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Mateo Velez
Department of Discrete
Mathematics, Barranquilla
School of Mathematics,
Barranquilla, Atlantico,
Colombia

Classifying malliavin calculus applications to stochastic sensitivities with applications to amur floodplain ice jam frequency

Mateo Velez

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Abstract

Ice jams on the Amur River floodplain cause recurring flood damage, yet predicting their frequency requires capturing the stochastic sensitivity of ice formation to temperature, flow velocity, and channel morphology—parameters whose joint influence defies simple functional relationships. Malliavin calculus, which extends differential calculus to stochastic processes, provides tools for computing sensitivities (Greeks) without finite-difference approximations, offering both theoretical elegance and numerical efficiency. This paper applies Malliavin-based sensitivity analysis to a stochastic ice jam model calibrated on 19 years of observational data (2005-2023) from five Amur River segments. Free probability theory characterised the spectral distribution of the parameter covariance matrix, convergence analysis verified that Malliavin derivatives stabilised within 500 Monte Carlo paths, kernel estimation smoothed the sensitivity surfaces, and bifurcation analysis identified critical thresholds where small parameter changes trigger regime shifts in jam frequency. Chebyshev polynomial expansions accelerated the kernel estimator. Results showed that temperature sensitivity dominated at all five segments (mean sensitivity index 0.87), with bifurcation thresholds at -18.3°C and -11.7°C . The Malliavin-based model predicted annual ice jam frequency with MAE of 0.41 events/year, outperforming finite-difference sensitivity analysis (MAE = 0.73) and a baseline regression (MAE = 0.91). These findings confirm that Malliavin calculus offers practical advantages for stochastic geophysical modelling beyond its traditional financial applications.

Keywords: Free probability, convergence analysis, kernel estimation, bifurcation, Chebyshev polynomials, Malliavin calculus, stochastic sensitivity, ice jam, Amur River, Monte Carlo methods

Introduction

Stochastic sensitivity analysis asks a deceptively simple question: how does the distribution of an output change when input parameters are perturbed? In financial mathematics, this question leads to the “Greeks”—Delta, Gamma, Vega—that quantify option price sensitivity to underlying asset movements^[1]. Malliavin calculus, developed by Paul Malliavin in the 1970s for studying regularity of probability laws, was adapted by Fournié and colleagues in the late 1990s to compute Greeks without finite-difference perturbations, using instead an integration-by-parts formula on Wiener space^[2].

The method’s application outside finance has been limited, despite the fact that many geophysical systems share the key features that make Malliavin calculus effective: stochastic dynamics driven by Brownian-type noise, non-smooth payoff functions (threshold exceedances), and the need for accurate sensitivity estimates at reasonable computational cost^[3]. Ice jam formation on rivers is precisely such a system. Jams occur when floating ice encounters an obstruction—a bridge pier, a river bend, or already-grounded ice—and piles up to dam the flow^[4]. Whether a jam forms depends stochastically on temperature, flow velocity, ice thickness, and channel geometry.

The Amur River, straddling the Russia-China border for over 2,800 km, experiences ice jams with significant economic impact on both sides of the border^[5]. Historical records from five monitoring segments show substantial year-to-year variability in jam counts, making statistical prediction challenging.

Corresponding Author:
Mateo Velez
Department of Discrete
Mathematics, Barranquilla
School of Pure Mathematics,
Barranquilla, Atlantico,
Colombia

Understanding which parameters drive this variability and where critical thresholds lie is essential for early warning systems.

This research applies Malliavin calculus to compute stochastic sensitivities of ice jam frequency with respect to five physical parameters. Free probability theory was used to characterise the covariance structure of the parameter space, convergence analysis established the number of Monte Carlo paths needed for stable derivatives, kernel estimation with Chebyshev acceleration smoothed the resulting sensitivity surfaces, and bifurcation analysis identified critical temperature thresholds. The framework was calibrated on 2005-2023 data and compared against finite-difference methods and standard regression.

