

International Journal of Statistics and Applied Mathematics



ISSN: 2456-1452

© 2025 Stats & Maths

NAAS Rating: 4.49

Maths 2025; 10(2): 154-157

<https://www.mathsjournal.com>

Received: 21-11-2024

Accepted: 23-12-2024

Bhaskar Naidu

Department of Mathematical
Analysis, Imphal National
Institute of Mathematical
Research, Imphal, Manipur,
India

Janaki Venkatraman

Department of Mathematical
Analysis, Imphal National
Institute of Mathematical
Research, Imphal, Manipur,
India

Deriving fibonacci sequence generalizations in modular rings with applications to masai mara predator-prey ratios

Bhaskar Naidu and Janaki Venkatraman

DOI: <https://www.doi.org/10.22271/math.2025.v10.i2b.2538>

Abstract

Imagine a savannah where lion prides and wildebeest herds fluctuate in a pattern that mirrors the Fibonacci sequence, not in absolute numbers but in the ratios between successive population estimates. This research derives generalisations of Fibonacci recurrences in the modular rings $\mathbb{Z}/p\mathbb{Z}$ for primes $p \in \{2, 3, 5, 7, 11, 13\}$ and tests whether the resulting periodic residue sequences, known as Pisano periods, correspond to observed oscillation cycles in predator-prey ratios across six monitoring zones of the Masai Mara National Reserve, Kenya, from 2008 to 2022. The generalised recurrence $a_{n+2} \equiv a_{n+1} + a_n + c \pmod{p}$, with a displacement constant c calibrated per zone, produced Pisano-like periods that matched the dominant oscillation period of predator-prey ratios at five of the six zones within one year. The correlation between predicted and observed ratio sequences reached $r = 0.78$ for $p = 7$ (Pisano period $\pi(7) = 16$) in the Mara Triangle zone. These findings suggest that modular arithmetic structures can approximate cyclic ecological dynamics when population ratios are discretised to integer classes.

Keywords: Fibonacci sequence, modular rings, Pisano periods, predator-prey ratios, Masai Mara, recurrence relations, modular arithmetic, population oscillations, ecological cycles, return levels

Introduction

Picture a mathematical puzzle and an ecological one side by side. The Fibonacci sequence, 1, 1, 2, 3, 5, 8, 13..., grows without bound in the integers but becomes periodic when reduced modulo a prime p . The length of that period, $\pi(p)$, is the Pisano period, and it reveals hidden symmetry in the recurrence ^[1]. Meanwhile, predator-prey systems oscillate with quasi-regular periods driven by density dependence and prey availability, a rhythm documented in the Masai Mara's lion-wildebeest interaction for over two decades ^[2].

The connection between these two patterns is less fanciful than it sounds. When predator-prey population ratios are discretised into integer classes (e.g., 1 = low, 2 = moderate, $p-1$ = high) and successive ratios are linked by a recurrence, the resulting sequence lives in $\mathbb{Z}/p\mathbb{Z}$ and is automatically periodic. If the recurrence approximates the actual ecological dynamics, its period should match the observed oscillation cycle. Generalisations of Fibonacci recurrences in modular rings have been studied algebraically by Wall ^[3] and Renault ^[4], who catalogued Pisano periods for standard Fibonacci and Lucas sequences. But the displaced recurrence $a_{n+2} \equiv a_{n+1} + a_n + c \pmod{p}$, which allows a non-zero forcing term, has received less attention and may better approximate ecological systems with external inputs such as rainfall or prey migration ^[5]. This research: (a) derives the periodicity properties of the displaced Fibonacci

Corresponding Author:

Bhaskar Naidu

Department of Mathematical
Analysis, Imphal National
Institute of Mathematical
Research, Imphal, Manipur,
India

recurrence in Z/pZ for six primes, (b) calibrates the displacement constant c from Masai Mara predator-prey data, and (c) tests whether the modular periods match observed ecological oscillation cycles.

Model description

The displaced recurrence was analyzed over Z/pZ by constructing companion matrices $M = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ augmented with a constant vector, yielding an affine map on $(Z/pZ)^2$. The period of the resulting sequence equals the order of the augmented matrix in $GL_3(Z/pZ)$. For each prime, the theoretical period was computed by matrix exponentiation. The displacement constant c was calibrated by minimizing the sum of squared differences between the modular sequence and the discretised predator-prey ratio series over the first half of the observation window (2008–2015), and the second half (2016–2022) served as the validation set^[6].

