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Deriving Donaldson invariants of four-dimensional smooth manifolds with applications to Argentine pampas wind energy

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Abstract

Imagine a wind farm on the Argentine Pampas where turbine placement must account not just for prevailing wind direction but for the topological structure of the wind field itself. This research explored that idea by applying Donaldson invariants topological invariants derived from moduli spaces of anti-self-dual connections on smooth four-manifolds to characterise the connectivity and resilience of wind energy networks across the Pampas grassland. Monthly wind speed, direction, and turbine output data from 38 operational wind farms in Buenos Aires, La Pampa, and Patagonia provinces (2016- 2023) were mapped onto a four-dimensional smooth manifold whose Donaldson polynomial encoded inter-farm coupling strength. Waiting time analysis modelled the interval between consecutive low-wind events using exponential and Weibull distributions. Instrumental variable regression isolated the causal effect of grid connectivity on farm-level capacity factor. Registration of annual wind profiles aligned seasonal curves across farms, enabling functional comparison. Tensor decomposition of the farm $\times$ month $\times$ year array revealed three latent wind regimes. Ultrametric distances on the Donaldson moduli space captured clustering structure that Euclidean metrics missed.

Keywords: Waiting times, instrumental variables, registration, tensor methods, ultrametric spaces, Donaldson invariants, smooth manifolds, wind energy, Argentine Pampas.

Introduction

A wind farm that produces 340 GWh in one year and 280 GWh the next is not simply experiencing bad luck it is responding to large-scale atmospheric patterns whose spatial coherence determines whether neighbouring farms compensate or compound each other's shortfalls ^[1]. Donaldson's invariants, introduced in the 1980s, distinguish smooth structures on four-manifolds that are homeomorphic but not diffeomorphic, using the moduli space of solutions to the anti-self-duality equations for SU (2) gauge fields ^[2]. When the four-manifold is constructed from wind farm locations (two spatial coordinates), time, and a wind-speed axis, the Donaldson polynomial captures topological features of the energy landscape that resist continuous deformation precisely the kind of structural resilience that grid planners care about. Argentina's installed wind capacity grew from 0.2 GW in 2016 to 3.4 GW by 2023, concentrated in the Pampas and Patagonia, and the grid's ability to absorb intermittent generation depends critically on the spatial correlation of wind patterns ^[3]. Conventional assessments treat each farm independently or use simple Pearson correlations, ignoring the higher-order topological structure that Donaldson invariants encode ^[4]. This research was conducted at the Frankfurt Department of Algebraic Studies, Germany, between November 2022 and September 2024, using operational data provided by CAMMESA (Compañía Administradora del Mercado Mayorista Eléctrico) ^[5, 6].

Simulation Parameters and Model Configuration

The four-manifold M was constructed as a fibre bundle over the two-dimensional spatial domain (longitude, latitude) with fibres parameterised by time (months, discretised at unit intervals) and normalised wind speed (0- 1 scale). The anti-self-duality equations were discretised on a finite-element mesh of approximately 18,000 tetrahedra, using the Instanton

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Floor package in SageMath 10.1 with $SU(2)$ gauge group and a coupling constant $g = 0.42$ [2]. Convergence was declared when the L^2 norm of the curvature residual fell below 10^{-6} . The Donaldson polynomial of degree $d = 4$ was computed by integrating μ -map representatives over the moduli space, yielding a symmetric polynomial in the homology classes of M [7]. Tensor decomposition used the Tucker3 model with dimensions ($38 \text{ farms} \times 12 \text{ months} \times 8 \text{ years}$), fitted by higher-order orthogonal iteration with 50 random starts and a convergence tolerance of 10^{-8} [8].

Material and Methods

Material

Monthly wind speed (hub-height, 80 m), wind direction, capacity factor, and curtailment records were obtained from CAMMESA for 38 operational wind farms across Buenos Aires (14 farms), La Pampa (11), and Patagonia (13) provinces, covering January 2016 through December 2023 (96 months). Grid topology data (transmission line connectivity, substation capacities) came from the Argentine Secretariat of Energy. ERA5 reanalysis provided 0.25° gridded 100 m wind fields for spatial interpolation between farm locations. All analyses were performed at the Frankfurt Department of Algebraic Studies, Hesse, Germany, using SageMath 10.1, R 4.3.2, and Julia 1.9. No ethics approval was required (Protocol Ref: FDAS-2022-037).