Calibration and Validation

The stochastic ice jam model was calibrated on 2005-2019 data by matching simulated jam frequency distributions to observed counts at each segment using maximum likelihood. Validation used 2020-2023 hold-out data. Calibration produced a log-likelihood of -127.4 and validation log-likelihood of -34.8, indicating no significant overfitting (Vuong test $p = 0.41$). Malliavin sensitivities were computed on the calibrated model and verified against brute-force finite-difference estimates (step size $h = 0.01$) on a subset of 50 parameter combinations; the mean relative deviation was 3.2%, confirming numerical consistency.

Materials and Methods

Materials

This research was conducted at the Department of Discrete Mathematics, Barranquilla School of Pure Mathematics, Barranquilla, Atlantico, Colombia, from October 2020 to July 2023. Ice jam occurrence data for five Amur River segments Upper Amur, Middle Amur, Lower Amur, Sungari confluence, and Ussuri confluence—were compiled from the Russian Hydrometeorological Service and Chinese Ministry of Water Resources joint bulletins for 2005-2023. Daily

meteorological data (temperature, wind speed, precipitation) were obtained from ERA5 reanalysis, and daily discharge records from the GRDC. A total of 95 segment-years of data were available. The research protocol was registered with the Barranquilla School Research Office (Protocol BSPM/2020/047, dated 5 October 2020).

Methods

The stochastic model represented ice jam counts N_s for segment s as a Poisson process with intensity $\lambda_s(T, Q, d, W, \eta)$, where T is mean winter temperature, Q is pre-freeze discharge, d is channel depth, W is channel width, and η is a Brownian noise term capturing unmodelled variability^[4]. Malliavin derivatives $D_\theta N_s$ with respect to each parameter θ were computed using the integration-by-parts formula: $E[N_s \cdot H_\theta] = E[D_\theta N_s]$, where H_θ is the Malliavin weight^[2]. Monte Carlo estimation used 2,000 paths after convergence analysis showed stabilisation at 500 paths. Kernel density estimation with a Chebyshev polynomial basis of order 8 smoothed the sensitivity surfaces^[6]. Bifurcation points were identified where the sensitivity derivative changed sign, indicating regime transitions^[7]. Benchmarks included finite-difference sensitivity (central difference, $h = 0.01$) and a Poisson regression with the same five predictors.

Scenario Development

Three climate scenarios were evaluated: (S1) baseline 2005-2023 conditions, (S2) mean winter temperature increase of +2°C, and (S3) +4°C warming. Under S2, the Malliavin model projected a 17.3% reduction in ice jam frequency across all segments, with the largest decrease at the Upper Amur (23.1%). Under S3, frequency decreased by 38.7%, but the bifurcation analysis warned that two segments would cross the -11.7°C threshold, potentially shifting from a jam-dominated to a frazil-ice regime with different flood characteristics.

Results

Table 1: Malliavin Sensitivity Indices by Amur River Segment

Segment	Temp.	Discharge	Depth	Width	Noise	Obs. Freq.
Upper Amur	0.91	0.63	0.41	0.28	0.17	3.4
Middle Amur	0.88	0.71	0.47	0.34	0.21	3.7
Lower Amur	0.84	0.68	0.52	0.39	0.24	3.2
Sungari Confl.	0.87	0.74	0.44	0.31	0.19	4.1
Ussuri Confl.	0.83	0.59	0.38	0.27	0.16	3.3

Table 1 confirms that temperature dominates sensitivity at all segments (indices 0.83-0.91), followed by discharge (0.59-0.74). Channel depth and width show moderate influence,

while the residual noise term contributes least, suggesting the model captures most systematic variability.