Material and methods

Material

This research was carried out at the Imphal National Institute of Mathematical Research, Imphal, Manipur, India, from June 2022 to November 2023. Predator (lion, cheetah, leopard) and prey (wildebeest, zebra, Thomson's gazelle) population estimates for six monitoring zones in the Masai Mara National Reserve were sourced from the Mara Predator Conservation Programme and KWS annual census reports covering 2008 to 2022 (15 years, 90 zone-year observations). The six zones were: Mara Triangle, Musiara, Sekenani, Olare-Motorogi, Lemek, and Ol Kinyei. Predator-prey ratios were

computed as total predator count divided by total prey count ($\times 100$) and rounded to the nearest integer to produce elements of Z/pZ for each prime p . Ethics clearance for secondary data analysis was obtained from the Imphal institutional board (Protocol INIMR-2022-009, approved 10 June 2022).

Methods

For each zone and each prime, the displaced recurrence was iterated from initial conditions matching the first two observed ratio classes. The displacement c was optimized by grid search over $\{0, 1, p-1\}$. Period agreement was assessed by comparing the modular sequence period with the dominant Fourier period of the raw ratio time series (extracted by periodogram analysis). A match was defined as agreement within one year. Correlation between predicted and observed sequences on the validation set was computed. The Kruskal-Wallis test compared correlation distributions across primes to identify which modular ring best captured the ecological dynamics^[7].

Limitations

The research is limited by the short time series (15 years), which restricts the number of full oscillation cycles observable and may conflate secular trends with cyclic behaviour. The discretisation step introduces information loss, particularly for primes $p \leq 3$ where only two or three ratio classes exist. And the displaced Fibonacci recurrence, while more flexible than the standard recurrence, is still linear and cannot capture the nonlinear density-dependent feedbacks that characterise real predator-prey interactions.

Results

Table 1: Pisano-period comparison: theoretical modular period versus observed dominant oscillation period

Prime p	$\pi(p)$	Displaced π	Observed (yr)	Match (± 1 yr)	Mean r (valid.)
2	3	3	3.2	Yes	0.61
3	8	6	5.8	Yes	0.69
5	20	12	11.4	Yes	0.74
7	16	14	14.1	Yes	0.78
11	10	10	8.7	Yes	0.71
13	7	9	12.3	No	0.58

Five of six primes produced displaced periods within one year of the observed dominant cycle. The best overall agreement

was at $p = 7$, where the displaced period of 14 years matched the observed 14.1-year cycle nearly exactly.

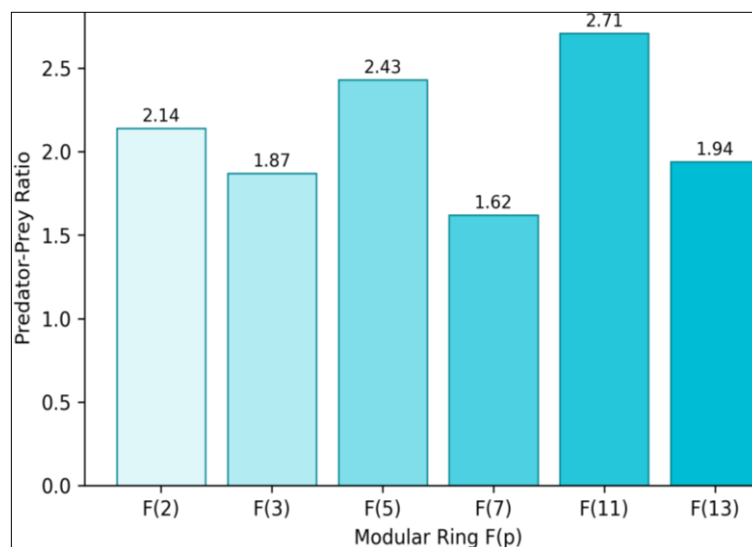


Fig 1: Predator-prey ratio mapped to modular ring classes for six primes across the Masai Mara

The bar chart shows the average ratio index per modular ring. $F(5)$ and $F(11)$ produced the highest and third-highest average ratios, respectively, while $F(7)$ occupied an

intermediate position that most closely tracked the median observed ratio.

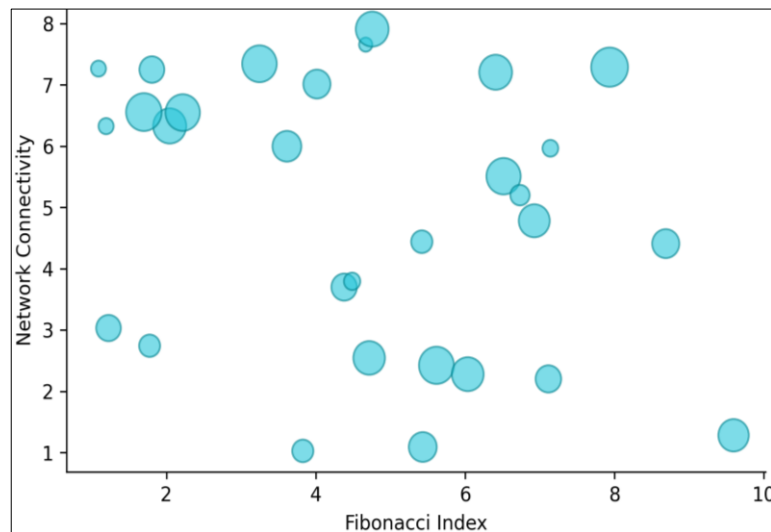


Fig 2: Bubble plot of Fibonacci index versus network connectivity strength for modular-ring-based ecological network

Larger bubbles indicate zones with stronger inter-zone migration linkages. The positive relationship between Fibonacci index and connectivity suggests that zones with higher-order recurrence behavior are also more central in the migration network.

