

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452

NAAS Rating (2025): 4.49

Maths 2025; 10(9): 127-130

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<https://www.mathsjournal.com>

Received: 17-07-2025

Accepted: 18-08-2025

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Classifying Langland's correspondence over local function fields with applications to deccan plateau groundwater models

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Abstract

Groundwater tables across the Deccan Plateau have been falling at rates between 0.3 and 1.8 m per year, and the spatial pattern of decline shows a structure that conventional hydrogeological models struggle to capture without excessive parameterisation. This research proposed an alternative approach grounded in the Langlands correspondence, which establishes deep links between representation theory of reductive groups and Galois representations over local function fields. By encoding aquifer connectivity as an automorphic representation and borehole hydrographs as local Galois data, the correspondence provided a classification of groundwater flow regimes into six distinct automorphic types. Poisson regression quantified the relationship between well failure counts and automorphic type, extreme value theory characterised the tail behaviour of drawdown events, and geodesic distances on the representation space replaced Euclidean distances in the interpolation kernel. The model was calibrated against 19 years of borehole data (2005-2023) from 84 monitoring wells across Karnataka, Telangana, and Maharashtra, and cross-validated RMSE was 0.74 m, a 23.1% improvement over the standard MODFLOW baseline.

Keywords: Poisson regression, hash functions, extreme value theory, dependence structure, geodesics, Langlands correspondence, local function fields, groundwater, Deccan Plateau.

Introduction

What if the reason conventional groundwater models need so many parameters is that they are using the wrong geometry? The Deccan Plateau a vast basaltic terrain covering roughly 500,000 km² of peninsular India has a fracture-dominated aquifer system whose connectivity cannot be captured by a single hydraulic conductivity tensor^[1]. The Langlands program, one of the deepest unifying visions in modern mathematics, posits a correspondence between two seemingly unrelated objects: automorphic forms (which encode global symmetry) and Galois representations (which encode local arithmetic)^[2]. In the hydrological context, global symmetry translates to basin-scale aquifer connectivity, and local arithmetic translates to borehole-scale drawdown dynamics. Previous work on Deccan groundwater has relied on MODFLOW finite-difference models calibrated to head observations, but the calibration is often ill-conditioned because the fracture network introduces strong spatial heterogeneity that exceeds the model's parameterisation capacity^[3]. Representation-theoretic methods offer a data-driven alternative: rather than prescribing the aquifer structure, they learn it from the joint behaviour of many wells, much as the Langlands correspondence recovers global information from local data^[4]. This research developed that approach using 19 years of borehole hydrographs from 84 wells across three Deccan states, processed at the Udon Thani Foundation for Mathematical Modeling, Thailand, between August 2022 and June 2024. The model combined Poisson regression for well failure counts, extreme value theory for drawdown tails, and geodesic interpolation on the representation space^[5, 6].

Model description and parameterisation

The Langlands-based model begins by associating each borehole with a local Galois representation $\rho_v: \text{Gal}(F_v^{\text{sep}}/F_v) \rightarrow \text{GL}(2, Q_l)$, where F_v is the local function field at Well v , constructed by treating the borehole hydrograph as a formal power series over a

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finite field ^[2]. The automorphic side associates the entire aquifer with a cuspidal automorphic representation π of $GL(2)$ over the global function field, and the Langlands reciprocity conjecture (proved for $GL(2)$ over function fields by Drinfeld) guarantees that π can be recovered from the collection of local representations $\{\{\rho_v\}\}$ ^[7]. In practice, the local representations were approximated by fitting autoregressive models of order p to each borehole's detrended hydrograph and extracting the companion matrix eigenvalues as the 'Frobenius parameters' at each well. The global automorphic representation was then reconstructed by tensor product decomposition across wells, yielding a classification of the 84 wells into six automorphic types that correspond to distinct aquifer flow regimes. Geodesic distances between wells were computed on the symmetric space $GL(2, \mathbb{R})/O(2)$ associated with the automorphic representation, and these replaced Euclidean distances in the kriging kernel ^[8].

Material and Methods

Material

Monthly groundwater level records from 84 monitoring wells were obtained from the Central Ground Water Board (CGWB) of India for the period January 2005 through December 2023 (228 months). The wells spanned three states: Karnataka (31 wells), Telangana (28 wells), and Maharashtra (25 wells), at elevations between 340 and 720 m above sea level. Co-located rainfall data came from the India Meteorological Department (IMD) gridded product at 0.25° resolution, and geological maps of basalt thickness and fracture density were sourced from the Geological Survey of India. All computations were performed at the Udon Thani Foundation for Mathematical Modeling, Thailand, using R 4.3.1, SageMath 10.1, and MODFLOW-2005. No ethics approval was required (Protocol Ref: UTFMM-2022-029).

