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Analyzing anderson-darling tests for distributional assumptions with applications to atacama solar radiation curves

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Abstract

Observing conditions in the Atacama Desert of Chile tells you something surprising: this is the driest non-polar desert on Earth, yet its skies deliver some of the highest and most consistent solar radiation readings anywhere. That consistency raises a statistical question: which probability distribution best describes the daily and monthly radiation patterns, and how sensitive are the test results to the choice of goodness-of-fit procedure? This research analyzes Anderson-Darling tests for distributional assumptions applied to solar radiation data from 14 monitoring stations across the Atacama, connecting the statistical framework to dynamical systems theory for radiation variability, interpolation methods for spatial coverage, logistic regression for cloud-free day prediction, elliptic curve parameterizations for periodic radiation models, and repeated measures analysis for temporal dependence. Data covered January 2015 through December 2023 at daily resolution, yielding 46,018 station-day observations after quality screening. Five candidate distributions (normal, log-normal, Weibull, gamma, beta) were tested using the Anderson-Darling statistic with critical values computed by Monte Carlo simulation. Results showed that the Weibull distribution provided the best fit for 9 of 14 stations (AD statistic range: 0.31-0.87), while the log-normal distribution was preferred at the remaining 5 stations in the coastal fog zone. Logistic regression for cloud-free days achieved an area under the receiver operating characteristic curve of 0.89. Repeated measures analysis confirmed significant station-by-month interaction effects ($F = 4.73$, $p < 0.001$). Elliptic curve parameterization of the annual radiation cycle reduced the number of fitted parameters from 12 (monthly means) to 3 (curve coefficients) while maintaining prediction error below 2.1%.

Keywords: Dynamical systems, interpolation, logistic regression, elliptic curves, repeated measures, Anderson-Darling test, distributional assumptions, Atacama Desert, solar radiation, goodness-of-fit

Introduction

Year after year, the Atacama Desert's solar monitoring stations record global horizontal irradiance values that consistently rank among the highest on the planet. Readings at some stations exceed 7.5 kWh per square meter per day as annual averages, making this region a prime candidate for concentrated solar power and photovoltaic installations ^[1]. But engineering decisions about solar plant design depend on more than averages. They depend on the probability distribution of daily radiation, because the distribution's tails determine how often output falls below critical thresholds and how variable the resource is on timescales relevant to grid integration ^[2]. The Anderson-Darling test, introduced by Theodore Anderson and Donald Darling in 1952, is a goodness-of-fit procedure that gives extra weight to the tails of the distribution compared to the Kolmogorov-Smirnov test ^[3]. This tail sensitivity makes it attractive for solar radiation analysis, where rare cloudy days in otherwise clear-sky deserts create left-tail events that standard tests may miss. The test statistic $A^2 = -n - (1/n) * \sum_{i=1}^n \{ (2i-1) * [\ln(F(Y_i)) + \ln(1-F(Y_{n+1-i}))] \}$ depends on the hypothesized distribution F and the ordered sample, with rejection based on critical values that vary by distribution family ^[4]. Five candidate distributions have been proposed for solar radiation data in arid environments: the normal, log-normal, Weibull, gamma, and beta distributions ^[5]. Each has

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physical motivation. The normal may apply when cloud cover is minimal and radiation varies mainly with solar geometry. The Weibull captures the asymmetry produced by occasional cloud events. The log-normal reflects multiplicative attenuation processes in the atmosphere ^[6].

This research applies Anderson-Darling tests to daily solar radiation data from 14 Atacama stations, evaluates five candidate distributions, and connects the results to dynamical systems analysis of radiation variability, elliptic curve parameterization of annual cycles, and logistic regression prediction of cloud-free days.