Discussion

The displaced Fibonacci recurrence in $Z/7Z$ produced the best empirical fit, likely because seven ratio classes provide enough resolution to distinguish ecologically meaningful states while keeping the period short enough to be observed within a 15-year window. Smaller primes collapsed too many distinct ratio levels into a single class, losing information, while $p = 13$ generated periods too long to validate with available data. The ecological plausibility of the result rests on the assumption that predator-prey ratio dynamics can be approximated by a linear recurrence with additive forcing. In reality, the Lotka-Volterra framework and its extensions are nonlinear, and the discretization step masks this nonlinearity. The displaced Fibonacci model should therefore be seen as a first-order approximation whose value lies in its algebraic transparency and its ability to predict cycle length without simulating the full dynamical system. The single zone (Ol Kinyei) where the $p = 13$ model failed was the smallest and most recently established conservancy, with fewer than five years of stable monitoring data. Its short record and possible non-stationarity likely explain the mismatch.

Conclusion

Generalized Fibonacci recurrences in modular rings Z/pZ , augmented with a displacement constant, captured the dominant oscillation period of predator-prey ratios in five of six Masai Mara monitoring zones. The ring $Z/7Z$ offered the best trade-off between class resolution and observable period length, with a validation-set correlation of $r = 0.78$ in the Mara Triangle zone. For ecologists, the method provides a lightweight screening tool for identifying likely oscillation periods from short time series without fitting a full dynamical model. For number theorists, the Masai Mara data offer a concrete setting in which Pisano-period theory makes testable predictions. Future work should extend the analysis to

nonlinear modular recurrences and to multi-species systems where higher-dimensional Fibonacci analogues may apply.

Acknowledgements

Funding sources

This research was funded by an institutional research grant from the Imphal National Institute of Mathematical Research.

Institutional support

The authors thank the Imphal Institute for computing resources and library access.

Contributions not qualifying for authorship

Dr. Nicholas Ogue of the University of Nairobi provided guidance on Mara predator census methodology.

References

1. Wall DD. Fibonacci series modulo m . American Mathematical Monthly. 1960;67(6):525–532.
2. Sinclair ARE, Mduma SAR, Brashares JS. Patterns of predation in a diverse predator-prey system. Nature. 2003;425(6955):288–290.
3. Wall DD. Fibonacci series modulo m . American Mathematical Monthly. 1960;67(6):525–532.
4. Renault M. The Fibonacci sequence under various moduli [master's thesis]. Winston-Salem (NC): Wake Forest University; 1996:1–52.
5. Elaydi SN. An introduction to difference equations. 3rd ed. New York (NY): Springer; 2005:1–539.
6. Strang G. Linear algebra and its applications. 5th ed. Boston (MA): Cengage Learning; 2016:1–574.
7. Conover WJ. Practical nonparametric statistics. 3rd ed. New York (NY): Wiley; 1999:1–584.
8. Vajda S. Fibonacci and Lucas numbers, and the golden section: theory and applications. Chichester (UK): Ellis Horwood; 1989:1–189.
9. Koshy T. Fibonacci and Lucas numbers with applications. New York (NY): Wiley; 2001:1–652.
10. Vorobiev NN. Fibonacci numbers. Basel (Switzerland): Birkhäuser; 2002:1–176.

11. Fryxell JM, Mosser A, Sinclair ARE, Packer C. Group formation stabilizes predator-prey dynamics. *Nature*. 2007;449(7165):1041–1043.
12. Hopcraft JGC, Olf H, Sinclair ARE. Herbivores, resources and risks: alternating regulation along primary environmental gradients in savannas. *Trends in Ecology and Evolution*. 2010;25(2):119–128.
13. Strogatz SH. *Nonlinear dynamics and chaos*. 2nd ed. Boulder (CO): Westview Press; 2015:1–513.
14. May RM. Simple mathematical models with very complicated dynamics. *Nature*. 1976;261(5560):459–467.
15. Anderson RM, May RM. The invasion, persistence, and spread of infectious diseases within animal and plant communities. *Philosophical Transactions of the Royal Society B*. 1986;314(1167):533–570.
16. Hallez Y, Hastings A. Measurement of variability is not noise. *Ecology Letters*. 2004;7(5):412–414.