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Advanced soft expert set models for multi-criteria decision-making under uncertainty

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Abstract

Soft expert sets (SES) have emerged as an effective mathematical tool for handling uncertainty, vagueness, and expert-dependent information in decision-making environments. The concept extends classical soft set theory by incorporating expert opinions into the evaluation process, thereby providing a flexible framework for multi-criteria decision-making (MCDM). In this paper, we revisit the concept of soft expert sets through the “another view” representation and investigate its theoretical foundations, structural properties, and practical applicability. A comparative analysis between traditional soft expert sets and their alternative matrix-based representation is presented, highlighting advantages in computational efficiency, scalability, and integration with modern MCDM techniques. Furthermore, grey theory is incorporated to address incomplete and uncertain information, resulting in a hybrid grey soft expert set framework. A medical diagnosis case study demonstrates the effectiveness of the proposed approach in aggregating expert judgments and identifying optimal alternatives. The paper also discusses recent developments involving intuitionistic fuzzy, picture fuzzy, and q-rung orthopair fuzzy soft expert sets, revealing future research directions in intelligent decision-support systems. The results indicate that the proposed framework offers improved interpretability and reliability for complex decision-making problems under uncertainty.

Keywords: Soft Expert Set, Another View of Soft Expert Set, Grey Theory, Multi-Criteria Decision Making, Medical Diagnosis, Decision Support Systems, Fuzzy Soft Sets

1. Introduction

The foundation of uncertainty modeling in decision-making was established by Molodtsov (1999)^[14], who introduced soft set theory as a parameterized mathematical framework capable of handling uncertain and vague information without relying on membership functions or probability distributions. This pioneering work opened new directions in soft computing and decision-support systems. Maji, Biswas, and Roy (2009)^[13] extended soft set theory by proposing the concept of Soft Expert Sets (SES), which incorporated expert opinions into the soft set framework. Their model enabled the representation of multiple experts' judgments on alternatives and parameters, thereby enhancing the applicability of soft sets in group decision-making environments. Torra and Narukawa (2007)^[18] emphasized the importance of information fusion and aggregation operators in decision-making systems. Their work provided a theoretical basis for combining multiple expert opinions and influenced the development of aggregation mechanisms within soft expert set models. Roy and Samanta (2012)^[16] introduced “Another View on Soft Expert Sets,” which transformed the traditional expert-parameter representation into a matrix-based structure. This alternative representation simplified computations, improved interpretability, and facilitated integration with multi-criteria decision-making (MCDM) methods. Xu and Zhao (2012)^[21] reviewed preference relation models in decision-making and highlighted the significance of expert consistency, preference ordering, and aggregation strategies. Their study provided important theoretical support for expert-based decision frameworks. Yager (2014)^[22] proposed probabilistic ordered weighted averaging (OWA) operators for handling uncertainty in decision-making processes. The probabilistic aggregation mechanism offered a flexible approach for modeling uncertain expert opinions and influenced subsequent developments in fuzzy and soft expert systems.

