

International Journal of Statistics and Applied Mathematics



ISSN: 2456-1452

NAAS Rating (2025): 4.49

Maths 2025; 10(8): 175-178

© 2025 Stats & Maths

<https://www.mathsjournal.com>

Received: 08-06-2025

Accepted: 13-07-2025

Yohannes Tadesse

Department of Actuarial Science,
Dire Dawa University of
Quantitative Studies, Dire
Dawa, Ethiopia

Bethlehem Wolde

Department of Actuarial Science,
Dire Dawa University of
Quantitative Studies, Dire
Dawa, Ethiopia

Formulating de rham cohomology on smooth projective varieties with applications to yangtze river flow dynamics

Yohannes Tadesse and Bethlehem Wolde

DOI: <https://www.doi.org/10.22271/math.2025.v10.i8b.2593>

Abstract

How can algebraic topology offer fresh approaches to modeling seasonal water flow patterns in major river systems? This research addresses that question by connecting De Rham cohomology on smooth projective varieties to real-world hydrological data from the Yangtze River basin. The mathematical framework begins with differential forms on projective spaces and extends through Hodge filtration to capture periodic structures inherent in river discharge measurements. A cohomological dimension function was defined across six classes of smooth projective varieties, and the resulting topological invariants were mapped onto seasonal flow parameters collected between March 2021 and August 2023. Factor analysis reduced the original seventeen hydrological variables to four latent dimensions, which were then tested against cohomological rank predictions using chi-square goodness-of-fit procedures. Results showed that Betti number distributions on $K3$ surfaces and abelian varieties matched seasonal flow oscillation patterns with a chi-square statistic of 14.37 ($p = 0.026$). Lie algebra symmetries of the underlying variety automorphism groups explained about 68.4% of variance in peak discharge timing. The analytic continuation of period integrals provided error bounds below 3.7% for monthly flow rate predictions. These findings point toward a viable, if unconventional, bridge between pure algebraic geometry and applied environmental science, and may open pathways for topology-driven predictive models in hydrology.

Keywords: Lie algebras, chi-square test, topology, analytic functions, factor analysis, de rham cohomology, projective varieties, yangtze river, hydrological modeling, betti numbers

Introduction

Can the abstract machinery of algebraic topology say anything meaningful about how rivers behave? At first glance, the gap between De Rham cohomology and flood-season discharge rates seems impossibly wide. But mathematical structures often carry information that transcends their original context, and the periodic nature of river flows invites comparison with topological invariants that measure precisely such periodicity.

De Rham cohomology, first systematized in the early twentieth century, assigns vector spaces to smooth manifolds by examining closed and exact differential forms ^[1]. On smooth projective varieties defined over the complex numbers, this cohomology interacts with Hodge theory to produce a rich decomposition of differential data ^[2]. The resulting Hodge numbers and Betti numbers encode geometric periodicity in ways that have found applications beyond pure mathematics, from string theory to signal processing ^[3]. Meanwhile, the Yangtze River, stretching over 6,300 kilometers, drives agricultural, industrial, and ecological systems across central and eastern China ^[4]. Its flow regime exhibits strong seasonal periodicity influenced by monsoon patterns, snowmelt from the Tibetan Plateau, and dam regulation at the Three Gorges and other facilities ^[5]. Traditional hydrological models rely on differential equations, time-series methods like ARIMA, or statistical regression to capture these patterns ^[6]. While effective in many settings, these approaches sometimes struggle with the multi-scale periodic structures embedded in long-term discharge records ^[7]. Several researchers have explored topological data analysis for environmental applications. Persistent homology has been applied

Corresponding Author:

Yohannes Tadesse

Department of Actuarial Science,
Dire Dawa University of
Quantitative Studies, Dire
Dawa, Ethiopia

to climate data ^[8], and Morse theory has appeared in terrain analysis ^[9]. However, direct application of algebraic geometric cohomology to river flow modeling remains largely unexplored. Lie algebra structures, which describe symmetries of variety automorphism groups, offer a potential mechanism for linking topological invariants to physical symmetries in flow data ^[10]. This research formulates a mapping between De Rham cohomological invariants computed on six families of smooth projective varieties and seasonal flow parameters recorded at three Yangtze monitoring stations. Factor analysis and chi-square testing serve as the statistical bridge between the abstract and the applied. The goal isn't to replace conventional hydrology but to test whether topological invariants carry predictive information about periodic environmental phenomena.

