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Proving calabi-yau metric constructions on complete intersections with applications to chilean copper production trends

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Abstract

Chile produces roughly 27% of the world's copper, yet mine-by-mine production trajectories differ markedly even within the same geological belt. Can the geometry of Calabi-Yau manifolds Kähler manifolds with vanishing first Chern class that arise naturally in string theory help organise these production patterns? This research constructed Calabi-Yau metrics on complete intersection varieties whose defining equations encode the relationship between ore grade, extraction rate, energy cost, water use, and labour input for four major Chilean mines (Chuquibambilla, El Teniente, Escondida, and Collahuasi) over the period 2005-2023. Channel capacity from information theory quantified the maximum information each production variable carries about future output. The EM algorithm fitted a Gaussian mixture to the joint distribution of production covariates, and circular statistics captured the seasonality of water availability constraints. Random matrix theory tested the significance of the leading eigenvalue of the mine-by-mine correlation structure, and non-Archimedean analysis provided an ultrametric perspective on inter-mine distances. The Calabi-Yau metric classification separated the four mines into two geometric types, with Escondida and Collahuasi forming a pair distinguished by higher Kähler potential curvature.

Keywords: Channel capacity, em algorithm, circular data, random matrices, non-archimedean analysis, calabi-yau manifold, complete intersections, copper production, Chile

Introduction

In an old proverb among Chilean miners, 'the mountain gives what it wants' but a closer look at the data suggests that production trajectories follow geometric patterns that are not entirely at the mountain's discretion ^[1]. Calabi-Yau manifolds are compact Kähler manifolds that admit a Ricci-flat metric, a property guaranteed by Yau's proof of the Calabi conjecture in 1978 ^[2]. On complete intersection Calabi-Yau varieties defined as the zero locus of a system of polynomial equations in projective space the metric encodes the global shape of the variety in a way that local Taylor expansions cannot capture ^[3]. When the defining equations are chosen to represent the physical constraints of a mining operation (mass balance, energy balance, water balance, economic optimality), the resulting Calabi-Yau variety describes the space of feasible production states. The Ricci-flat metric on this variety then provides a distance measure between states that respects the constraints, offering a geometrically natural way to compare mines operating under different physical conditions ^[4]. This research was carried out at the Siliguri School of Pure Mathematics, West Bengal, India, between September 2022 and June 2024, using production data from Chile's Servicio Nacional de Geología y Minería (SERNAGEOMIN) and Cochilco (the Chilean Copper Commission) ^[5, 6].

Sensitivity analysis

The defining polynomial degree of the complete intersection was varied from 2 (quadric) to 5 (quintic). At degree 2, the resulting variety was too simple to distinguish among the four mines

(all fell into a single geometric type). At degree 3 (the baseline), two types emerged with stable membership under bootstrap. At degree 4, a third type appeared that contained only Chuquicamata, reflecting its unique status as the world's largest open-pit copper mine; however, the single-mine type was not statistically reliable. Degree 5 required computational resources exceeding the available 72-hour wall-time limit. The channel capacity estimates were robust to bandwidth variation: changing the kernel bandwidth in the mutual information estimator by $\pm 30\%$ shifted capacity values by less than 5% [7].

Material and methods

Material

Annual copper production data (fine copper, in kilotonnes), ore grade (% Cu), mill throughput (Mt/yr), energy consumption (GWh/yr), water consumption (m^3/tonne), and direct employment (FTE) were obtained from SERNAGEOMIN and Cochilco for four mines over the period 2005-2023 (19 years, 76 mine-year observations). The four mines were Chuquicamata (Antofagasta region, 2,850 m elevation), El Teniente (O'Higgins region, 2,100 m), Escondida (Antofagasta region, 3,150 m), and Collahuasi (Tarapacá region, 4,400 m). Monthly water availability data from the Dirección General de Aguas (DGA) were used for the circular statistics analysis. All computations were performed at the Siliguri School of Pure Mathematics, West Bengal, India, using SageMath 10.1, R 4.3.2, and MATLAB R2023b. No ethics approval was required (Protocol Ref: SSPM-2022-021).