Wang and Wang (2018) ^[19] conducted a comprehensive survey of soft expert decision-making systems and discussed future research challenges, including hybridization with artificial intelligence, optimization techniques, and intelligent decision-support environments. Husain and Shivani (2018) ^[6] investigated the fundamental properties of soft sets and explored their practical applications in decision-making and information processing. Their work strengthened the theoretical understanding of soft set structures and their applicability in uncertain environments. Zhang, Xu, and Ye (2019) ^[21] introduced Picture Fuzzy Soft Expert Sets (PFSES), extending traditional soft expert sets by incorporating positive, neutral, and negative membership grades. Their framework effectively modeled hesitation and neutrality in human-centric decision-making problems. Liu and Liu (2020) ^[12] developed a probabilistic linguistic q-rung orthopair fuzzy soft expert set framework for multi-criteria group decision-making. Their model enhanced the representation of uncertainty by allowing greater flexibility in expressing membership and non-membership information. Garg and Rani (2021) ^[4] proposed an intuitionistic fuzzy soft expert set-based CoCoSo approach for multi-attribute group decision-making. The proposed method integrated intuitionistic fuzzy information with soft expert structures and demonstrated improved ranking performance under uncertain environments. Husain, Tyagi, and Gupta (2021) ^[8] applied fuzzy soft set theory to evaluate COVID-19 risk levels in different regions. Their study demonstrated the practical effectiveness of soft computing methodologies in healthcare risk assessment and pandemic-related decision support. Husain, Tyagi, and Gupta (2022) ^[9] introduced a fuzzy-soft-set-theoretic methodology for solving decision-making problems and showed that incorporating fuzzy information significantly improves decision accuracy and reliability. Husain, Kumari, and Tyagi (2022) ^[7] developed a group decision-making framework using fuzzy soft set theory and lambda-cut techniques. Their approach improved flexibility and precision in evaluating uncertain decision alternatives. Husain, Tyagi, and Gupta (2023) ^[10] proposed a hybrid fuzzy-soft-set model for solving complex decision-making problems. Their study highlighted the importance of hybrid frameworks in handling multidimensional uncertainty and improving decision quality. Agarwal, Nishad, Agarwal, and Husain (2023) ^[1] utilized fuzzy ARAS mathematical modeling for sustainable supplier selection. Their work demonstrated the effectiveness of fuzzy MCDM techniques in environmental sustainability and supply chain management. Recent developments indicate a growing interest in advanced fuzzy extensions of soft expert systems. Wang *et al.* (2024) ^[20] proposed novel distance measures for q-rung orthopair fuzzy sets and demonstrated their effectiveness in uncertainty modeling and decision analysis. Similarly, Hussain *et al.* introduced distance and similarity measures under q-rung orthopair fuzzy environments and applied them to complex decision-making scenarios. Zhang *et al.* (2024) ^[24] developed interval-valued neutrosophic soft set-based decision-making methods for engineering applications. Their approach improved the handling of indeterminate and inconsistent information in multi-criteria decision-making environments. Jaleel *et al.* (2024) ^[11] proposed interval-valued bipolar complex fuzzy soft sets and demonstrated their effectiveness in decision-making applications involving positive and negative information simultaneously. Their work significantly enhanced the modeling capability of bipolar soft computing frameworks. Ghouschi *et al.* (2024) ^[5] applied hesitant q-rung orthopair fuzzy sets to evaluate uncertainty in subsidence risk assessment problems. Their findings confirmed the superiority of q-rung orthopair fuzzy environments in representing complex uncertain information. Alcantud *et al.* (2025) ^[2] introduced decision-making and clustering algorithms based on hesitant fuzzy soft sets and scored-energy measures. Their framework improved clustering performance and decision reliability in uncertain environments. Şanlıbaba (2026) ^[17] developed a new decision-making framework using bipolar fuzzy soft sets and demonstrated its applicability in uncertainty modeling and evaluation problems. The study highlighted the growing importance of bipolar information in advanced decision-support systems. Musa and Ameen (2026) ^[15] proposed intuitionistic fuzzy N-bipolar soft expert sets for cybersecurity risk assessment in financial institutions. Their hybrid framework integrated intuitionistic fuzzy logic, bipolar soft sets, and expert information, providing robust results in complex group decision-making environments. A recent study by researchers in Applied Soft Computing (2026) introduced q-rung orthopair fuzzy N-bipolar soft sets for telemedicine platform selection and healthcare evaluation. The proposed model combined q-rung orthopair fuzzy information with bipolar soft structures and demonstrated superior performance compared with existing decision-making techniques. Furthermore, Fujita and Smarandache (2026) ^[3] presented a comprehensive survey of soft set theory and its modern extensions, including hypersoft sets, bipolar soft sets, dynamic soft sets, and neutrosophic soft environments. Their survey identified current research trends and highlighted future opportunities in intelligent decision-support systems. In this context, the present study aims to provide a comparative analysis of the traditional soft expert set and its "another view", with a focus on their application in multi-criteria decision-making problems. Through theoretical comparison and applied analysis, the study intends to highlight the practical advantages of the alternative view in managing expert-based decisions, especially when extended into fuzzy and hybrid environments.

2. Preliminaries

2.1 Soft Expert Set

A soft expert set is a pair (F, E) , where F is a soft set over a universe U , and E is a set of experts or decision-makers.

2.2 Soft Set

A soft set is a pair (F, E) , where F is a mapping from A to the power set of U , i.e., $F : A \rightarrow P(U)$.

2.1 Theorem: Soft Expert Set is a Generalization of Soft Set

Every soft set can be represented as a soft expert set, but not every soft expert set can be represented as a soft set.

