$$

Case (ii): When n is even

The number of edges with the labels -1 and 1 are equal and it is $\frac{9n-6}{2}$ in each case, respectively.

$$\text{i.e., } e_{f*}(-1) = e_{f*}(1) = \frac{9n-6}{2}$$

In both the cases, the labeled edges always satisfy the relation $|e_{f*}(-1) - e_{f*}(1)| \leq 1$.

Thus, there exists a Liouville Difference Cordial labeling in $Spl(L_n)$, for all $n \geq 1$.

Theorem 2.4: For all $n > 1$, $Spl(S_n)$ is a Liouville Difference Cordial graph.

Proof: Let the vertex and edge sets of the splitting graph $Spl(S_n)$ be denoted by $V = \{u_i, u'_i; 1 \leq i \leq n\}$ and $E = \{u_1 u_i; 1 < i \leq n\} \cup \{u_1 u'_i; 1 < i \leq n\} \cup \{u'_1 u_i; 1 < i \leq n\}$, respectively.

$$\Rightarrow |V| = 2n \text{ and } |E| = 3n - 3.$$

Let the vertex labeling of the graph $Spl(S_n)$ be defined by the 1-1 function as follows:

$$f(u_i) = \begin{cases} 2^{2i} & \text{if } i = 1 \\ 2^{2i-1} & \text{if } 1 < i \leq n \end{cases}$$

$$\text{and } f(u'_i) = \begin{cases} 2^{2n} & \text{if } i = 1 \\ 2^2 + 3^{i-1} & \text{if } 1 < i \leq n \end{cases}$$

Then by the definition of the induced edge function, for every edge in E , the following results holds good: For all $1 < i \leq n$,

$$f * (u_1 u'_i) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{2} \\ -1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

$$f * (u_1 u_i) = \lambda(2^2) \lambda(|1 - 2^{2i-3}|) \rightarrow (1)$$

and

$$f * (u'_1 u_{n-i+2}) = \lambda(|2^{2n-2i+3}|) \lambda(|1 - 2^{2i-3}|) \rightarrow (2)$$

Case (1): Suppose if $1 - 2^{2i-3}$ is a prime or a product of primes where the sum of all the exponents is odd.

Then, $f * (u_1 u_i) = -1$ and $f * (u'_1 u_{n-i+2}) = 1$.

Case (2): Suppose if $1 - 2^{2i-3}$ is a product of primes where the sum of all the exponents is even.

Then, $f * (u_1 u_i) = 1$ and $f * (u'_1 u_{n-i+2}) = -1$.

Case (i): When n is odd

The number of edges labelled with the labels -1 and 1 are equal and it is $\frac{3n-3}{2}$ in each case respectively.

$$\text{i.e., } e_{f*}(-1) = e_{f*}(1) = \frac{3n-3}{2}$$

Case (ii): When n is even

The number of edges with the labels -1 and 1 are $\frac{3n-2}{2}$ and $\frac{3n-4}{2}$, respectively.

$$\text{i.e., } e_{f*}(-1) = \frac{3n-2}{2} \text{ and } e_{f*}(1) = \frac{3n-4}{2}$$

In both the above mentioned cases, it is clear that the labeled edges always satisfy the condition $|e_{f*}(-1) - e_{f*}(1)| \leq 1$.

Hence, for all $n > 1$, $Spl(S_n)$ is a Liouville Difference Cordial graph.

Theorem 2.5: For all $n \geq 3$, there exists a Liouville Difference Cordial labeling in $Spl(B_{n,n})$.

Proof: Let the vertex and edge sets of the splitting graph $Spl(B_{n,n})$ be denoted by $V = \{u_i, v_i, u'_i, v'_i; 1 \leq i \leq n\}$ and $E = \cup_{j=1}^9 E_j$ respectively, where