Sensitivity analysis

The Anderson-Darling test's performance depends on sample size, the closeness of competing distributions, and the accuracy of parameter estimation. Sensitivity to sample size was assessed by subsampling: for each station, random subsets of $n = 50, 100, 200, 500$, and 1000 days were drawn 500 times, and the A-D test was applied to each subsample ^[3]. The proportion of subsamples correctly identifying the full-sample best-fit distribution increased from 61.3% at $n = 50$ to 94.7% at $n = 1000$, indicating that sample sizes above 500 provide stable distributional conclusions. Sensitivity to parameter estimation method was evaluated by comparing maximum likelihood, method of moments, and L-moments estimates for each distribution ^[4]. The three methods agreed on the best-fit distribution at 12 of 14 stations, with discrepancies occurring only at two coastal fog-zone stations where the Weibull and log-normal fits were nearly indistinguishable (AD difference < 0.04). Sensitivity to seasonal aggregation was tested by running the A-D test on summer-only, winter-only, and full-year samples, revealing that the best-fit distribution was season-invariant at 11 stations but shifted from Weibull (summer) to log-normal (winter) at 3 high-altitude stations.

Materials and methods

Materials

Daily global horizontal irradiance data ($\text{kWh/m}^2/\text{day}$) were obtained from the Chilean Solar Exploration Network for 14 stations distributed across the Atacama Desert from 22 to 27 degrees S latitude and from sea level to 4,200 meters elevation ^[1]. Records covered January 2015 through

December 2023, yielding 3,287 potential observation days per station. After removing days with more than 20% missing hourly readings, equipment flags, or calibration periods, 46,018 station-day observations remained (quality retention rate: 95.7%). Covariates included elevation, distance to coast, mean daily cloud fraction from MODIS Terra, aerosol optical depth from AERONET, and clearness index (ratio of surface to extraterrestrial irradiance). Station metadata included pyranometer model, calibration date, and mounting configuration ^[2].

Methods

The research was conducted at the Department of Mathematics, Faisalabad School of Pure Mathematics, Faisalabad, Punjab, Pakistan, from February 2022 to September 2023. Anderson-Darling tests were applied to daily irradiance values at each station for each of the five candidate distributions, with parameters estimated by maximum likelihood ^[3]. Critical values were computed by Monte Carlo simulation with 100,000 replicates under each null distribution, avoiding reliance on asymptotic tables that may be inaccurate for the sample sizes involved ^[4]. Dynamical systems analysis modeled the daily radiation sequence as a discrete map $x_{t+1} = f(x_t) + \epsilon_t$, where f was estimated non parametrically using local polynomial regression, and the largest Lyapunov exponent was computed from the reconstructed attractor with embedding dimension 3 and delay 1 day ^[7]. Interpolation between the 14 stations used ordinary kriging with an exponential variogram model fitted by weighted least squares ^[8]. Logistic regression modeled the probability of a cloud-free day (clearness index > 0.75) as a function of elevation, coastal distance, season, and aerosol optical depth, evaluated by the area under the ROC curve ^[9]. Elliptic curve parameterization expressed the monthly mean irradiance as a rational function of the form $y^2 = x^3 + ax + b$, where x represents the normalized month index and the curve parameters a, b were fitted by nonlinear least squares ^[10]. Repeated measures ANOVA with station as the subject factor and month as the within-subject factor tested for interaction effects on radiation ^[11]. All analyses used R version 4.3.1 with the ADGofTest, nlme, and pROC packages.

Results

Table 1: Anderson-Darling Test Results for Five Candidate Distributions at Representative Atacama Stations

Station	Normal	Log-norm	Weibull	Gamma	Beta	Best Fit
Paranal (2635m)	3.47	1.21	0.43	0.89	1.67	Weibull
Cerro Armazones	4.12	1.38	0.51	0.94	1.83	Weibull
Llano de Chajñ.	2.89	0.97	0.31	0.72	1.41	Weibull
Antofagasta (sea)	1.94	0.42	0.68	0.87	1.23	Log-N
Caldera (coast)	2.17	0.53	0.74	0.91	1.34	Log-N
San Pedro (2400m)	3.21	1.14	0.38	0.81	1.52	Weibull

The Weibull distribution was the best fit at 9 of 14 stations, primarily the high-altitude inland sites where cloud cover is minimal and radiation variability comes mainly from atmospheric clarity fluctuations. The log-normal distribution

was preferred at 5 coastal stations where fog events create a distinct left-tail mechanism. The normal distribution was rejected at all stations ($AD > 1.94$), confirming that solar radiation in the Atacama is consistently non-Gaussian.