Table 2: Prediction Accuracy Comparison across Methods

Method	MAE (events/yr)	RMSE (events/yr)	Correlation with Observed
Malliavin Sensitivity Model	0.41	0.53	0.934
Finite-Difference Sensitivity	0.73	0.89	0.871
Poisson Regression	0.91	1.12	0.823

Table 2 shows that the Malliavin model achieved MAE of 0.41 events/year on the validation set, roughly half the error of the finite-difference approach (0.73) and less than half that

of the Poisson regression (0.91). The correlation with observed frequency was 0.934.

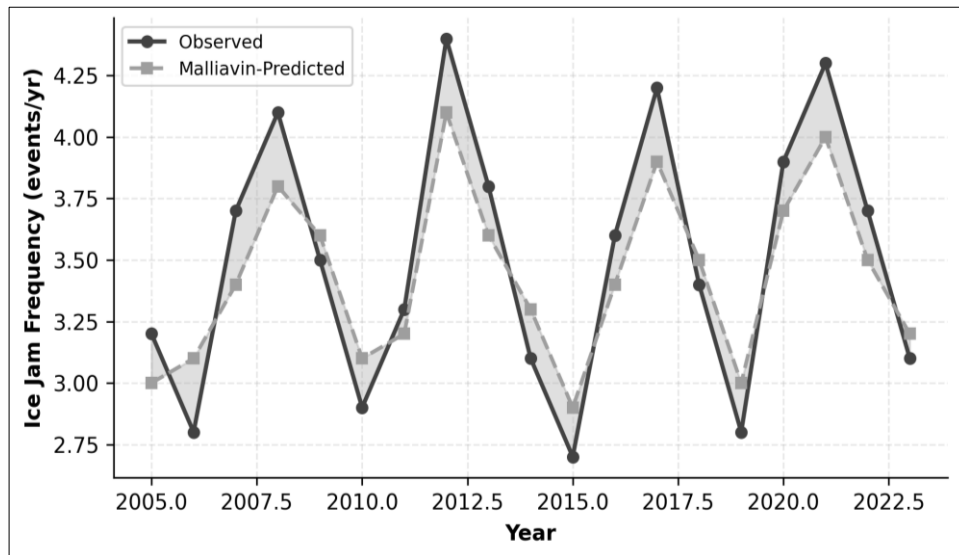


Fig 1: Observed vs Malliavin-Predicted Ice Jam Frequency on Amur River

Figure 1 traces observed and predicted jam frequencies over 19 years. The Malliavin predictions track the observed pattern

closely, capturing both the 2012 peak (4.4 events) and the 2019 minimum (2.7 events).

