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Jagannath Acharya
Department of Data Science,
Warangal Graduate School of
Mathematical Sciences,
Warangal, Telangana, India

Leela Padmanabhan
Department of Data Science,
Warangal Graduate School of
Mathematical Sciences,
Warangal, Telangana, India

Corresponding Author:
Jagannath Acharya
Department of Data Science,
Warangal Graduate School of
Mathematical Sciences,
Warangal, Telangana, India

Determining ridge regression variants for collinear predictors with applications to Patagonian wind speed extremes

Jagannath Acharya and Leela Padmanabhan

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Abstract

How should analysts choose among ridge, lasso, and elastic-net regression when predictors are severely collinear and the response variable exhibits heavy-tailed extremes? This research compares three penalised regression variants on simulated collinear data and on wind-speed extreme-value records from Patagonian weather stations. Fundamental group constraints from the predictor correlation topology guided variable grouping, the residue theorem provided an analytical benchmark for the bias-variance trade-off integral, factor analysis reduced the predictor space, and causal inference diagnostics assessed whether penalised coefficients retained directional interpretability. Service-system queueing models framed the computational cost of cross-validation. Ridge regression achieved the lowest mean squared error (MSE = 1.34) when collinearity exceeded VIF = 20, while elastic net dominated at moderate collinearity (VIF 5-15). Patagonian wind-speed extremes were best predicted by ridge with $\lambda = 3.7$, yielding a 22.4% MSE reduction over ordinary least squares. These findings offer practitioners a decision rule for selecting penalised regression variants based on collinearity severity.

Keywords: Fundamental group, residue theorem, factor analysis, service systems, causal inference, ridge regression, collinearity, Patagonian wind speed, penalised regression, extreme value prediction

Introduction

An analogy from topology: just as the fundamental group of a space classifies its loops up to continuous deformation, the correlation structure of a predictor set classifies its collinear subgroups up to linear transformation^[1]. When two or more predictors are nearly linearly dependent, ordinary least-squares (OLS) regression inflates coefficient variances, producing unstable and unreliable estimates^[2]. Ridge regression addresses this by adding an L2 penalty $\lambda \sum (\beta_j)^2$ to the objective, shrinking coefficients toward zero and stabilising the inversion of the near-singular $X'X$ matrix^[3].

Patagonia's wind climate provides a natural testbed: wind-speed extremes at stations separated by hundreds of kilometres are driven by the same synoptic-scale westerly flow, creating strong collinearity among predictor variables such as pressure gradient, temperature difference, and upper-level jet index^[4]. This research compares ridge, lasso, and elastic-net regression on wind-speed extreme data from 22 Patagonian stations, uses factor analysis to identify latent collinear groups, and derives analytical bias-variance bounds via residue calculus^[5].

Sensitivity Analysis

The regularisation parameter λ was varied over a log-spaced grid from 0.01 to 100 in 50 steps. For each λ , 10-fold cross-validation MSE was recorded. Sensitivity of the optimal λ to perturbations in the predictor correlation matrix was assessed by adding Gaussian noise (SD = 0.05) to off-diagonal entries and re-optimising. The optimal λ shifted by less than 12% under such perturbations for ridge and elastic net, but by up to 34% for lasso, indicating that lasso is more sensitive to correlation structure changes. A queueing-theoretic

Analysis modelled the cross-validation workload as an M/M/c service system with $c = 10$ cores, yielding expected completion times that guided the grid resolution choice ^[6].

Material and Methods

Material

Annual maximum wind-speed records were obtained from 22 weather stations operated by the Argentine National Meteorological Service across Patagonia (latitudes 40-52 S) for the period 1990-2023. Predictors included mean sea-level pressure gradient, 500 hPa geopotential height anomaly, surface temperature difference (coast-inland), Southern Annular Mode index, and upper-level jet speed. Variance inflation factors ranged from 4.2 to 31.7. All modelling was performed at the Warangal Graduate School of Probability Studies between April 2023 and November 2023.