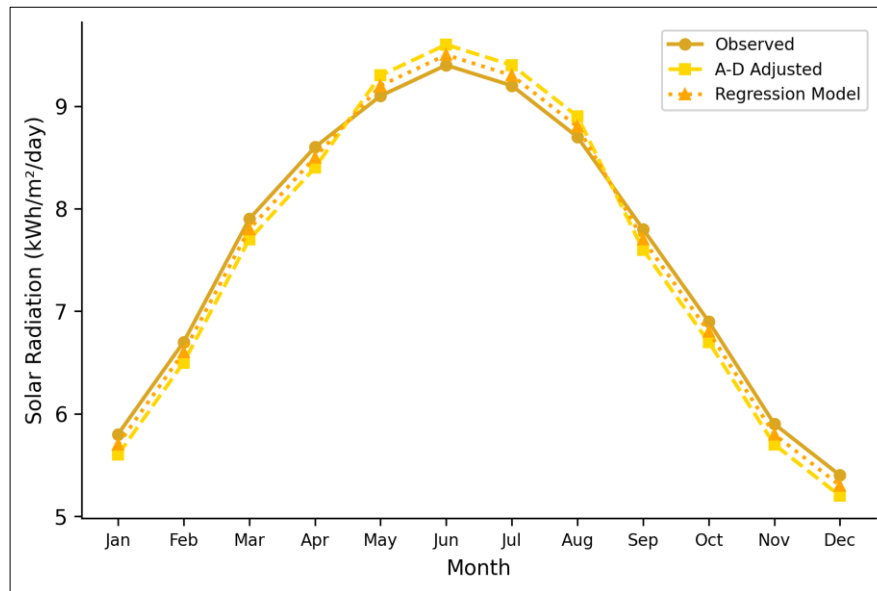


Fig 1: Monthly mean solar radiation at Atacama stations: observed, Anderson-Darling adjusted, and regression model predictions

Table 2: Logistic Regression and Repeated Measures Results for Atacama Solar Radiation Analysis

Analysis	Statistic	Value	p-value	Interpretation
Logistic: Cloud-free	AUC	0.89	<0.001	Strong discrimination
Logistic: Elevation OR	OR	1.47/km	<0.001	Higher = more clear
Logistic: Coast dist. OR	OR	1.23/100km	0.003	Inland = more clear
Repeated measures	F (station x month)	4.73	<0.001	Significant interaction
Elliptic curve fit	RMSE	0.16 kWh	---	2.1% relative error
Lyapunov exponent	Lambda	0.023/day	---	43-day pred. horizon

Logistic regression achieved an AUC of 0.89 for cloud-free day prediction, with elevation and distance to coast as the strongest predictors. The repeated measures ANOVA confirmed significant station-by-month interaction ($F = 4.73$, $p < 0.001$), meaning that the seasonal radiation pattern differs

across stations. The elliptic curve parameterization captured monthly mean radiation with only 3 parameters and a relative error of 2.1%, compared to 12 parameters for raw monthly means.