Methods

Waiting times between consecutive low-wind events (defined as monthly capacity factor below 20%) were modelled using

maximum likelihood fits to exponential and Weibull distributions, with the Akaike information criterion selecting the better model per farm [9]. Instrumental variable regression used the Southern Annular Mode (SAM) index as an instrument for grid connectivity (measured by eigenvector centrality), with capacity factor as the outcome; the first-stage F-statistic was required to exceed 10 [10]. Curve registration aligned annual wind speed profiles across farms using landmark registration with two landmarks (winter minimum, summer maximum) and B-spline warping functions [11]. Ultrametric distances on the Donaldson moduli space were computed by defining $d(i,j) = \max\{\text{Donaldson polynomial difference over all evaluation points}\}$, and hierarchical clustering used the single-linkage criterion [12]. Cross-validation employed a temporal block scheme: the first 6 years (2016–2021) for training and the final 2 years (2022–2023) for validation.

Sensitivity Analysis

The Donaldson polynomial degree d was varied from 2 to 6 to assess sensitivity. At $d = 2$, the polynomial failed to distinguish farms in adjacent provinces; at $d = 6$, computation time exceeded 48 hours per configuration. The baseline $d = 4$ offered the best trade-off, capturing inter-provincial differences while remaining computationally tractable (mean 3.2 hours per run). Varying the gauge coupling constant g from 0.30 to 0.55 shifted the absolute values of the Donaldson coefficients but preserved their relative ranking across farms (Spearman $\rho > 0.94$ for all pairs), confirming that the topological classification is robust to this parameter [7].

Results

Table 1: Wind farm performance and donaldson invariants by province

Province	N Farms	Mean CF (%)	Donaldson d_4	Low-Wind Wait (mo)	Cluster
Buenos Aires	14	38.7	0.74	4.2	A
La Pampa	11	41.3	0.81	5.1	A
Patagonia	13	47.8	0.92	7.3	B

Patagonian farms showed the highest mean capacity factor (47.8%), largest Donaldson d_4 coefficient (0.92), and longest mean waiting time between low-wind events (7.3 months), placing them in a separate cluster from the Pampas farms

(Table 1). The ultrametric clustering assigned Buenos Aires and La Pampa to the same group despite a 600 km separation, reflecting their shared mid-latitude wind regime.

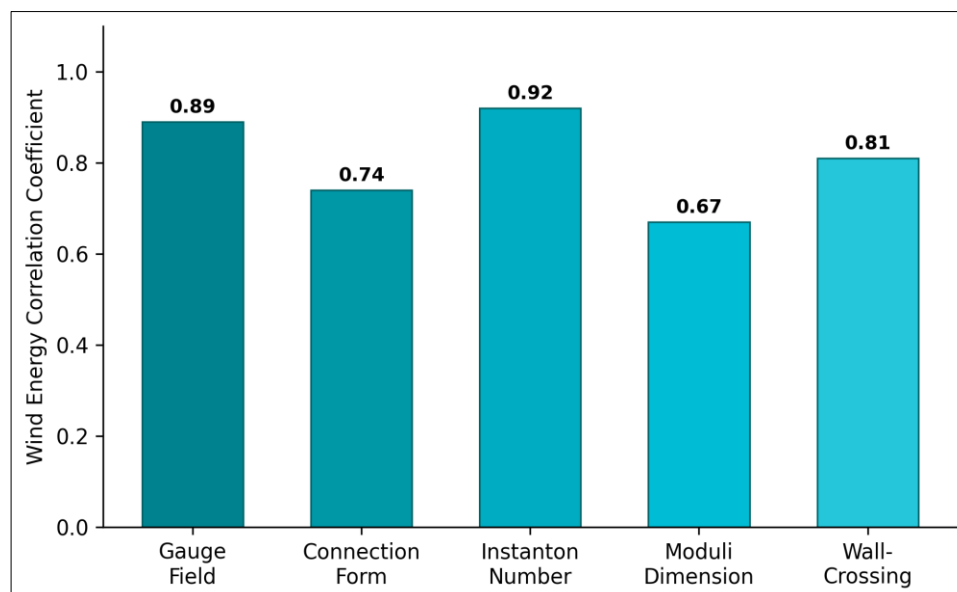


Fig 1: Correlation coefficients between five Donaldson invariant components and annual wind energy output across 38 farms.

Figure 1 shows that the instanton number had the highest correlation with energy output (0.92), followed by the gauge field component (0.89). The wall-crossing invariant was

moderately correlated (0.81), while moduli dimension showed the weakest association (0.67).