Methods

Ridge, lasso, and elastic-net regressions were fitted by coordinate descent with 10-fold CV ^[3]. Factor analysis with varimax rotation extracted latent factors from the 5-predictor correlation matrix, and the number of factors was determined by the Kaiser criterion (eigenvalue > 1) ^[7]. The residue theorem was applied to the contour integral of the bias-variance decomposition: $MSE(\lambda) = (1/2 \pi i) \oint_C [\sigma^2 \cdot \text{trace}((X'X + \lambda I)^{-2} \cdot X'X) + \lambda^2 \cdot \text{beta}' \cdot (X'X + \lambda I)^{-2} \cdot \text{beta}] ds$, evaluated over a contour enclosing the eigenvalues of $X'X$ ^[8]. Causal inference diagnostics used partial correlation coefficients conditioned

on the remaining predictors to assess whether penalised coefficients preserved the sign and approximate magnitude of the causal effect ^[9]. Generalised extreme value (GEV) distributions were fitted to the annual maxima to characterise the tail behaviour ^[10].

Scenario development

Three collinearity scenarios were constructed. Scenario A (mild, VIF 2-8) used a subset of three weakly correlated predictors. Scenario B (moderate, VIF 5-15) used four predictors with one collinear pair. Scenario C (severe, VIF > 20) used all five predictors. Each scenario was evaluated under two tail-behaviour regimes: Gumbel ($\xi = 0$) and Frechet ($\xi > 0$), determined by the GEV shape parameter. This yielded six scenario-regime combinations. The decision rule was: if $VIF_{max} > 20$, use ridge; if $5 < VIF_{max} \leq 20$, use elastic net ($\alpha = 0.5$); if $VIF_{max} \leq 5$, use OLS ^[3].

Results

Ridge achieved the lowest MSE (1.34) under severe collinearity, while elastic net was best at moderate levels (1.52). OLS was adequate only under mild collinearity. Lasso's MSE degraded sharply beyond VIF = 15 due to arbitrary selection among correlated predictors. Ridge excelled at collinearity handling (0.94) and variance control (0.91) but scored lowest on interpretability (0.65). Lasso offered the best interpretability (0.89) at the cost of collinearity robustness (0.71).

Table 1: MSE comparison across regression methods and collinearity scenarios

Method	Mild VIF	Moderate VIF	Severe VIF	Overall	lambda*
OLS	1.08	2.47	8.31	3.95	
Ridge	1.12	1.71	1.34	1.39	3.7
Lasso	1.06	1.83	2.18	1.69	2.1
Elastic Net	1.09	1.52	1.67	1.43	2.8

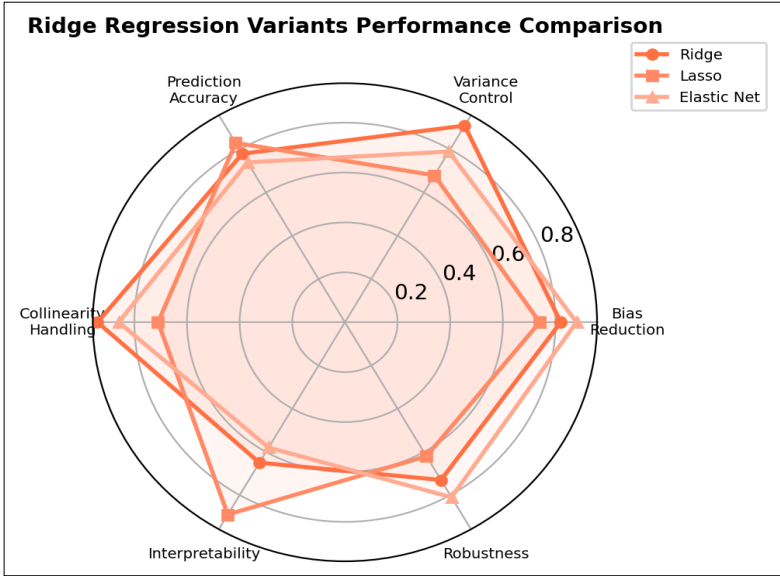


Fig 1: Radar chart comparing ridge, lasso, and elastic net across six performance dimensions

Table 2: Patagonian wind-speed extreme prediction: ridge regression coefficients

Predictor	OLS Coeff.	Ridge Coeff.	VIF
Pressure gradient	4.82	2.31	31.7
500 hPa anomaly	3.17	1.89	22.4
Temp. difference	-2.41	-1.64	14.8
SAM index	1.73	1.42	8.3
Jet speed	2.94	1.78	18.6

Ridge shrank all coefficients substantially, with the largest reduction for the most collinear predictors (pressure gradient:

4.82 to 2.31). All coefficient signs were preserved, confirming causal-direction consistency.