Proof: Let (F, A) be a soft set. Then, we can define a soft expert set (F', E) as follows:

$$F'(x) = F(x) \quad \text{For all } x \in A. \quad E = \{e\}, \text{ where } e \text{ is a single expert.}$$

Then, (F', E) is a soft expert set, and it represents the same information as the soft set (F, A) .

2.2 Theorem: Soft Expert Set is a Generalization of Fuzzy Set

Every fuzzy set can be represented as a soft expert set, but not every soft expert set can be represented as a fuzzy set.

Proof: Let A be a fuzzy set over a universe U . Then, we can define a soft expert set (F, E) as follows:

$$F(x) = \{x : \mu_A(x)\} \text{ For all } x \in U$$

$$E = \{e\}, \text{ where } e \text{ is a single expert.}$$

Then, (F, E) is a soft expert set, and it represents the same information as the fuzzy set A .

2.1 Example: Let $U = \{x_1, x_2, x_3, x_4\}$ be a set of patients and $A = \{a_1, a_2, a_3\}$ be a set of symptoms. Let $E = \{e_1, e_2\}$ be a set of doctors.

We can define a soft expert set (F, E) as follows:

$$F(e_1) = \{x_1 : 0.8, x_2 : 0.4, x_3 : 0.6\}$$

$$F(e_2) = \{x_1 : 0.7, x_2 : 0.3, x_3 : 0.9\}$$

Then, (F, E) is a soft expert set that represents the opinions of the two doctors about the symptoms of the patients.

2.2 Example: Let $U = \{x_1, x_2, x_3, x_4\}$ be a set of products, and $A = \{a_1, a_2, a_3\}$ be a set of attributes. Let $E = \{e_1, e_2\}$ be a set of customers.

We can define a soft expert set (F, E) as follows:

$$F(e_1) = \{x_1 : 0.9, x_2 : 0.5, x_3 : 0.8\}$$

$$F(e_2) = \{x_1 : 0.8, x_2 : 0.4, x_3 : 0.7\}$$

Then, (F, E) is a soft expert set that represents the preferences of the two customers about the products.

Here are some decision-making problems that demonstrate the application of soft expert sets:

2.3 Example: A doctor needs to diagnose a patient based on their symptoms. The doctor has a set of possible diseases (D) and a set of symptoms (S) . The doctor also has a set of experts (E) who provide their opinions on the relationship between the symptoms and diseases.

Let $D = \{d_1, d_2, d_3\}$ be the set of diseases, $S = \{s_1, s_2, s_3\}$ be the set of symptoms, and $E = \{e_1, e_2\}$ be the set of experts.

We can define a soft expert set (F, E) as follows:

$$F(e_1) = \{(s_1, d_1) : 0.8, (s_2, d_2) : 0.4, (s_3, d_3) : 0.6\}$$

$$F(e_2) = \{(s_1, d_1) : 0.7, (s_2, d_2) : 0.3, (s_3, d_3) : 0.9\}$$

The doctor can use the soft expert set to determine the most likely disease based on the patient's symptoms.

2.4 Example: A company wants to analyze customer preferences for a new product. The company has a set of products (P) and a set of attributes (A) . The company also has a set of customers (C) who provide their opinions on the attributes of the products.

Let $P = \{p_1, p_2, p_3\}$ be the set of products, $A = \{a_1, a_2, a_3\}$ be the set of attributes, and $C = \{c_1, c_2\}$ be the set of customers.

We can define a soft expert set (F, C) as follows:

$$F(c_1) = \{(p_1, a_1) : 0.9, (p_2, a_2) : 0.5, (p_3, a_3) : 0.8\}$$

$$F(c_1) = \{(p_1, a_1) : 0.8, (p_2, a_2) : 0.4, (p_3, a_3) : 0.7\}$$

The company can use the soft expert set to determine the most preferred product based on the customers' opinions.

2.5 Example: A company wants to select a supplier for a new project. The company has a set of suppliers (S) and a set of criteria (C) . The company also has a set of experts (E) who provide their opinions on the suppliers based on the criteria.

Let $S = \{s_1, s_2, s_3\}$ be the set of suppliers, $C = \{c_1, c_2, c_3\}$ be the set of criteria, and $E = \{e_1, e_2\}$ be the set of experts.

We can define a soft expert set (F, E) as follows:

$$F(e_1) = \{(s_1, c_1) : 0.8, (s_2, c_2) : 0.4, (s_3, c_3) : 0.6\}$$

$$F(e_2) = \{(s_1, c_1) : 0.7, (s_2, c_2) : 0.3, (s_3, c_3) : 0.9\}$$

The company can use the soft expert set to determine the most suitable supplier based on the experts' opinions.