$$E_1 = \{u_1 u_i; 1 < i \leq n\};$$

$$E_2 = \{v_1 v_i; 1 < i \leq n\};$$

$$E_3 = \{u_1 u'_i; 1 < i \leq n\};$$

$$E_4 = \{u'_1 u_i; 1 < i \leq n\};$$

$$E_5 = \{v_1 v'_i; 1 < i \leq n\};$$

$$E_6 = \{v'_1 v_i; 1 < i \leq n\};$$

$$E_7 = \{u_1 v_1\};$$

$$E_8 = \{u_1 v'_1\};$$

$$E_9 = \{u'_1 v_1\}$$

$$\Rightarrow |V| = 4n \text{ and } |E| = 6n - 3.$$

Let us consider the vertex labeling of the splitting graph $Spl(B_{n,n})$ as follows:

$$f(u_i) = \begin{cases} 2^5 & \text{if } i = 1 \\ 2^{2i+3} & \text{if } 1 < i \leq n \end{cases}$$

$$f(v_i) = \begin{cases} 2 & \text{if } i = 1 \\ 2^{2i+2} & \text{if } 1 < i \leq n \end{cases}$$

$$f(u'_i) = \begin{cases} 2^2 & \text{if } i = 1 \\ 2^5 + 3^i & \text{if } 1 < i \leq n \end{cases}$$

$$\text{and } f(v'_i) = \begin{cases} 2^4 & \text{if } i = 1 \\ 2 + 3^{i-1} & \text{if } 1 < i \leq n \end{cases}$$

Then by the definition of the induced edge function, every edge in E , satisfies the following results:

$$f * (u_1 v_1) = f * (u'_1 v_1) = -1$$

$$f * (u_1 v'_1) = 1$$

For all $1 < i \leq n$,

$$f * (u_1 u'_i) = \begin{cases} -1 & \text{if } i \equiv 1 \pmod{2} \\ 1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

$$\text{and } f * (v_1 v'_i) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{2} \\ -1 & \text{if } i \equiv 0 \pmod{2} \end{cases}$$

Now consider for all $1 < i \leq n$,

$$f * (u_1 u_i) = \lambda(2^5) \lambda(|1 - 2^{2i-2}|) \rightarrow (3)$$

and

$$f * (v'_1 v_i) = \lambda(2^4) \lambda(|1 - 2^{2i-2}|) \rightarrow (4)$$

Case (1): Suppose if $1 - 2^{2i-2}$ is a prime or a product of primes where the sum of all the exponents is odd.

Then from (3) and (4), $f * (u_1 u_i) = 1$ and $f * (v'_1 v_i) = -1$.

Case (2): Suppose if $1 - 2^{2i-2}$ is a product of primes where the sum of all the exponents is even.

Then (3) and (4) implies that $f * (u_1 u_i) = -1$ and $f * (v'_1 v_i) = 1$.

And consider for all $1 < i \leq n$,

$$f * (v_1 v_i) = \lambda(2) \lambda(|1 - 2^{2i+1}|) \rightarrow (5)$$

and

$$f * (u'_1 u_i) = \lambda(2^2) \lambda(|1 - 2^{2i+1}|) \rightarrow (6)$$

Case (1): Suppose if $1 - 2^{2i+1}$ is a prime or a product of primes where the sum of all the exponents is odd.

Then, from (5) and (6), $f * (v_1 v_i) = 1$ and $f * (u'_1 u_i) = -1$.
Case (2): Suppose if $1 - 2^{2i+1}$ is a product of primes where the sum of all the exponents is even.

Then, from (5) and (6), $f * (v_1 v_i) = -1$ and $f * (u'_1 u_i) = 1$.
Thus, the number of edges labeled with the labels -1 and 1 are $3n - 1$ and $3n - 2$, respectively.

$$\text{i.e., } e_{f^*}(-1) = 3n - 1 \text{ and } e_{f^*}(1) = 3n - 2$$

It is evident that the labeled edges always satisfy the condition $|e_{f^*}(-1) - e_{f^*}(1)| \leq 1$.

Hence, there exists a Liouville Difference Cordial labeling in $Spl(B_{n,n})$, for all $n \geq 3$.

Conclusion

This paper establishes that the splitting graphs of the Path, Comb, Ladder, Star, and Bistar admit Liouville Difference Cordial Labeling. The constructions presented here extend the known family of Liouville Difference Cordial graphs and demonstrate that the splitting operation preserves this property for several important graph classes. The present study suggests several directions for further research, particularly the investigation of LDCL for other graph families and graph operations.

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