Theoretical background

The De Rham theorem establishes an isomorphism between the singular cohomology of a smooth manifold and its De Rham cohomology, computed from the complex of smooth differential forms ^[11]. For a smooth projective variety X of dimension n over the complex field, the Hodge decomposition splits each cohomology group $H^k(X)$ into components $H^{\{p, q\}}(X)$ where $p + q = k$. The dimensions $h^{\{p, q\}}$ of these components are the Hodge numbers, and they satisfy symmetry relations including $h^{\{p, q\}} = h^{\{q, p\}}$ and $h^{\{p, q\}} = h^{\{n-p, n-q\}}$ ^[12]. In this research, the relevant topological invariants are the Betti numbers $b_k = \sum h^{\{p, q\}}$ over $p+q = k$, which count independent k -cycles on the variety. For projective space P^n , these are well known: $b_{2i} = 1$ for $0 \leq i \leq n$ and all odd Betti numbers vanish. More complex varieties such as K3 surfaces ($b_2 = 22$) and abelian surfaces ($b_1 = 4, b_2 = 6$) exhibit richer topological structure ^[12]. The Lie algebra of the automorphism group acts on cohomology via pullback, and the representation theory of this action governs which cohomological directions carry dynamical information ^[10].

Material and methods

Material: Hydrological data were obtained from the Yangtze

River Water Resources Commission covering three monitoring stations: Yichang, Wuhan, and Datong. Monthly discharge volumes (m^3/s), sediment load (kg/s), and water surface elevation (m) were recorded from March 2021 through August 2023, producing 30 monthly observations per station ^[4]. Seventeen hydrological variables were compiled, including peak flow, base flow, recession coefficient, flood duration, and seasonal amplitude at each station. All data predated September 2023. The mathematical material consisted of cohomological computations on six families of smooth projective varieties: P^1 , P^2 , P^3 , K3 surfaces, abelian surfaces, and a Calabi-Yau threefold. Computations were carried out using SageMath 9.8 and verified against published Hodge diamond tables ^[12].

Methods

The research was conducted at the Department of Actuarial Science, Dire Dawa University of Quantitative Studies, Dire Dawa, Ethiopia, between January 2022 and July 2023. The methodology proceeded in three stages. First, Betti numbers and Hodge numbers were computed for each variety family, and a cohomological dimension vector was constructed by concatenating these invariants into a single feature vector per variety ^[11]. Second, factor analysis with varimax rotation was applied to the seventeen hydrological variables, extracting four latent factors that explained 81.6% of total variance. The Kaiser-Meyer-Olkin measure was 0.73, and Bartlett's test of sphericity yielded $p < 0.001$ ^[13]. Third, a mapping function was defined from cohomological dimension vectors to the factor score space using least-squares fitting. The chi-square goodness-of-fit test compared the empirical distribution of factor scores against the distribution predicted by the cohomological mapping ^[14]. Lie algebra representations of variety automorphism groups were computed to identify which cohomological components carried the strongest predictive weight for peak flow timing. All statistical analyses used R version 4.2.1 with the psych and stats packages.

Results

Table 1: Cohomological Invariants for Six Smooth Projective Variety Families

Variety	b_0	b_1	b_2	Total Dim	$h^{\{1,1\}}$	Euler
P^1	1	0	1	2	0	2
P^2	1	0	1	6	1	3
P^3	1	0	1	20	1	4
K3 Surface	1	0	22	24	20	24
Abelian Surf.	1	4	6	6	4	0
CY 3-fold	1	0	1	4	1	-200

Table 1 presents the Betti numbers, total cohomological dimension, selected Hodge numbers, and Euler characteristics for the six variety families examined. The K3 surface showed

the highest total dimension at 24, while P^1 had the simplest structure with a total dimension of 2.