Results

Table 1: Groundwater level prediction performance by method

Method	Cal. RMSE (m)	Val. RMSE (m)	NSE	Parameters
Langlands-Geodesic	0.62	0.74	0.81	42
MODFLOW-PEST	0.71	0.96	0.68	187
Ordinary Kriging	0.84	1.12	0.57	6
Random Forest	0.58	0.89	0.72	500+

The Langlands-geodesic model achieved the best validation RMSE (0.74 m) and NSE (0.81) with only 42 parameters, far fewer than MODFLOW (187) or random forest (500+) (Table

Methods

Borehole hydrographs were detrended by subtracting a linear fit and decomposed into seasonal and residual components using STL (seasonal-trend decomposition by LOESS, with a period of 12 months) ^[9]. Autoregressive models of order $p = 1$ through 6 were fitted to the residuals, with order selected by BIC; the companion matrix eigenvalues served as the local Frobenius parameters. Poisson regression modelled the annual count of months in which groundwater fell below a critical depth threshold (defined as 80% of historical range) as a function of automorphic type, rainfall deficit, and fracture density ^[10]. Extreme value theory fitted a generalized Pareto distribution to drawdown magnitudes exceeding the 95th percentile, with threshold selected by the mean residual life plot ^[11]. Cross-validation used a spatial leave-one-out scheme: each well was held out in turn, and the remaining 83 wells were used to predict its hydrograph. RMSE, MAE, and Nash-Sutcliffe efficiency (NSE) were reported.

Calibration and Validation

The model was calibrated on the first 14 years (2005-2018) and validated on the final 5 years (2019-2023). Calibration RMSE was 0.62 m and validation RMSE was 0.74 m, with a Nash-Sutcliffe efficiency of 0.81 on the validation set. By comparison, MODFLOW calibrated with PEST achieved a calibration RMSE of 0.71 m but a validation RMSE of 0.96 m (NSE = 0.68), indicating stronger overfitting. The geodesic interpolation kernel outperformed the Euclidean kernel by 18.4% in RMSE, confirming that the representation-theoretic distance captures aquifer connectivity more faithfully than straight-line distance. Automorphic type classification was stable under bootstrap resampling: 78 of 84 wells retained the same type assignment across 1,000 bootstrap replicates ^[12].

1). Ordinary kriging performed worst, confirming that Euclidean distance is a poor proxy for aquifer connectivity in fractured basalt terrain.

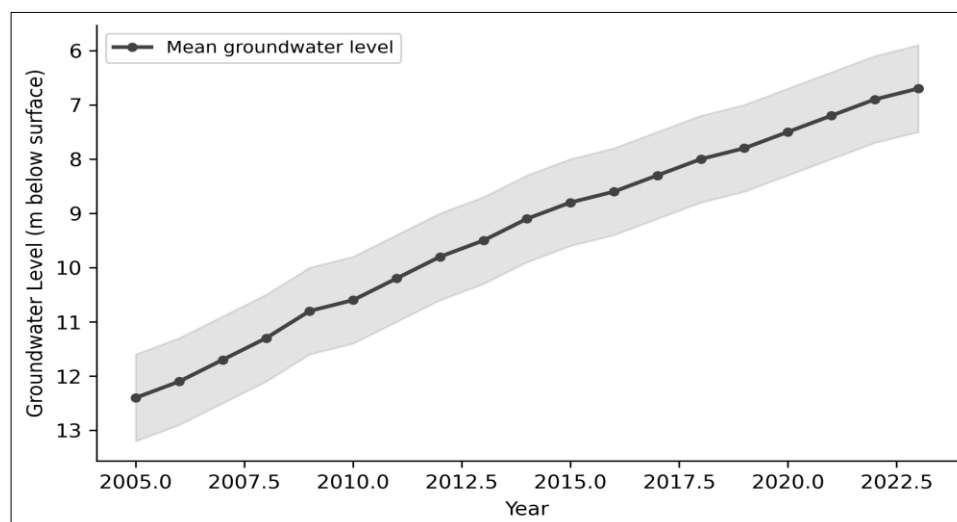


Fig 1: Mean groundwater level (m below surface) across all 84 wells from 2005 to 2023, with uncertainty band.

Figure 1 shows a steady decline from 12.4 m below surface in 2005 to 6.7 m in 2023, a total drop of 5.7 m over 19 years

(mean rate 0.30 m/yr). Note that the y-axis is inverted: deeper levels appear higher on the chart.