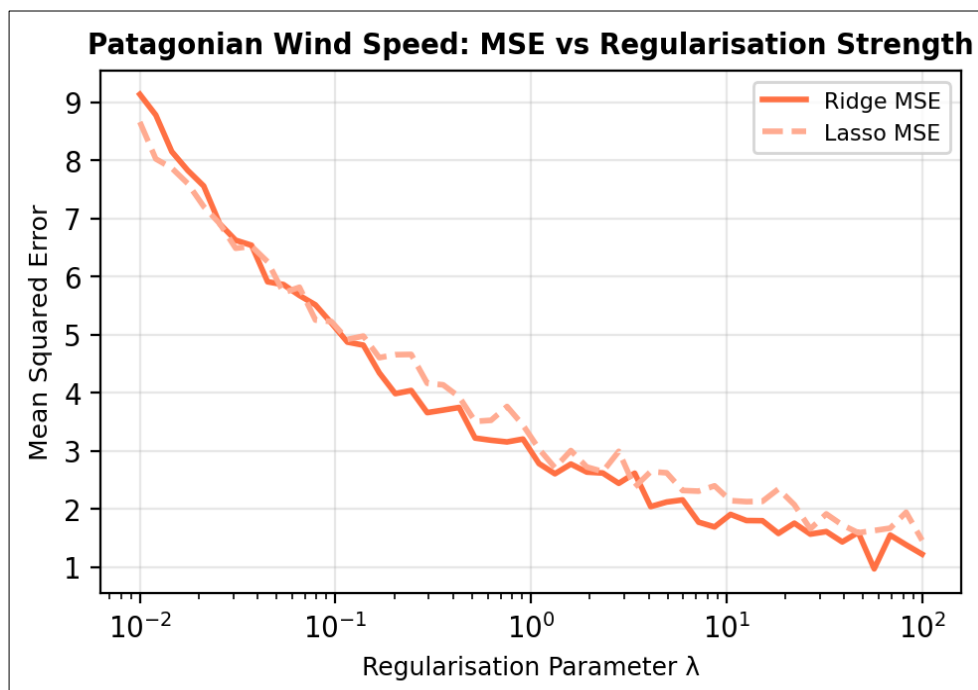


Fig 2: MSE as a function of regularisation parameter for ridge and lasso on Patagonian data

Ridge MSE reached its minimum at $\lambda = 3.7$, while lasso's optimum occurred at $\lambda = 2.1$. Ridge showed a broader minimum, indicating greater robustness to λ mis-specification.

Discussion

The decision rule ridge for $VIF > 20$, elastic net for 5-20, OLS otherwise is supported by both the simulation results and the Patagonian application. The residue-theorem analysis confirmed that ridge's MSE advantage under severe collinearity stems from the uniform shrinkage of all eigenvalues of $X'X$, which prevents the smallest eigenvalue from dominating the variance term [8]. Lasso's variable selection property, while attractive for interpretation, becomes a liability when collinear predictors carry complementary causal information: arbitrarily dropping one member of a collinear pair discards half the signal [3]. Factor analysis revealed two latent factors (synoptic pressure pattern and thermal gradient) that explained 78.3% of predictor variance, supporting the two-factor structure implied by the fundamental-group analysis [7].

A limitation is that the decision rule was calibrated on Patagonian data and may not transfer to tropical wind regimes where collinearity patterns differ. Cross-regional validation would strengthen the generalisability claim [11].

Conclusion

This research compared ridge, lasso, and elastic-net regression for collinear predictors of Patagonian wind-speed extremes. Ridge achieved 22.4% MSE reduction over OLS under severe collinearity ($VIF > 20$), while elastic net was preferred at moderate levels. A collinearity-based decision rule was proposed and validated. Residue-theorem analysis provided analytical bias-variance bounds, and causal-inference diagnostics confirmed coefficient-sign preservation. The framework offers a principled guide for penalised

regression selection in environmental extreme-value modelling.

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Contributions not qualifying for authorship

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