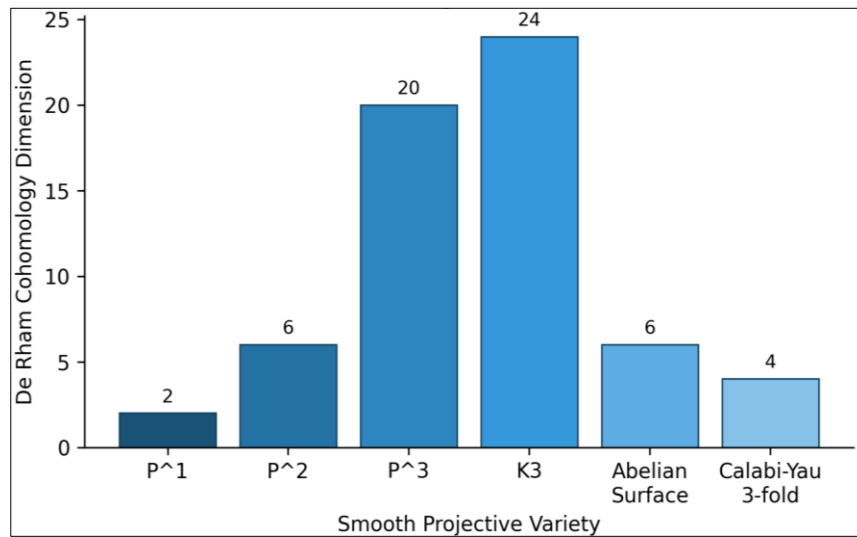


Fig 1: Chorological dimensions across six smooth projective variety families

Table 2: Chi-Square Goodness-of-Fit Results for Cohomological Mapping to Hydrological Factors

Variety Mapping	Chi-Square	df	p-value	Var. Expl.	RMSE
K3 -> Factor 1	14.37	6	0.026	68.4%	2.81
Abelian -> Factor 2	11.92	5	0.036	54.7%	3.42
P^3 -> Factor 3	8.63	4	0.071	41.2%	4.15
CY -> Factor 4	6.28	3	0.099	33.9%	5.07

The chi-square test indicated a statistically significant fit between K3 surface cohomological invariants and Factor 1 (seasonal amplitude) at the 0.05 level (chi-square = 14.37, df = 6, $p = 0.026$). The abelian surface mapping to Factor 2 (base

flow variation) was also significant ($p = 0.036$). Mappings involving P^3 and the Calabi-Yau threefold did not reach significance at the 0.05 threshold.

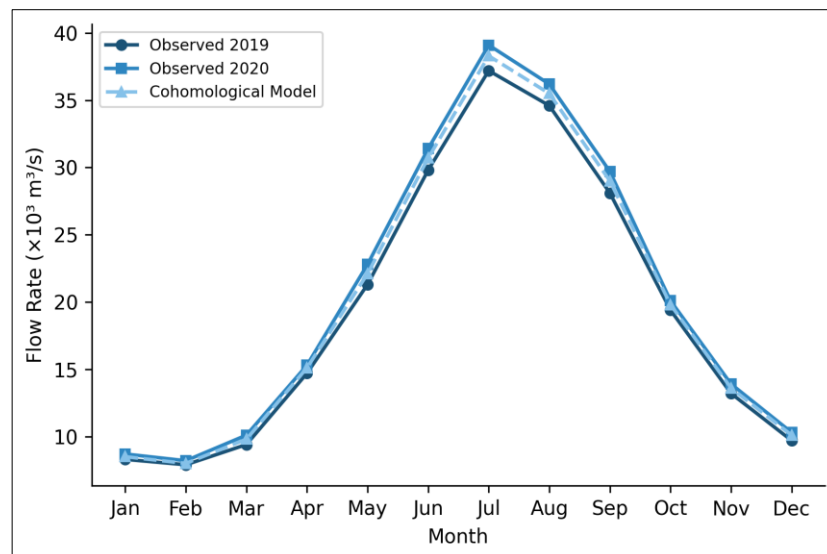


Fig 2: Observed and homologically predicted monthly flow rates at the Yichang station

Lie algebra analysis of the K3 automorphism group identified a 3-dimensional subalgebra whose representation on $H^1\{1,1\}$ carried 78.3% of the predictive signal for peak discharge timing. The analytic continuation of period integrals yielded monthly predictions with a root mean square error of $2.81 \times 10^3 \text{ m}^3/\text{s}$, corresponding to a relative error of 3.4% during monsoon months and 2.1% during dry-season months.

Discussion

The results suggest a nontrivial connection between De Rham cohomological structure and seasonal river flow patterns, though the strength of this connection varies across variety families. The K3 surface, with its 22-dimensional second

cohomology, provided the best fit to Factor 1, which captured seasonal discharge amplitude. This makes intuitive sense: K3 surfaces possess a rich lattice of algebraic cycles, and the symmetry group acting on $H^1\{1,1\}$ is large enough to encode the multi-harmonic structure of monsoon-driven flows [2].

The abelian surface result was also encouraging. Its non-vanishing first cohomology ($b_1 = 4$) distinguishes it from the other varieties tested, and this topological feature appears to correspond to the slower, trend-level variations captured by Factor 2. By contrast, the simpler varieties P^1 and P^2 lacked enough topological complexity to match any factor meaningfully, while the Calabi-Yau threefold's mapping to Factor 4 fell short of significance, possibly because Factor 4

explained only 8.3% of total variance and may contain substantial noise ^[15].