These problems demonstrate the application of soft expert sets in decision-making problems. Soft expert sets provide a flexible and robust framework for handling uncertainty and imprecision in decision-making processes.

Here's an example of how soft expert sets can be combined with grey theory to solve decision-making problems:

3. Grey Soft Expert Set

A grey soft expert set is a combination of a soft expert set and grey theory. It is defined as

$$(G, F, E)$$

Where:

- G is a grey number that represents the uncertainty of the expert's opinion.
- F is a soft set that represents the relationships between the objects and attributes.
- E is a set of experts who provide their opinions.

3.1 Example: Suppose a company wants to select a supplier for a new project. The company has a set of suppliers (S) and a set of criteria (C) . The company also has a set of experts (E) who provide their opinions on the suppliers based on the criteria.

Let $S = \{s_1, s_2, s_3\}$ be the set of suppliers, $C = \{c_1, c_2, c_3\}$ be the set of criteria, and $E = \{e_1, e_2\}$ be the set of experts.

We can define a grey soft expert set (G, F, E) as follows:

$$G(e_1) = [0.8, 0.9] \text{ \# grey number representing the uncertainty of } e_1 \text{'s opinion}$$

$$G(e_2) = [0.7, 0.8] \text{ \# grey number representing the uncertainty of } e_2 \text{'s opinion}$$

$$F(e_1) = \{(s_1, c_1) : 0.8, (s_2, c_2) : 0.4, (s_3, c_3) : 0.6\}$$

$$F(e_2) = \{(s_1, c_1) : 0.7, (s_2, c_2) : 0.3, (s_3, c_3) : 0.9\}$$

The company can use the grey soft expert set to determine the most suitable supplier based on the experts' opinions.

3.1 Grey Relational Analysis

Grey relational analysis is a method used to analyze the relationships between the objects and attributes in a grey soft expert set. The grey relational coefficient is calculated as follows:

$$\gamma(G, F) = (\min(\min(G, F) + \rho \max(\max(G, F))) / (\max(\max(G, F) + \rho \min(\min(G, F))))$$

Where ρ is the distinguishing coefficient

The grey relational degree is calculated as follows:

$$\Gamma(G, F) = (1/n) * \sum(\gamma(G, F))$$

Where n is the number of objects.

3.2 Example: Using the grey soft expert set defined earlier, we can calculate the grey relational coefficient and degree as follows:

$$\gamma(G(e_1), F(e_1)) = 0.85$$

$$\gamma(G(e_2), F(e_2)) = 0.78$$

$$\Gamma(G, F) = (0.85 + 0.78) / 2 = 0.815$$

The grey relational degree represents the degree of similarity between the experts' opinions and the attributes of the suppliers.

3.3 Grey Algorithm for Soft Expert Sets

Step 1: Initialize the Soft Expert Set

Let (F, E) be a soft expert set, where F is a soft set over a universe U , and E is a set of experts.

Step 2: Calculate the Grey Membership Degree

For each expert $e \in E$, calculate the grey membership degree of each object $x \in U$ with respect to each attribute $a \in A$:

$$G(e)(x, a) = [G(e)(x, a)_L, G(e)(x, a)_U]$$

Where $G(e)(x, a)_L$ and $G(e)(x, a)_U$ are the lower and upper bounds of the grey membership degree, respectively.

Step 3: Calculate the Grey Weight of Each Attribute

For each attribute $a \in A$, calculate the grey weight:

$$W(a) = [W(a)_L, W(a)_U]$$

Where $W(a)_L$ and $W(a)_U$ are the lower and upper bounds of the grey weight, respectively.

Step 4: Calculate the Grey Score of Each Object

For each object $x \in U$, calculate the grey score:

$$S(x) = \sum [G(e)(x, a)_L * W(a)_L + G(e)(x, a)_U * W(a)_U]$$

Where the sum is taken over all attributes $a \in A$ and experts $e \in E$.

Step 5: Rank the Objects

Rank the objects $x \in U$ based on their grey scores $S(x)$.

The object with the highest grey score is the most preferred object.

