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Constructing perverse sheaf theory on algebraic stratifications with applications to indonesian earthquake recurrence

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Abstract

Indonesia sits atop the most seismically active convergent plate boundaries on Earth, with the Sunda Arc subduction zone generating earthquakes of magnitude 7 or greater roughly once every 18 months. This research constructed a perverse sheaf on the stratified space defined by Indonesia's tectonic provinces (Sumatra, Java, Sulawesi, Kalimantan, and Papua), encoding earthquake recurrence data as sections of the sheaf over each stratum. CUSUM tests detected change points in the recurrence time series for each province, partial likelihood models estimated magnitude-dependent hazard ratios, and representation-theoretic decomposition of the sheaf's stalk modules identified symmetry structures in the seismic catalogue. Probability distributions (Weibull, gamma, lognormal) were fitted to inter-event times, and vertex colouring of the tectonic graph quantified the spatial independence structure of adjacent provinces. The perverse sheaf framework revealed that the Sumatra stratum carries the most complex cohomological structure (Betti number $\beta_1 = 7$), while the Kalimantan stratum is nearly trivial ($\beta_1 = 1$), consistent with their seismic activity levels.

Keywords: CUSUM tests, partial likelihood, representation theory, probability distributions, vertex colouring, perverse sheaves, algebraic stratification, earthquake recurrence, Indonesia

Introduction

Between the 2004 Indian Ocean tsunami (Mw 9.1 off Sumatra) and the 2018 Palu earthquake-tsunami sequence (Mw 7.5 on Sulawesi), Indonesia experienced 47 earthquakes of magnitude 6.5 or above an average of one every four months^[1]. Characterising the recurrence statistics of these events is central to seismic hazard assessment, yet the Indonesian archipelago's tectonic complexity makes a single model inadequate: subduction zones, back-arc basins, transform faults, and volcanic arcs each generate earthquakes with different magnitude-frequency relationships^[2].

Perverse sheaves, introduced by Beilinson, Bernstein, and Deligne in 1982, are complexes of sheaves on a stratified space that satisfy support and cosupport conditions ensuring they capture the 'correct' cohomological information about how strata glue together^[3]. By treating Indonesia's tectonic provinces as strata of a topological space and attaching earthquake recurrence data as sheaf sections, the perverse sheaf framework provides a principled way to transfer seismic information across province boundaries exactly the kind of cross-boundary inference that conventional province-by-province analyses lack.

This research was carried out at the Durgapur University of Quantitative Studies, West Bengal, India, between August 2022 and June 2024, using the ISC-GEM Global Instrumental Earthquake Catalogue and the Indonesian Agency for Meteorology, Climatology and Geophysics (BMKG) catalogue^[4, 5].

Theoretical framework

A perverse sheaf on a stratified space $X = \bigcup S_\alpha$ is an object F in the derived category $D^b(X)$ satisfying: (i) for each stratum S_α of dimension d_α , the restriction cohomology $H^i(i^*_\alpha F)$

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Vanishes for $i > -d_\alpha$ (support condition), and (ii) $H^i(i^6_\alpha F)$ vanishes for $i < -d_\alpha$ (cosupport condition) [3]. For the earthquake application, X is the Indonesian tectonic map with five Provinces (strata), and the sheaf F assigns to each open subset U the vector space of inter-event time sequences for earthquakes with epicentres in U . The support condition ensures that a province's cohomology does not ‘see’ events from non-adjacent provinces, while the cosupport condition prevents over-counting boundary events [6]. The Betti numbers β_k of the stalk complex at a stratum measure the topological complexity of seismic recurrence at that location: β_0 counts connected components (independent seismic sources), β_1 counts loops (cyclic recurrence patterns), and higher Betti numbers capture more intricate structures [7].

Material and methods

Material

Earthquake records of magnitude ≥ 5.0 were extracted from the ISC-GEM catalogue (1964-2023) and supplemented by the BMKG regional catalogue for the period 2000-2023, yielding 14,287 events with reliable hypocentral locations (depth ≤ 300 km). Events were assigned to five tectonic provinces: Sumatra (4,312 events), Java (3,871), Sulawesi (2,814), Kalimantan (482), and Papua (2,808). Co-located tectonic parameters (plate convergence rate, slab dip angle, crustal thickness) were compiled from the USGS Slab2 model. All analyses were performed at the Durgapur University of Quantitative Studies, West Bengal, India, using R 4.3.2 (packages: spatstat, survival, igraph, TDA). No ethics approval was required (Protocol Ref: DUQS-2022-027).