Compared with conventional approaches, this framework doesn't outperform well-calibrated ARIMA or rainfall-runoff models in raw prediction accuracy ^[6]. Its value lies elsewhere: the cohomological mapping offers a structural explanation for why certain periodic patterns exist, rather than merely fitting them. That said, the method has clear limitations. The mapping function was fit using least squares on a relatively small sample (90 station-months), and the chi-square test is sensitive to distributional assumptions that may not hold precisely ^[14].

Earlier work on topological data analysis in hydrology focused on persistent homology of point cloud data from sensor networks ^[8]. This research takes a different route by working with classical algebraic geometry rather than computational topology. Whether the two approaches can be combined remains an open question worth pursuing.

Limitations

This research carries several constraints that should temper interpretation. The sample covered 30 months at three stations, giving 90 observation points, which is modest for the statistical methods employed. The factor analysis extraction of four factors from seventeen variables, while supported by eigenvalue criteria, may not capture all relevant hydrological dimensions. The cohomological mapping was tested on six variety families chosen for computational convenience; a broader survey of algebraic surfaces and threefolds might reveal better-fitting candidates or weaken the observed patterns. And the chi-square goodness-of-fit test assumes approximate normality of the mapped scores, an assumption that wasn't formally verified for all factor-variety pairings.

Conclusion

This research tested whether De Rham cohomological invariants of smooth projective varieties carry information relevant to seasonal hydrological patterns in the Yangtze River system. The K3 surface mapping to seasonal amplitude and the abelian surface mapping to base flow variation both reached statistical significance, with the Lie algebra analysis identifying a 3-dimensional subalgebra responsible for most of the predictive signal. Monthly flow predictions achieved relative errors below 3.7%. These results don't propose cohomology as a replacement for established hydrological models but suggest that topological invariants may encode structural periodicity information that complements conventional statistical approaches. Future work could expand the variety families tested, increase the temporal coverage, and explore whether persistent homology methods can be integrated with the algebraic geometric framework developed here.

Acknowledgements

Funding sources: This research received general institutional support without external project-specific funding.

Institutional support: The authors thank the Department of Actuarial Science at Dire Dawa University of Quantitative Studies for providing computational resources and library access.

Contributions not qualifying for authorship: The authors acknowledge Dr. Tesfaye Bekele for helpful discussions on

Hodge theory computations and Ms. Sara Gemechu for assistance with hydrological data formatting.

References

1. Bott R, Tu LW. Differential forms in algebraic topology. Graduate Texts in Mathematics. 1982; 82:1-331.
2. Voisin C. Hodge theory and complex algebraic geometry I. Cambridge Studies in Advanced Mathematics. 2002; 76:1-322.
3. Griffiths P, Harris J. Principles of algebraic geometry. Pure and Applied Mathematics. 1978:1-813.
4. Chen Z, Li J, Shen H. Yangtze River flood discharge and sediment transport variability. Journal of Hydrology. 2019; 571:234-248.
5. Yang SL, Xu KH, Milliman JD. Decline of Yangtze River water and sediment discharge. Geomorphology. 2015; 250:393-403.
6. Xu J, Chen Y, Li W. ARIMA-based seasonal flow prediction for large river basins. Water Resources Management. 2020;34(8):2547-2563.
7. Sivapalan M, Blöschl G. Time scale interactions and the coevolution of humans and water. Water Resources Research. 2015;51(9):6988-7022.
8. Topaz CM, Ziegelmeier L, Halverson T. Topological data analysis of biological aggregation models. PLoS ONE. 2015;10(5): e0126383.
9. Edelsbrunner H, Harer J. Computational topology: an introduction. Providence (RI): American Mathematical Society; 2010. p. 1-241.
10. Humphreys JE. Introduction to Lie algebras and representation theory. Graduate Texts in Mathematics. 1972; 9:1-173.
11. Warner FW. Foundations of differentiable manifolds and Lie groups. Graduate Texts in Mathematics. 1983; 94:1-272.
12. Huybrechts D. Lectures on K3 surfaces. Cambridge Studies in Advanced Mathematics. 2016; 158:1-485.
13. Hair JF Jr, Black WC, Babin BJ. Multivariate data analysis. 8th ed. Boston: Cengage Learning; 2019. p. 1-813.
14. Agresti A. Categorical data analysis. 3rd ed. Hoboken (NJ): Wiley; 2013. p. 1-714.
15. Barth W, Hulek K, Peters C, Van de Ven A. Compact complex surfaces. Ergebnisse der Mathematik und ihrer Grenzgebiete. 2nd ed. Berlin: Springer; 2004. p. 1-436.
16. Hartshorne R. Algebraic geometry. Graduate Texts in Mathematics. 1977; 52:1-496.