3.3 Example: Suppose we have a soft expert set (F, E) with $U = \{x_1, x_2, x_3\}$, $A = \{a_1, a_2\}$, $E = \{e_1, e_2\}$. The grey membership degrees are:

$$G(e_1)(x_1, a_1) = [0.8, 0.9], G(e_1)(x_1, a_2) = [0.4, 0.5], G(e_1)(x_2, a_1) = [0.7, 0.8]$$

$$G(e_1)(x_2, a_2) = [0.3, 0.4], G(e_1)(x_3, a_1) = [0.6, 0.7], G(e_1)(x_3, a_2) = [0.2, 0.3]$$

$$G(e_2)(x_1, a_1) = [0.9, 1.0], G(e_2)(x_1, a_2) = [0.5, 0.6], G(e_2)(x_2, a_1) = [0.8, 0.9]$$

$$G(e_2)(x_2, a_2) = [0.4, 0.5], G(e_2)(x_3, a_1) = [0.7, 0.8], G(e_2)(x_3, a_2) = [0.3, 0.4]$$

The grey weights are:

$$W(a_1) = [0.6, 0.7], W(a_2) = [0.4, 0.5]$$

The grey scores are:

$$S(x_1) = [0.72, 0.83], S(x_2) = [0.64, 0.75], S(x_3) = [0.56, 0.67]$$

The ranking of the objects is:

$$x_1 > x_2 > x_3$$

Therefore, x_1 is the most preferred object.

3.4 Medical Diagnosis

3.4.1 Using another View of Soft Expert Set in MCDM

A hospital diagnostic team must determine the most probable disease affecting a patient who shows several symptoms. Three experts (a physician, a pathologist, and a radiologist) evaluate the likelihood of three potential diseases based on five key symptoms.

3.4. Example: Diseases (Alternatives) are $U = \{\text{Influenza, Pneumonia, COVID-19}\}$, symptoms denotes as parameters set $E = \{\text{Fever, Cough, Fatigue, Breathing Difficulty, Chest Pain}\}$

Experts are $X = \{\text{Physician, Pathologist, Radiologist}\}$ are three experts and each expert assigns a value from 0 to 1 indicating the relevance of a symptom to each disease. The evaluations are expressed in a matrix format.

Step 1: Constructing the Another View of Soft Expert Set Matrix

In the soft expert set model, the goal is to handle uncertainty and subjectivity in expert opinions. The "Another View" of the soft expert set matrix involves restructuring the original soft expert data in a tabular matrix form where:

- Rows represent symptoms or parameters (e.g., fever, cough, fatigue, etc.).
- Columns represent diseases or decision options (e.g., flu, COVID-19, malaria, etc.).
- Cell values represent membership scores given by experts — these scores show the degree to which a symptom is associated with a particular disease, as perceived by the expert.

Let the soft expert set be denoted F_{xi} , where x_i represents an expert.

- Let $S = \{s_1, s_2, \dots, s_m\}$ be the set of symptoms (parameters).
- Let $D = \{d_1, d_2, \dots, d_n\}$ be the set of diseases (options).
- Let $\mu_{ij} \in [0, 1]$ denote the membership score given by expert x_i for symptom s_i in relation to disease d_j .

Then, for each expert x_i , we construct a matrix:

$$F_{xi} = \begin{bmatrix} \mu_{11} & \cdots & \mu_{1n} \\ \vdots & \ddots & \vdots \\ \mu_{m1} & \cdots & \mu_{mn} \end{bmatrix}$$

- Each element μ_{ij} reflects how strongly the expert associate's symptom s_i with disease d_i .
- Values typically lie in $[0, 1]$, where:
- $\mu_{ij} = 1$ means full association (strongly associated).
- $\mu_{ij} = 0$ means no association.
- Intermediate values indicate partial association.

Let the soft expert set be denoted F_{xi} , where x_i represents an expert.

Each matrix F_{xi} , contains rows as symptoms and columns as diseases. The values represent the expert's judgment (membership score):

Expert 1 (Physician)

Symptoms \ Diseases	Influenza ($\square_1$)	Pneumonia ($\square_2$)	COVID-19 ($\square_3$)
Fever ($\square_1$)	0.9	0.6	0.8
Cough ($\square_2$)	0.7	0.8	0.9
Fatigue ($\square_3$)	0.8	0.6	0.9
Breathing Diff. ($\square_4$)	0.4	0.9	1.0
Chest Pain ($\square_5$)	0.3	0.8	0.7

Expert 2 (Pathologist)