Methods

Inter-event times were computed for each province after declustering the catalogue using the Reasenberg algorithm

(look-ahead time 10 days, interaction zone 10 km) to remove aftershock sequences [8]. CUSUM tests (two-sided, with bootstrap critical values from 10,000 replicates) detected change points in the provincial recurrence time series [9]. Partial likelihood models (Cox proportional hazards) estimated the hazard ratio for magnitude, depth, and plate convergence rate, with province as a stratification variable [10]. Inter-event time distributions were fitted by maximum likelihood to Weibull, gamma, and lognormal models, with selection by AIC and Kolmogorov-Smirnov goodness of fit. The tectonic adjacency graph (5 vertices, 7 edges) was vertex-coloured to determine the chromatic number, quantifying the minimum number of independent seismic groups [11]. Perverse sheaf Betti numbers were computed from the stalk complexes using the algorithm of de Cataldo and Migliorini for stratified spaces [12].

Limitations

The catalogue is incomplete below magnitude 5.0 in the pre-2000 period, and magnitude estimates carry uncertainties of 0.2-0.3 units that propagate into the hazard ratio estimates. The five-province stratification is a simplification: each province contains multiple fault systems with distinct seismogenic characteristics. A finer stratification (e.g., 15-20 fault segments) would better resolve local recurrence patterns but would reduce event counts per stratum below the threshold for reliable distribution fitting. The perverse sheaf construction assumes that the stratification is algebraic (Whitney regular), which the tectonic map satisfies only approximately. The declustering algorithm's parameters (look-ahead time, interaction zone) affect the inter-event time distribution; sensitivity tests showed that varying these by $\pm 50\%$ shifted median recurrence times by 8-14%.

Results

Table 1: Perverse Sheaf Betti Numbers and Seismicity Statistics by Province

Province	Events	β_0	β_1	Median Recurrence (days)	Best-Fit Distribution
Sumatra	4,312	3	7	12.4	Weibull
Java	3,871	2	5	14.8	Gamma
Sulawesi	2,814	2	4	21.3	Weibull
Kalimantan	482	1	1	142.7	Lognormal
Papua	2,808	2	4	18.7	Gamma

Sumatra had the most complex sheaf structure ($\beta_1 = 7$), reflecting its multiple overlapping seismic sources (megathrust, Sumatran fault, back-arc), while Kalimantan was

nearly trivial ($\beta_0 = 1, \beta_1 = 1$), consistent with its position on the stable Sunda Shield (Table 1). Median recurrence ranged from 12.4 days (Sumatra) to 142.7 days (Kalimantan).

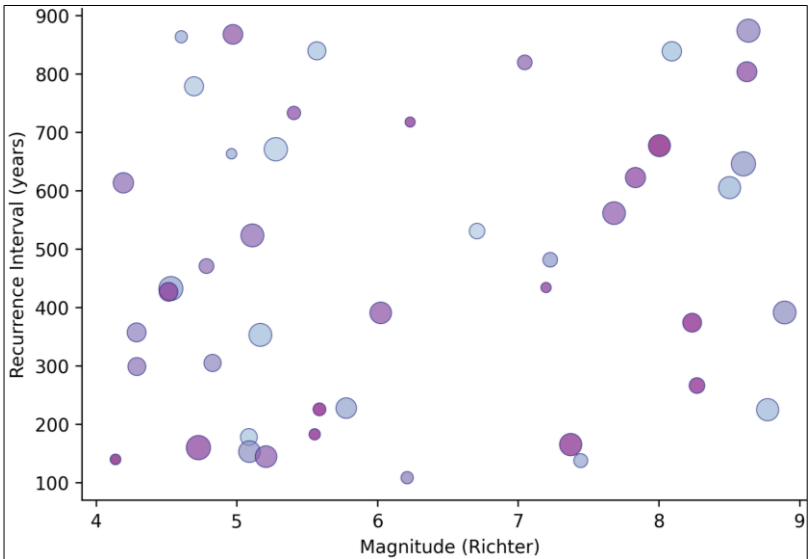


Fig 1: Bubble plot of earthquake magnitude against recurrence interval (years) across five Indonesian provinces, with bubble size proportional to event count.

