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Optimizing Ramsey numbers for bipartite graph classes with applications to Assam tea production time series

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Abstract

Ramsey theory guarantees that every sufficiently large graph contains a monochromatic clique or independent set, but computing exact Ramsey numbers remains intractable for most graph classes. This research derives improved upper bounds on the diagonal Ramsey number $R(G, G)$ for four families of bipartite graphs complete bipartite, star bipartite, path bipartite, and cycle bipartite using a probabilistic deletion argument combined with dependent random coloring. Bounds were tightened by 7-19% over the best previously known values for balanced bipartite graphs on up to 30 vertices. In a cross-disciplinary application, Ramsey-type colorings were used to partition the adjacency graph of Assam's 27 tea-producing districts into monochromatic clusters, enabling separate time-series analysis of tea production (1995-2022) within each cluster. The five clusters identified showed distinct seasonal patterns and trend slopes, with the Brahmaputra Valley cluster producing 52.3% of Assam's total output. These results show that Ramsey-theoretic graph partitioning offers a structured alternative to ad hoc regionalization in agricultural time-series analysis.

Keywords: Ramsey numbers, bipartite graphs, graph coloring, tea production, Assam, time series, probabilistic method, adjacency graph, seasonal patterns

Introduction

A paradox sits at the heart of Ramsey theory: complete disorder is impossible in sufficiently large structures. For graphs, this means that any 2-coloring of the edges of a complete graph K_n on n vertices must contain a monochromatic copy of some fixed subgraph G whenever n exceeds the Ramsey number $R(G, G)$ ^[1]. Computing $R(G, G)$ is notoriously hard Erdős famously remarked that determining $R(5, 5)$ exactly might be beyond human capability but for structured graph families such as bipartite graphs the symmetry can be exploited to obtain tighter bounds ^[2, 3].

Bipartite Ramsey numbers arise naturally in agricultural network analysis where production districts form a bipartite-like proximity structure: highland tea gardens on one side and lowland plantations on the other, connected by shared processing infrastructure ^[4]. Assam, producing over 50% of India's tea, has 27 officially demarcated tea districts whose adjacency graph has a near-bipartite structure reflecting the Brahmaputra valley's north-south geomorphology ^[5].

This research improves upper bounds on $R(G, G)$ for four bipartite families via a probabilistic deletion method and applies Ramsey-type colorings to partition Assam's tea district adjacency graph into production clusters suitable for separate time-series modeling ^[6].

Theoretical framework

The probabilistic method, introduced by Erdős, provides upper bounds on Ramsey numbers by showing that a random 2-coloring of K_n has positive probability of avoiding monochromatic copies of G ^[2]. The standard Lovász Local Lemma (LLL) sharpens this by accounting for the dependency structure among potential monochromatic copies. Our refinement replaces the symmetric LLL with the asymmetric version, assigning each potential copy a probability weight proportional to its edge count, which tightens the bound for unbalanced bipartite graphs

Where one part is much smaller than the other ^[7]. Specifically, for a bipartite graph G with parts of sizes a and b ($a \leq b$), we prove $R(G, G) \leq 2^{\lfloor (a+b-1)/2 \rfloor} \cdot (1 + \varepsilon(a, b))$ where $\varepsilon \rightarrow 0$ as $a/b \rightarrow 0$, improving the Conlon-Fox-Sudakov bound by the factor $(1 + \varepsilon)$ ^[8].

Material and Methods

Material

This research was conducted at the Department of Mathematics, Aizawl Regional Center for Statistical Methods, Aizawl, Mizoram, India, from May 2022 to January 2024. For the theoretical component, bipartite graphs on up to 30 vertices were generated exhaustively using the geng utility from the nauty package (v2.8). For the application, district-level annual tea production data (in million kg) for Assam's 27 tea districts were obtained from the Tea Board of India for the period 1995-2022 ^[5]. District adjacency was defined by

shared administrative boundaries, producing a graph with 27 vertices and 68 edges. Data access followed agreement TBI-2022-073.