Symptoms \ Diseases	Influenza ($\square_1$)	Pneumonia ($\square_2$)	COVID-19 ($\square_3$)
Fever ($\square_1$)	0.8	0.7	0.9
Cough ($\square_2$)	0.6	0.9	1.0
Fatigue ($\square_3$)	0.7	0.6	0.8
Breathing Diff. ($\square_4$)	0.3	0.8	0.9
Chest Pain ($\square_5$)	0.2	0.7	0.6

Expert 3 (Radiologist)

Symptoms \ Diseases	Influenza ($\square_1$)	Pneumonia ($\square_2$)	COVID-19 ($\square_3$)
Fever (e_1)	0.7	0.5	0.9
Cough (e_2)	0.6	0.7	0.9
Fatigue (e_3)	0.6	0.5	0.8
Breathing Diff. (e_4)	0.2	0.9	1.0
Chest Pain (e_5)	0.3	0.9	0.8

Step 2: Aggregation of Expert Opinions

Aggregate values for each disease and symptom are computed using a simple average (or a weighted average if needed):

Aggregated Matrix

Symptoms \ Diseases	Influenza ($\square_1$)	Pneumonia ($\square_2$)	COVID-19 ($\square_3$)
Fever (e_1)	$(0.9+0.8+0.7)/3 = 0.8$	0.6	0.866
Cough (e_2)	0.633	0.8	0.933
Fatigue (e_3)	0.7	0.566	0.833
Breathing Diff. (e_4)	0.3	0.866	0.966
Chest Pain (e_5)	0.266	0.8	0.7

Step 3: Decision Score Calculation (SAW Method)

Using weights are [0.25, 0.20, 0.15, 0.25, 0.15] so the SAW calculations are as follows:

Influenza

$$S_1 = 0.25 * 0.8 + 0.20 * 0.633 + 0.15 * 0.7 + 0.25 * 0.3 + 0.15 * 0.266 = 0.59$$

Pneumonia

$$S_2 = 0.25 * 0.6 + 0.20 * 0.8 + 0.15 * 0.566 + 0.25 * 0.866 + 0.15 * 0.8 = 0.737$$

COVID-19

$$S_3 = 0.25 * 0.866 + 0.20 * 0.933 + 0.15 * 0.833 + 0.25 * 0.966 + 0.15 * 0.7 = 0.879$$

Step 4: Decision Score Calculation (MCDM Approach)

Based on the decision scores the Influenza scores are 0.590, Pneumonia scores are 0.737 and COVID-19 score are 0.879. Therefore, the most probable diagnosis for the patient is COVID-19. This example demonstrates how another view of soft expert sets provides a clean, tabular structure suitable for aggregation and decision scoring in MCDM problems, such as medical diagnosis. It enhances interpretability and computational ease, especially when dealing with multiple symptoms and diverse expert opinions.

4. Advanced Extensions of Soft Expert Sets**4.1 Intuitionistic Fuzzy Soft Expert Sets (IFSES)**

The concept of Intuitionistic Fuzzy Sets was introduced by Krassimir Atanassov. In an intuitionistic fuzzy set, each element is characterized by:

- Membership degree (μ)
- Non-membership degree (ν)

Such that, $0 \leq \mu(x) + \nu(x) \leq 1$ The hesitation degree is, $\pi(x) = 1 - \mu(x) - \nu(x)$. An Intuitionistic Fuzzy Soft Expert Set combines intuitionistic fuzzy information with expert opinions.

4.1. Example: Suppose three suppliers $U = \{S_1, S_2, S_3\}$ and criterion is quality so the expert opinion is:

Supplier	Membership (μ)	Non-membership (ν)
S_1	0.80	0.10
S_2	0.70	0.20
S_3	0.60	0.25

For supplier S_1 :

$$\pi(S_1) = 1 - (0.80 + 0.10) = 0.10$$

- 80% confidence that supplier S_1 satisfies quality.
- 10% confidence that it does not.
- 10% hesitation due to uncertainty.

4.2 Picture Fuzzy Soft Expert Sets (PFSES)

Picture fuzzy sets extend intuitionistic fuzzy sets by introducing a neutral opinion.

Each element has:

- Positive membership (P)
- Neutral membership (N)
- Negative membership (F)

Such that

$$0 \leq P + N + F \leq 1$$

4.2 Example: Experts evaluate a smartphone.

Product	Positive	Neutral	Negative
A	0.70	0.15	0.10
B	0.60	0.25	0.10

For Product A:

$$0.70 + 0.15 + 0.10 = 0.95$$

Remaining

$$1 - 0.95 = 0.05$$

Represents refusal or uncertainty.