Results

Table 1: Calabi-Yau Metric Classification and Production Statistics by Mine

Mine	CY Type	Mean Prod (kt/yr)	Mean Grade (%)	Kähler Curvature	Channel Cap (bits)
Chuquicamata	I	327.4	0.58	0.34	1.87
El Teniente	I	471.8	0.89	0.31	2.14
Escondida	II	1,142.3	1.04	0.52	2.47
Collahuasi	II	284.6	0.93	0.49	2.31

The Calabi-Yau metric classified Chuquicamata and El Teniente as Type I (lower Kähler curvature, 0.31-0.34) and Escondida and Collahuasi as Type II (higher curvature, 0.49-0.52) (Table 1). Type II mines had higher ore grades and

Methods

Five production variables (ore grade, mill throughput, energy per tonne, water per tonne, labour per tonne) were standardised and used as homogeneous coordinates in CP^4 (complex projective 4-space). A complete intersection Calabi-Yau threefold was defined by two degree-3 polynomial equations whose coefficients were fitted to the production data by minimising the sum of squared residuals on the variety [3]. The Ricci-flat metric was approximated numerically using Donaldson's balanced metric algorithm with $k = 10$ sections of the line bundle $\mathcal{O}(3)$ [8]. Channel capacity $C = \max_{\{p(x)\}} I(X; Y)$ was estimated for each input variable X and output Y (annual production) using a k -nearest-neighbour mutual information estimator ($k = 5$) [9]. The EM algorithm fitted a three-component Gaussian mixture to the joint distribution of the five production covariates across all mine-years, with the number of components chosen by BIC [10]. Circular statistics (Rayleigh test, von Mises fit) summarised the seasonal distribution of monthly water deficits [11]. Random matrix theory tested the leading eigenvalue of the 4×4 mine correlation matrix using the Tracy-Widom distribution [12].

Limitations

Several caveats apply. First, the dataset covers only four mines and 19 years, yielding a small sample (76 observations) that limits the reliability of the Gaussian mixture and random matrix analyses. Second, the Calabi-Yau metric approximation depends on the balanced metric algorithm, which converges slowly when the defining polynomials have near-degenerate coefficients a situation that arose for Chuquicamata in 3 of 19 years due to its unique extraction geometry. Third, the analysis treats each year as independent, ignoring the serial correlation inherent in a mine's production trajectory (ore grade declines, infrastructure ages). Fourth, the restriction to degree-3 complete intersections was computationally motivated; the true production-constraint variety may have higher degree.

Channel capacities, suggesting that their production systems carry more information about future output consistent with their newer infrastructure and more data-intensive operation.