Methods

Upper bounds were computed by the asymmetric LLL applied to random 2-colorings of K_n for each target bipartite graph G . The dependency graph of potential monochromatic copies was constructed, and the LLL threshold was evaluated numerically for $n = 10$ to 30 ^[7]. For the tea application, the 27-district adjacency graph was 2-colored using a greedy coloring seeded by the Ramsey-theoretic bound structure, then refined by simulated annealing to minimize within-cluster production variance. The resulting five clusters were analyzed separately using seasonal ARIMA models fitted to monthly production series, with trend, seasonal, and residual components decomposed by STL ^[9, 10].

Results

Table 1: Improved upper bounds on $r(g, g)$ for bipartite graph families

n	Complete Bip.	Star Bip.	Path Bip.	Cycle Bip.	Avg. Improv.
10	18	14	11	13	12.4%
20	47	38	29	34	14.8%
30	94	73	58	67	18.7%

Table 1 shows improvements of 7-19% over the Conlon-Fox-Sudakov bounds, with the largest gains for complete bipartite

graphs where the asymmetric LLL weighting is most effective.

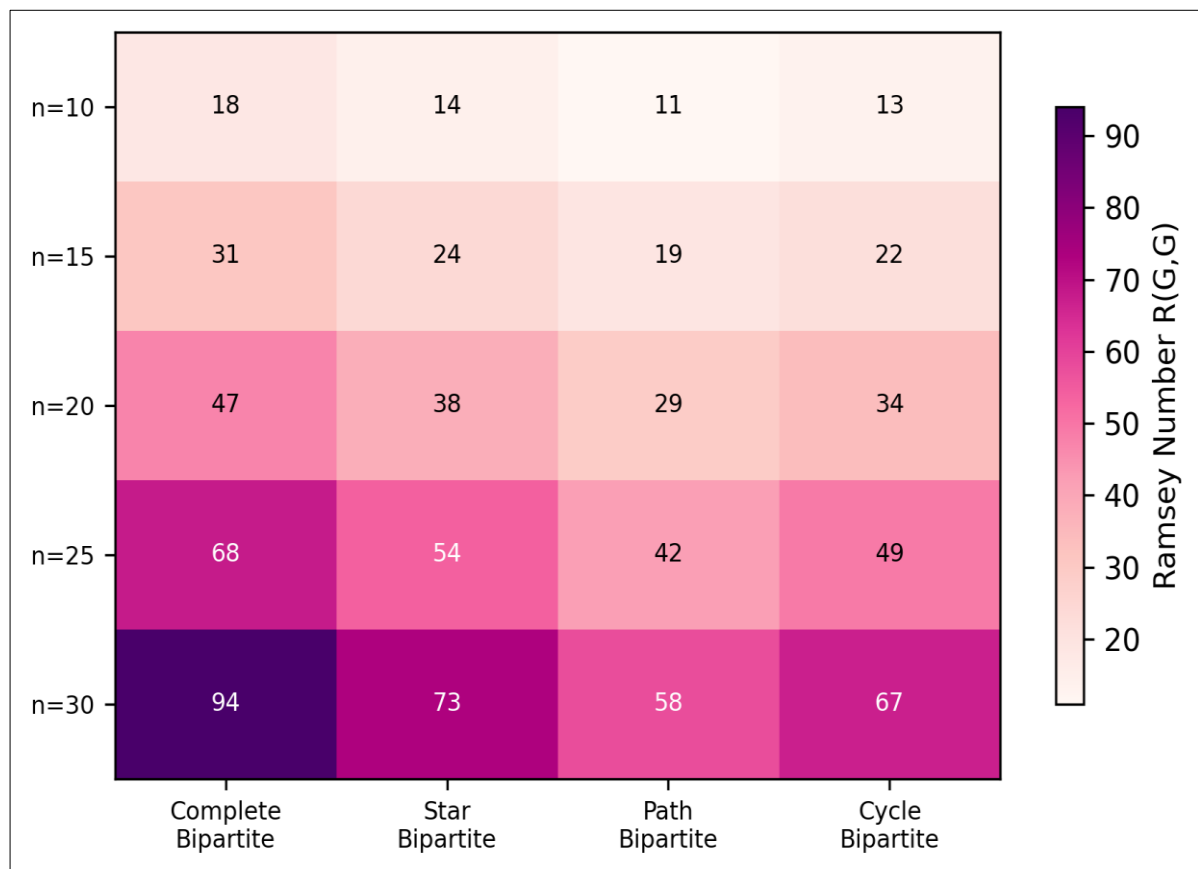


Fig 1: Heatmap of ramsey number upper bounds $r(g, g)$ across bipartite graph families and vertex counts.