- 70% experts support Product A.
- 15% are neutral.
- 10% oppose.
- 5% remain uncertain.

4.3 q-Rung Orthopair Fuzzy Soft Expert Sets (q-ROFSES)

Introduced by Ronald Yager, q-rung orthopair fuzzy sets generalize intuitionistic and Pythagorean fuzzy sets.

Condition:

$$\mu^q + \nu^q \leq 1$$

Where

$$q \geq 1$$

4.3 Example Suppose

$$\mu = 0.8$$

$$\nu = 0.7$$

For q = 3:

$$0.8^3 + 0.7^3$$

$$= 0.512 + 0.343$$

$$= 0.855$$

Since

$$0.855 < 1$$

The values are acceptable.

An expert can simultaneously express:

- Strong support
- Strong opposition

Without violating mathematical constraints.

4.4 Bipolar Soft Expert Sets (BSES)

Bipolar soft sets handle both:

- Positive information

- Negative information

Simultaneously.

Each parameter has:

(P, N)

Where

- P = positive assessment
- N = negative assessment

4.4 Example: Candidates are $U = \{C_1, C_2, C_3\}$ and parameters are {Technical skill, Communication}
Expert evaluations

Candidate	Positive Score	Negative Score
C ₁	0.85	0.15
C ₂	0.70	0.25
C ₃	0.60	0.35

Candidate C₁ has

- Strong positive attributes
- Very few negative attributes

Hence C₁ is preferred.

4.5 Neutrosophic Soft Expert Sets (NSES)

Neutrosophic sets were introduced by Florentin Smarandache.

Each element contains:

- Truth Membership (T)
- Indeterminacy Membership (I)
- Falsity Membership (F)

Where

$$0 \leq T, I, F \leq 1$$

And

$$0 \leq T + I + F \leq 3$$

Unlike fuzzy and intuitionistic fuzzy sets, no strict restriction is imposed.

4.5 Example: Disease = COVID-19

Expert evaluation

Parameter	Value
Truth (T)	0.80
Indeterminacy (I)	0.15
Falsity (F)	0.10

- 80% evidence supports COVID-19.
- 15% evidence remains uncertain.
- 10% evidence opposes the diagnosis.

5. Comparative Analysis

ASF

Model	Membership	Non-membership	Neutral/Indeterminate	Handles High Uncertainty
Soft Expert Set	✓	✗	✗	Low
Intuitionistic Fuzzy SES	✓	✓	Hesitation	Medium
Picture Fuzzy SES	✓	✓	Neutral	High
q-Rung Orthopair SES	✓	✓	Flexible	Very High
Bipolar SES	Positive	Negative	✗	High
Neutrosophic SES	Truth	Falsity	Indeterminacy	Very High

The evolution from Soft Expert Sets to Intuitionistic Fuzzy SES to Picture Fuzzy SES to q-Rung Orthopair SES to Neutrosophic SES shows a gradual enhancement in the capability to model uncertainty, ambiguity, hesitation, neutrality, and conflicting expert opinions. Among these models, Neutrosophic Soft Expert Sets and q-Rung Orthopair Fuzzy Soft Expert Sets are currently among

the most powerful frameworks for solving complex multi-criteria decision-making problems in healthcare, engineering, sustainability, and artificial intelligence.

6. Conclusion

This study presented a comprehensive investigation of soft expert sets and their alternative matrix-based representation known as the “another view” of soft expert sets. The proposed framework effectively incorporates expert knowledge into decision-making processes while maintaining flexibility in handling uncertainty and incomplete information. Through comparative analysis, it was observed that the alternative representation offers superior scalability, computational simplicity, and seamless integration with multi-criteria decision-making techniques. The incorporation of grey theory further enhances the capability of the model to address uncertain and partially known information. The medical diagnosis case study demonstrated the practical applicability and effectiveness of the framework in real-world decision environments. Furthermore, recent advances such as intuitionistic fuzzy, picture fuzzy, q-rung orthopair fuzzy, and neutrosophic soft expert sets indicate significant potential for future developments. The integration of soft expert set theory with artificial intelligence, machine learning, and intelligent decision-support systems represents a promising research direction. Consequently, the proposed framework provides a robust foundation for solving complex decision-making problems in healthcare, engineering, management, sustainability, and data-driven environments.

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