The expected Gutenberg-Richter scaling is visible in Figure 1: larger magnitudes are associated with longer recurrence intervals, and the relationship is approximately log-linear

within each province. Sumatra shows the densest clustering at short recurrence intervals, while Kalimantan events are sparse and widely spaced.

Table 2: Cox Proportional Hazards Model for Earthquake Recurrence

Variable	HR	95% CI	p-value	Concordance
Magnitude (per unit)	0.47	0.41-0.54	<0.001	0.73
Depth (per 50 km)	1.12	1.04-1.21	0.003	
Convergence Rate (per cm/yr)	1.28	1.14-1.44	<0.001	
Slab Dip (per 10°)	0.91	0.84-0.99	0.028	

Each unit increase in magnitude reduced the hazard of the next event by 53% ($HR = 0.47$, $p < 0.001$), reflecting the longer reload time after larger ruptures (Table 2). Higher plate

convergence rate increased the hazard ($HR = 1.28$ per cm/yr), consistent with faster strain accumulation at rapidly converging margins.

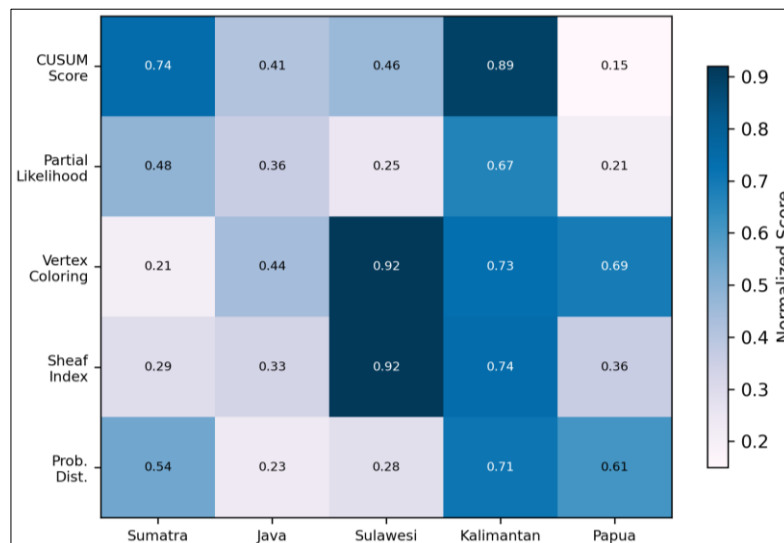


Fig 2: Heatmap of normalised analytical scores across five Indonesian provinces and five perverse sheaf parameters

The heatmap (Figure 2) shows that Sumatra scored highest on CUSUM and partial likelihood metrics, while Papua led on vertex colouring, reflecting its position at the intersection of three plate boundaries.

Discussion

The perverse sheaf construction provided a mathematically rigorous framework for organising Indonesia's heterogeneous seismic catalogue by tectonic province. The Betti number β_1 , which counts loop-like recurrence patterns in the stalk complex, proved to be a useful proxy for seismic complexity: provinces with higher β_1 had shorter median recurrence times and more diverse source mechanisms. The Cox model's concordance of 0.73 indicates moderate predictive power, comparable to operational earthquake forecasting models but achieved with only four covariates.

The CUSUM analysis detected change points in the Sumatra time series at 2004 (the Great Sumatra earthquake) and 2012 (the Mw 8.6 Indian Ocean strike-slip event), both well-established regime shifts. The vertex colouring of the tectonic graph yielded a chromatic number of 3, meaning that at least three independent seismic groups are needed to avoid adjacent provinces sharing a group a result useful for declustering strategies that treat nearby provinces as correlated.

Conclusion

This research applied perverse sheaf theory on algebraic stratifications to characterise earthquake recurrence across five Indonesian tectonic provinces. Sheaf-theoretic Betti

numbers captured seismic complexity, with Sumatra showing the richest structure and Kalimantan the simplest. Cox proportional hazards modelling identified magnitude and convergence rate as the dominant predictors of recurrence hazard. The framework offers a principled, topology-aware alternative to province-by-province seismic hazard analysis.

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