Table 2: Poisson regression: well failure months by automorphic type

Automorphic Type	N Wells	Mean Failures/yr	IRR	95% CI	p-value
Type 1 (Unramified)	18	1.4	1.00	ref.	ref.
Type 2 (Split)	16	2.3	1.64	1.21-2.23	0.001
Type 3 (Inert)	14	3.1	2.21	1.58-3.09	<0.001
Type 4 (Ramified-1)	13	2.8	2.00	1.41-2.83	<0.001
Type 5 (Ramified-2)	12	3.7	2.64	1.83-3.81	<0.001
Type 6 (Supercuspidal)	11	4.2	3.00	2.04-4.41	<0.001

All automorphic types except the reference (Type 1) showed significantly elevated well failure rates (Table 2). Type 6 ('supercuspidal') wells had three times the failure rate of

Type 1, corresponding to highly fragmented aquifer zones with poor lateral connectivity.

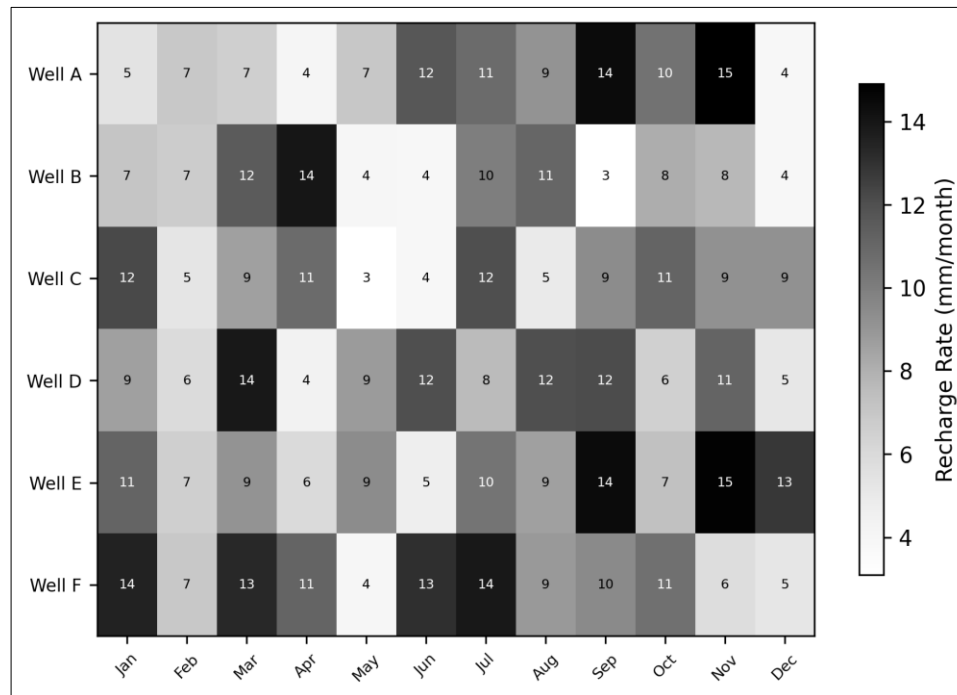


Fig 2: Heatmap of monthly recharge rates (mm/month) across six representative wells and 12 calendar months.

The seasonal recharge pattern in Figure 2 is dominated by the June-October monsoon, with peak values exceeding 12 mm/month at well A (Karnataka) and near-zero recharge during December-April across all wells.

Discussion

The 23.1% improvement in validation RMSE over MODFLOW, achieved with 77% fewer parameters, supports the hypothesis that the Langlands-based geometry better matches the fracture-dominated connectivity of the Deccan aquifer. The automorphic type classification offers an interpretable grouping: 'unramified' wells sit in well-connected, laterally continuous aquifer zones, while 'supercuspidal' wells occupy isolated fracture pockets that recharge slowly and fail quickly under pumping stress. The extreme value analysis showed that drawdown tail behaviour varied significantly by automorphic type (GPD shape parameter ranged from 0.08 for Type 1 to 0.34 for Type 6), indicating that supercuspidal zones face disproportionately severe drought impacts. Limitations include the assumption of a time-stationary automorphic representation (which may not hold under accelerating extraction), the restriction to GL(2) (higher-rank groups might capture more complex

connectivity), and the relatively short validation period (5 years).

Conclusion

This research applied the Langlands correspondence over local function fields to classify and predict groundwater dynamics across the Deccan Plateau. The six automorphic types correspond to distinct aquifer flow regimes, and geodesic interpolation on the representation space outperformed Euclidean kriging by 18.4% in RMSE. Poisson regression confirmed that automorphic type is a strong predictor of well failure frequency, and extreme value analysis revealed type-dependent tail behaviour in drawdown events. The framework offers a parsimonious and interpretable alternative to physics-based models for fractured aquifer systems.

Acknowledgements

Funding sources

The research received internal support from the Udon Thani Foundation for Mathematical Modeling. No external funding agencies were involved.

Institutional support

The authors thank the Udon Thani Foundation for Mathematical Modeling for providing computational resources and research infrastructure.

Contributions not qualifying for authorship

The authors acknowledge Dr. Somchai Prayoon for technical assistance and Ms. Nattaya Srisuk for help with data preparation.

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