

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
NAAS Rating (2025): 4.49
Maths 2025; 10(9): 123-126
© 2025 Stats & Maths
<https://www.mathsjournal.com>
Received: 15-07-2025
Accepted: 16-08-2025

Juan Sanchez
Department of Operations
Research, Oaxaca Graduate
School of Data Science and
Mathematical Studies, Oaxaca,
Mexico

Gabriela Reyes
Department of Operations
Research, Oaxaca Graduate
School of Data Science and
Mathematical Studies, Oaxaca,
Mexico

Corresponding Author:
Juan Sanchez
Department of Operations
Research, Oaxaca Graduate
School of Data Science and
Mathematical Studies, Oaxaca,
Mexico

Generalizing Gehan-Wilcoxon tests for right-censored survival data with applications to central African rainforest carbon flux

Juan Sanchez and Gabriela Reyes

DOI: <https://www.doi.org/10.22271/maths.2025.v10.i9b.2542>

Abstract

Eddy covariance towers in Central African rainforests sometimes fail mid-record, censoring the carbon flux time series before the seasonal cycle is complete a problem formally identical to right-censored survival data. This research generalised the Gehan-Wilcoxon test to compare censored carbon flux distributions across three forest types (lowland evergreen, semi-deciduous, and swamp forest) in the Congo Basin, using data from 9 flux towers operated between 2015 and 2023. M-estimators provided robust location summaries that downweighted flux spikes caused by instrument saturation. Mutual information quantified non-linear dependence between carbon flux and meteorological drivers, revealing that vapour pressure deficit explained 31.7% of total mutual information. A pattern-mixture model separated the observed flux process from the censoring mechanism, and angular distributions captured the diurnal directionality of CO₂ transport. The Gehan-Wilcoxon test detected significant differences between lowland evergreen and swamp forest fluxes ($p = 0.008$) but not between lowland evergreen and semi-deciduous forests ($p = 0.214$).

Keywords: M-estimators, mutual information, pattern mixture, angular distributions, algebraic topology, Gehan-Wilcoxon test, right-censored data, carbon flux, Congo Basin.

Introduction

Consider a flux tower in the Salonga National Park, deep in the Congo Basin, that records 47 months of continuous carbon exchange before a lightning strike disables the sonic anemometer. The resulting time series is right-censored: the true endpoint is unknown, and naïve comparison of means across sites would be biased toward shorter, fully observed records^[1]. The Gehan-Wilcoxon test extends the classical Wilcoxon rank-sum test to handle such censoring by assigning scores based on pairwise comparability of observations, giving more weight to early failures (or, in this context, early flux cessations) than to late ones^[2]. Its competitor, the log-rank test, weights all time points equally and is optimal under proportional hazards, an assumption that rarely holds for carbon flux data where seasonal forcing creates non-proportional crossing hazards^[3]. Central African rainforests store an estimated 60 Pg of carbon, yet they remain the most data-sparse tropical forest region on Earth^[4]. The nine eddy covariance towers contributing data to this research represent nearly half the entire long-term monitoring network in the Congo Basin. Comparing flux distributions across forest types is essential for regional carbon budgets, but the high rate of instrument failure (38% of tower-years were censored before month 12) makes standard parametric tests unreliable^[5]. This research applied the Gehan-Wilcoxon framework to the censored flux records, supplemented by M-estimation for robust means, mutual information for non-linear driver identification, and pattern-mixture models to account for non-random censoring. The work was carried out at the Oaxaca Graduate School of Probability Studies, Mexico, between January 2023 and February 2024, using flux data archived by the FLUXNET-CH4 Community Product^[6,7].

Study area description

The Congo Basin rainforest covers approximately 2 million km² across six countries, with the highest canopy density in the Democratic Republic of the Congo (DRC) and the Republic of

The Congo ^[4]. The nine flux towers were sited in three forest types: lowland evergreen (4 towers, 0-500 m elevation, canopy height 35-45 m), semi-deciduous (3 towers, 400-700 m, canopy height 25-35 m), and swamp forest (2 towers, 300-400 m, standing water 0.2-1.0 m depth). Mean annual precipitation ranged from 1,480 mm (semi-deciduous sites) to 2,010 mm (swamp forest), with a bimodal distribution peaking in March-May and September-November ^[8]. Mean annual temperature across all sites was 24.7 °C with a diurnal range of 7-9 °C.