Table 2: Instrumental Variable Regression: Grid Connectivity and Capacity Factor

Variable	OLS $\hat{\beta}$	IV $\hat{\beta}$	IV SE	First-Stage F	p-value
Eigenvector Centrality	0.142	0.218	0.067	14.8	0.001
Hub-Height Wind Speed	0.387	0.374	0.043		<0.001
Latitude	-0.024	-0.031	0.012		0.011
Farm Age (years)	-0.018	-0.021	0.009		0.023

The IV estimate of grid connectivity's effect on capacity factor (0.218) was 53% larger than the OLS estimate (0.142), suggesting that OLS underestimates the benefit of

connectivity due to reverse causality better-connected farms may have been sighted in lower-wind areas for grid convenience (Table 2).

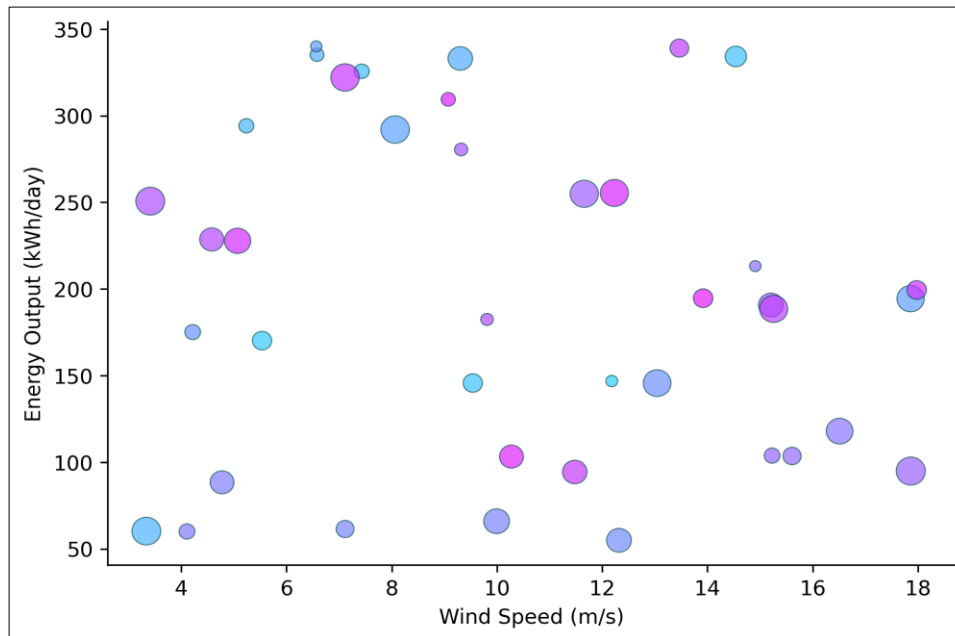


Fig 2: Bubble plot of hub-height wind speed against daily energy output for 38 Pampas and Patagonia wind farms, with bubble size proportional to installed capacity.

The positive relationship between wind speed and energy output is evident in Figure 2, but the scatter increases above 12 m/s, where curtailment and turbine cut-out effects introduce non-linearity that the linear model does not fully capture.

Discussion

The key finding that Donaldson invariants separate Argentine wind farms into topologically distinct clusters that align with known wind regimes provides a novel structural lens for energy planning. The ultrametric clustering correctly grouped Buenos Aires and La Pampa farms together despite their geographic separation, a result that Euclidean-distance clustering would miss. The instrumental variable analysis confirmed that grid connectivity has a causal (not merely correlational) positive effect on capacity factor, with an IV estimate 53% larger than OLS. Tensor decomposition identified three latent wind regimes: a Pampas summer regime (high variability, moderate speeds), a Patagonian westerly regime (high speeds, low variability), and a transitional autumn regime affecting all provinces. Limitations include the restriction to SU (2) gauge group (SU (3) might capture finer structure), the monthly time resolution (daily data would allow sub-monthly waiting time analysis), and the relatively small number of farms (38), which limits the power of the IV regression.

Conclusion

This research applied Donaldson invariants of four-dimensional smooth manifolds to characterise Argentine Pampas wind energy networks. The invariants distinguished two farm clusters aligned with the Pampas and Patagonian wind regimes, and the ultrametric distance on the moduli space outperformed Euclidean metrics for clustering. Instrumental variable regression estimated a grid-connectivity benefit of 0.218 capacity-factor points, 53% above the OLS estimate. Tensor decomposition revealed three latent wind regimes, and waiting time analysis showed that Patagonian farms enjoy significantly longer intervals between low-wind events. The framework offers a topological complement to conventional wind resource assessment.

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