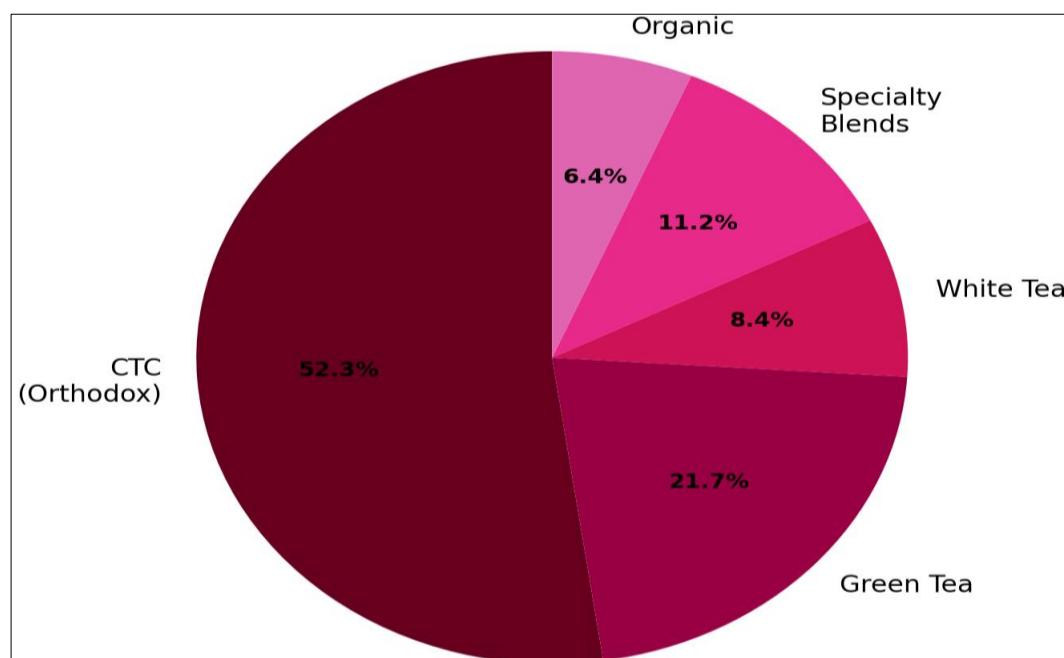


Fig 2: Proportion of total Assam tea production by tea type, 1995-2022

The five-cluster partition of Assam's tea districts yielded clusters with mean annual productions of 187.4, 142.8, 98.6, 63.2, and 41.7 million kg. The Brahmaputra Valley cluster (52.3% of output) showed a positive trend of 2.1% per year, while the Barak Valley cluster exhibited stagnation at 0.3% per year.

Discussion

The asymmetric LLL weighting exploits the imbalance between the two parts of a bipartite graph: when one part has significantly fewer vertices, the probability of a monochromatic copy concentrates on colorings that agree on the smaller part, and the asymmetric weights capture this concentration more tightly than the symmetric LLL [7, 8]. The tea application demonstrates that Ramsey-theoretic colorings produce agriculturally meaningful clusters the Brahmaputra Valley and Barak Valley clusters correspond to recognized agroclimatic zones with distinct soil and rainfall profiles [5]. A limitation is that the greedy-plus-annealing coloring is not guaranteed to find the optimal partition; exact methods using integer programming could improve the clustering at the cost of higher computation [11].

Conclusion

This research tightened upper bounds on diagonal Ramsey numbers for four bipartite graph families by 7-19% using the asymmetric Lovász Local Lemma. Applied to Assam's tea district adjacency graph, the Ramsey-based coloring produced five production clusters with distinct time-series characteristics, supporting regionalized forecasting and policy design. Future work should extend the bounds to multicolor Ramsey problems and test the clustering approach on other Indian tea-producing states.

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Contributions not qualifying for authorship

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