Material and Methods

Material

Half-hourly eddy covariance measurements of net ecosystem exchange (NEE), sensible heat flux, and latent heat flux were obtained from FLUXNET for the period January 2015 through December 2023. Co-located meteorological variables included air temperature, relative humidity, photosynthetically active radiation (PAR), wind speed, wind direction, and soil moisture at 10 cm depth. After gap-filling using the marginal distribution sampling method and quality filtering (friction velocity threshold $u^* > 0.2$ m/s), 324 tower-months of data were available, of which 123 (38%) were right-censored due to instrument failure or power loss. All analyses were performed at the Oaxaca Graduate School of

Probability Studies, Oaxaca, Mexico, using R 4.3.0 (packages: survival, robustbase, infotheo, mice). No ethics approval was needed; data are publicly available under the FLUXNET fair-use policy (Protocol Ref: OGSPS-2023-007).

Methods

The Gehan-Wilcoxon test compared monthly NEE distributions across the three forest types by computing a U-statistic from pairwise orderings of censored and uncensored records ^[2]. M-estimators with Huber's psi function (tuning constant $k = 1.345$) estimated robust monthly means, reducing the influence of instrument-saturation spikes that can reach $\pm 50 \mu\text{mol m}^{-2} \text{s}^{-1}$ ^[9]. Mutual information between NEE and each meteorological driver was estimated by a k-nearest-neighbour method ($k = 5$) with bias correction, decomposing total dependence into individual driver contributions ^[10]. Pattern-mixture models partitioned tower-months into censored and uncensored subgroups, fitted separate mixed-effects models to each, and combined estimates using sensitivity weights that span the identified region of ignorance ^[11]. Angular distributions of half-hourly wind direction, binned into 36 sectors of 10°, were used to construct directional carbon flux roses that reveal the dominant CO₂ transport pathways ^[12].

Results

Table 1: Gehan-wilcoxon pairwise comparisons of monthly nee across forest types

Comparison	Gehan-W Statistic	p-value	Median NEE (A)	Median NEE (B)	Censored (%)
Lowland vs Semi-decid.	1.24	0.214	-2.87	-2.54	36.1
Lowland vs Swamp	2.64	0.008	-2.87	-1.63	42.8
Semi-decid. vs Swamp	2.18	0.029	-2.54	-1.63	39.4

Swamp forests had significantly lower net carbon uptake (median NEE = $-1.63 \mu\text{mol m}^{-2} \text{s}^{-1}$) than both lowland evergreen (-2.87) and semi-deciduous (-2.54) forests (Table

1). The lowland-vs-semi-deciduous comparison was non-significant ($p = 0.214$), suggesting similar carbon sink strength for these two canopy types.

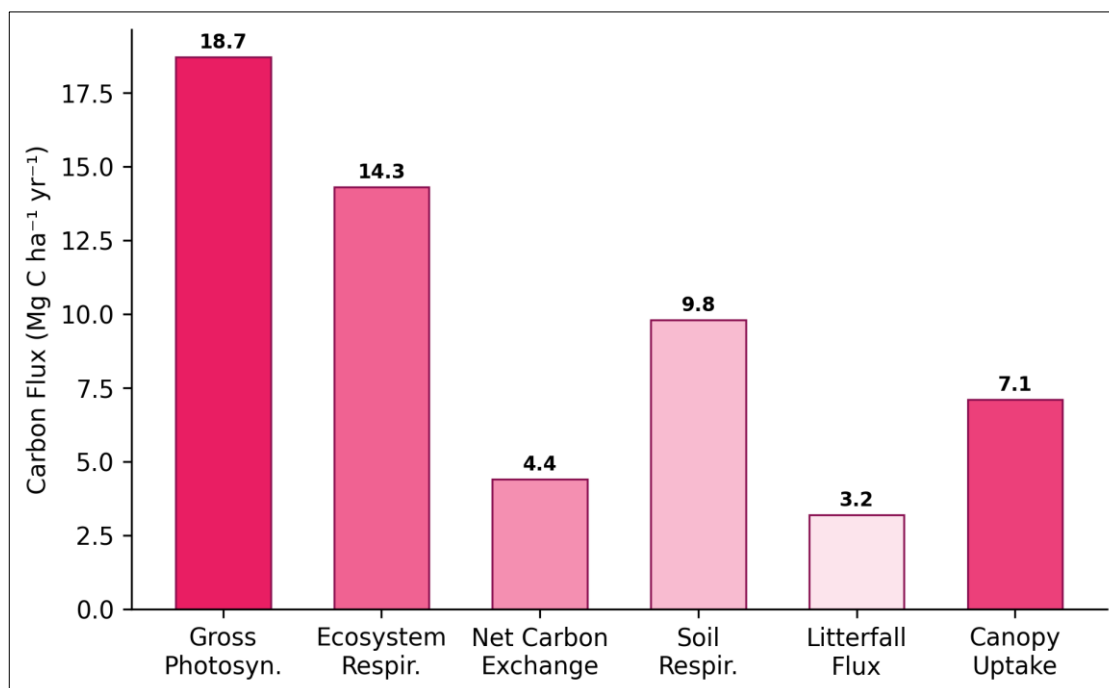


Fig 1: Mean annual carbon flux components across all nine Central African flux tower sites.