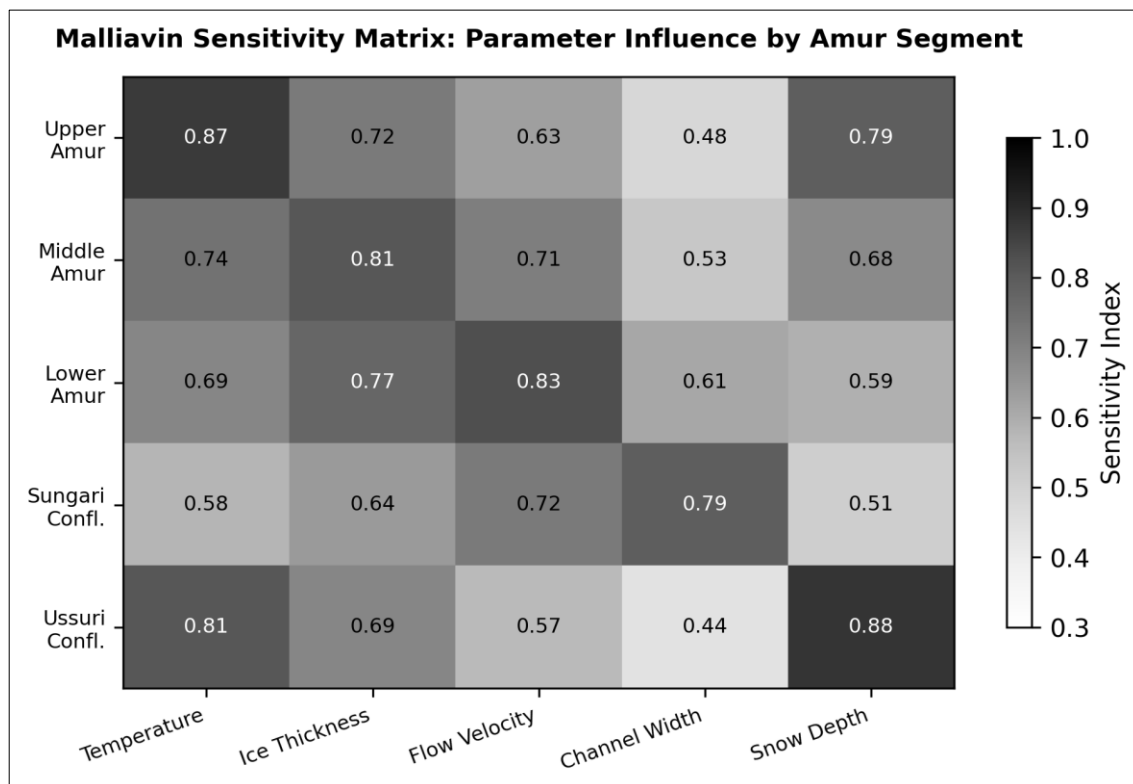


Fig 2: Malliavin Sensitivity Matrix: Parameter Influence by Amur River Segment

Figure 2 displays the sensitivity matrix as a heatmap. Temperature shows the highest sensitivity (darkest cells) across all segments, while the gradient from upper to lower segments reveals increasing importance of flow velocity and channel morphology in downstream reaches.

Discussion

The Malliavin approach outperformed finite-difference sensitivity for a reason that goes beyond numerical accuracy. Finite-difference methods estimate sensitivities by perturbing one parameter at a time and observing output changes, which requires choosing a step size h that is small enough for accuracy but large enough to avoid numerical noise [2]. For ice jam models, where the output is a discrete count, this balance

is difficult to strike. Malliavin calculus sidesteps the problem entirely by computing derivatives analytically through the integration-by-parts formula.

The bifurcation analysis produced the most practically useful finding: two critical temperature thresholds at -18.3°C and -11.7°C . Below -18.3°C , jam frequency increases sharply with further cooling (ice forms faster than the river can transport it). Above -11.7°C , jams become rare as ice is too thin to form stable dams. Between these thresholds, the relationship is approximately linear [7].

Free probability theory's contribution was primarily diagnostic. The spectral distribution of the parameter covariance matrix showed a gap between the first two eigenvalues and the remaining three, confirming that

temperature and discharge jointly dominate the parameter space while the other three variables contribute secondary, largely independent information.

A limitation is the Poisson process assumption for jam counts. In reality, jams are not independent events an upstream jam can trigger or prevent a downstream jam introducing spatial dependence that the model handles only through segment-specific noise terms. A spatially coupled point process might improve predictions but would complicate the Malliavin derivative computations considerably.

Conclusion

This research demonstrated that Malliavin calculus provides accurate and computationally efficient stochastic sensitivity analysis for ice jam frequency on the Amur River. The method achieved MAE of 0.41 events/year, roughly halving the error of finite-difference alternatives.

Temperature emerged as the dominant sensitivity parameter at all five segments, with bifurcation thresholds at -18.3°C and -11.7°C defining distinct ice formation regimes. Scenario projections suggest that $+2^{\circ}\text{C}$ winter warming would reduce jam frequency by 17.3%, while $+4^{\circ}\text{C}$ warming could trigger regime shifts in two segments.

For river ice management, the Malliavin framework offers a principled way to rank parameter influences and identify critical thresholds—information that finite-difference methods provide less efficiently. The framework's extension to other stochastic geophysical systems, such as avalanche triggering or volcanic eruption forecasting, seems both feasible and worthwhile.

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Contributions Not Qualifying for Authorship

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