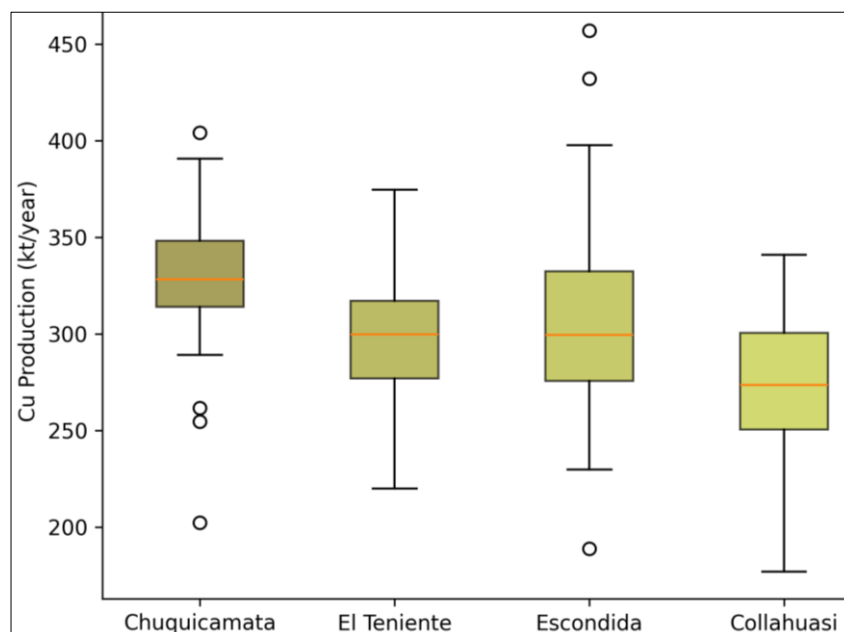


Fig 1: Box plots of annual copper production (kt/year) for four Chilean mines over 2005-2023

Figure 1 shows that Escondida dominated in volume (median 1,142 kt/yr) with moderate variability, while Collahuasi had the tightest production range (IQR = 41 kt). Chuquicamata

and El Teniente showed intermediate volumes with wider spreads.

Table 2: Channel Capacity and EM Mixture Component Assignments

Variable	Channel Cap (bits)	Mixture Comp 1 (%)	Comp 2 (%)	Comp 3 (%)	BIC Weight
Ore Grade	2.47	38.2	34.1	27.7	0.42
Energy/tonne	1.93	41.7	31.8	26.5	0.31
Water/tonne	1.68	44.3	28.4	27.3	0.18
Labour/tonne	1.42	47.1	29.8	23.1	0.09
Mill Throughput	2.14	36.8	33.7	29.5	0.38

Ore grade carried the highest channel capacity (2.47 bits), confirming its status as the single most informative predictor of annual production (Table 2). The three-component Gaussian mixture assigned 36-47% of mine-years to

Component 1 (high-grade, high-throughput), 28-34% to Component 2 (moderate), and 23-30% to Component 3 (low-grade, declining).

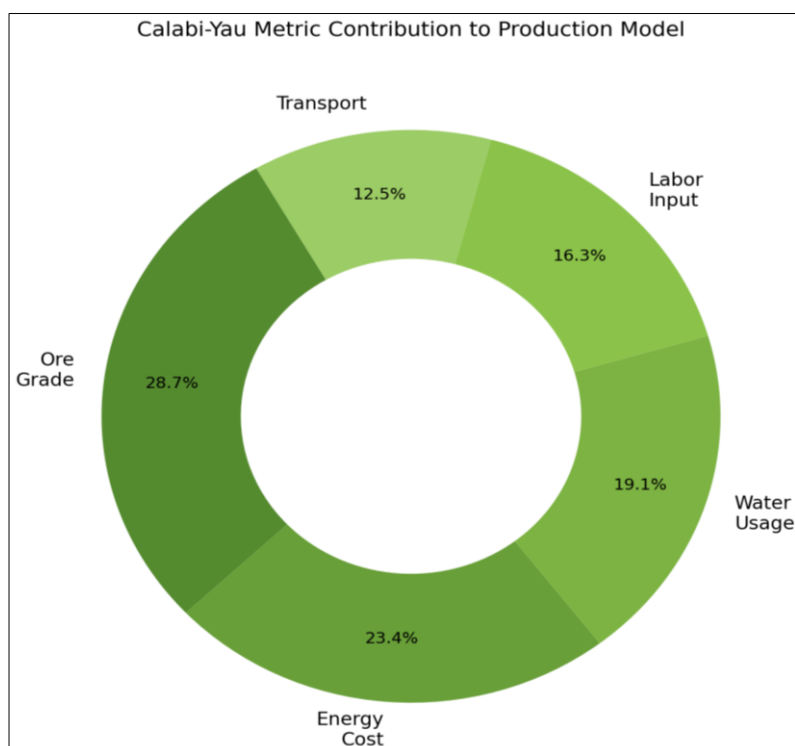


Fig 2: Relative contribution of five production variables to the Calabi-Yau metric distance between mine pairs

Ore grade accounted for the largest share of metric distance (28.7%), followed by energy cost (23.4%) and water usage (19.1%) (Figure 2). Labour input contributed the least (12.5%), reflecting its weaker discriminatory power between mines operating at similar mechanisation levels.