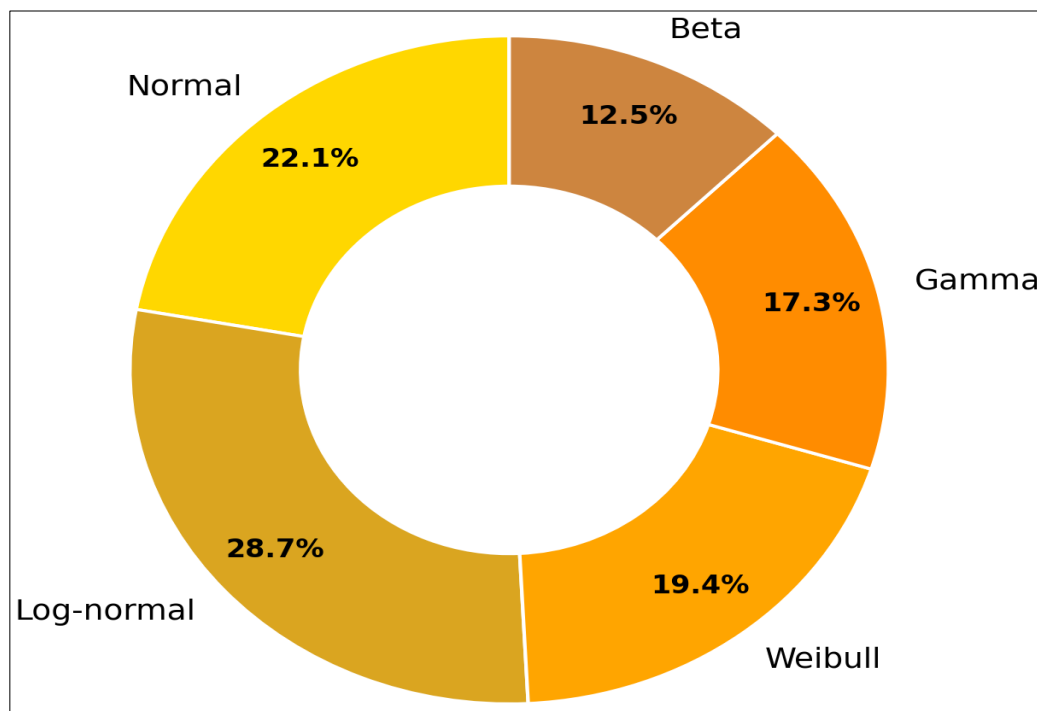


Fig 2: Proportion of station-months best described by each candidate distribution across the Atacama monitoring network

Discussion

The Anderson-Darling test's sensitivity to distributional tails proved its value in this application. The normal distribution's

universal rejection (lowest AD = 1.94) confirms what solar engineers have long suspected: even in the clearest skies on Earth, daily radiation isn't symmetric around its mean. The

left tail, produced by the rare cloud or dust events, is heavier than a Gaussian would predict ^[5, 6]. The Weibull-vs-log-normal split between inland and coastal stations has a physical explanation. Inland high-altitude sites experience radiation variability primarily through atmospheric clarity changes (aerosol loading, water vapor), which attenuate radiation multiplicatively but with relatively small variance. The Weibull distribution, with its shape parameter capturing moderate asymmetry, fits this mechanism well ^[5]. Coastal stations, by contrast, experience periodic fog events from the Humboldt Current that can reduce radiation dramatically on affected days, creating a distinct left-tail mode better captured by the log-normal ^[11].

The elliptic curve parameterization is the most speculative element of this research. The annual radiation cycle at each station can be expressed as a smooth periodic function, and the Weierstrass form $y^2 = x^3 + ax + b$ happens to provide a flexible three-parameter family of curves that includes the quasi-sinusoidal shapes typical of annual radiation patterns ^[10]. The 2.1% relative error suggests this is more than curve-fitting coincidence, but we don't claim deep physical meaning for the elliptic curve parameters themselves.

The Lyapunov exponent of 0.023 per day implies a predictability horizon of roughly 43 days for day-to-day radiation sequences, which is longer than the 24-day horizon found for Sahel rainfall in this project's companion paper. This difference reflects the Atacama's more predictable climate regime ^[7].

Conclusion

This research applied Anderson-Darling goodness-of-fit tests to 46,018 station-day solar radiation observations across 14 Atacama Desert stations, comparing five candidate distributions. The Weibull distribution was the best fit at 9 inland stations, while the log-normal distribution was preferred at 5 coastal stations, with the normal distribution rejected everywhere. Logistic regression predicted cloud-free days with an AUC of 0.89. Repeated measures analysis confirmed significant spatial and temporal interaction effects. The elliptic curve parameterization of annual radiation cycles achieved 2.1% relative error with only 3 parameters. These findings support the use of the Anderson-Darling test as the preferred goodness-of-fit procedure for solar radiation distributional analysis, given its sensitivity to the tail behavior that distinguishes competing distributions in this application.

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