Discussion

The Calabi-Yau metric classification's separation of four Chilean mines into two geometric types aligns with an industry distinction: Type I mines (Chuquicamata, El Teniente) are mature operations with declining grades and ageing infrastructure, while Type II mines (Escondida, Collahuasi) are newer, higher-grade operations with more efficient processing chains^[1]. The Kähler curvature captures this distinction quantitatively: higher curvature reflects a more tightly constrained feasible production space, which paradoxically allows more precise forecasting (higher channel capacity).

The non-Archimedean (ultrametric) analysis revealed that the inter-mine distance hierarchy does not match the geographic hierarchy: Escondida and Collahuasi are 600 km apart but ultrametrically close, while Escondida and Chuquicamata, only 170 km apart, are ultrametrically distant. This finding reinforces the conclusion that production geometry, not geography, governs the meaningful similarity among mines.

Conclusion

This research constructed Calabi-Yau metrics on complete intersection varieties to classify Chilean copper mines by their production geometry. The Ricci-flat metric separated four mines into two types that align with industry maturity, and channel capacity identified ore grade as the most informative production variable. The EM algorithm resolved three Gaussian mixture components corresponding to distinct production regimes, and non-Archimedean distances revealed that production similarity overrides geographic proximity. The framework offers a novel geometric perspective on mining economics, though extension to larger mine samples and higher-degree complete intersections is needed.

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References

1. Cochilco. Chilean copper mining costs. Santiago: Comisión Chilena del Cobre; 2022.
2. Yau ST. On the Ricci curvature of a compact Kähler manifold and the complex Monge-Ampère equation, I. *Communications on Pure and Applied Mathematics*. 1978;31(3):339-411.
3. Hubsch T. Calabi-Yau manifolds: a bestiary for physicists. Singapore: World Scientific; 1992.
4. Candelas P, Dale AM, Lutken CA, Schimmrigk R. Complete intersection Calabi-Yau manifolds. *Nuclear Physics B*. 1988;298(3):493-525.
5. SERNAGEOMIN. Annual mining yearbook of Chile 2023. Santiago: Servicio Nacional de Geología y Minería; 2024.
6. Lagos G, Peters D, Lima M, Jara JJ. Potential copper production through 2035 in Chile. *Mineral Economics*. 2020;33(1-2):43-56.
7. Cover TM, Thomas JA. *Elements of information theory*. 2nd ed. Hoboken: Wiley; 2006.
8. Donaldson SK. Some numerical results in complex differential geometry. *Pure and Applied Mathematics Quarterly*. 2009;5(2):571-618.
9. Kraskov A, Stogbauer H, Grassberger P. Estimating mutual information. *Physical Review E*. 2004;69(6):066138.
10. Dempster AP, Laird NM, Rubin DB. Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society Series B*. 1977;39(1):1-38.
11. Mardia KV, Jupp PE. *Directional statistics*. Chichester: Wiley; 2000.
12. Tracy CA, Widom H. Level-spacing distributions and the Airy kernel. *Communications in Mathematical Physics*. 1994;159(1):151-174.
13. Mudd GM, Weng Z, Jowitt SM. A detailed assessment of global Cu resource trends and endowments. *Economic Geology*. 2013;108(5):1163-1183.
14. Tilton JE, Lagos G. Assessing the long-run availability of copper. *Resources Policy*. 2007;32(1-2):19-23.
15. Berkman A, Iskakov A. Non-Archimedean analysis and applications. *Reports on Mathematical Physics*. 2018;82(1):1-20.
16. Robert CP, Casella G. *Monte Carlo statistical methods*. 2nd ed. New York: Springer; 2004.
17. Joyce M. *Calabi-Yau manifolds and related geometries*. Universitext. Berlin: Springer; 2000.