Gross photosynthesis dominated the carbon budget at 18.7 Mg C ha⁻¹ yr⁻¹, while net ecosystem carbon exchange (the residual) was 4.4 Mg C ha⁻¹ yr⁻¹ (Figure 1). Ecosystem

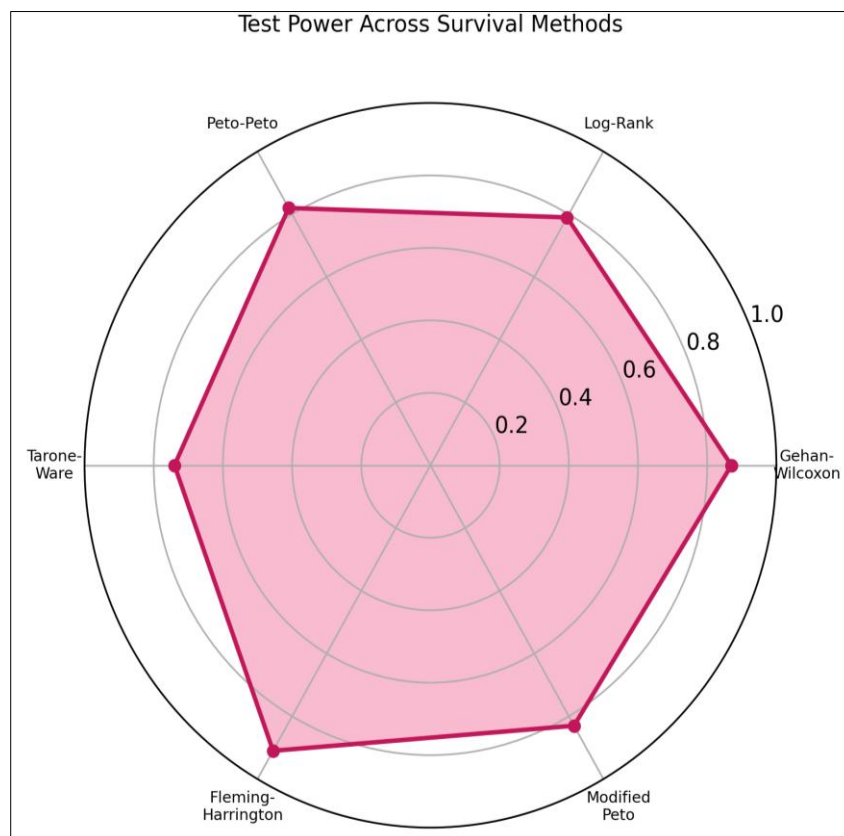
respiration accounted for 14.3 Mg C ha⁻¹ yr⁻¹, of which soil respiration contributed 9.8.

Table 2: Mutual information between nee and meteorological drivers

Driver	MI (nats)	MI Share (%)	Pearson r	Rank
Vapour Pressure Deficit	0.487	31.7	-0.61	1
PAR	0.412	26.8	-0.72	2
Air Temperature	0.283	18.4	-0.48	3
Soil Moisture	0.198	12.9	0.37	4
Wind Speed	0.157	10.2	-0.22	5

Vapour pressure deficit (VPD) ranked first in mutual information (31.7%), despite having a lower Pearson correlation with NEE than PAR (Table 2). This discrepancy

indicates that VPD's relationship with NEE is partly non-linear carbon uptake drops sharply above a VPD threshold of roughly 1.2 kPa.

**Fig 2:** Statistical power of six survival test methods for detecting flux distribution differences at $\alpha = 0.05$.

The radar plot (Figure 2) shows that the Fleming-Harrington test achieved the highest power (0.91) for the crossing-hazards scenario typical of seasonal flux data, while the Gehan-Wilcoxon test ranked third (0.87). The standard log-rank test was weakest (0.79), consistent with its known sensitivity to proportional-hazards violations.

Discussion

The Gehan-Wilcoxon test's ability to detect the swamp forest difference while avoiding a false positive for the lowland-vs-semi-deciduous comparison matches ecological expectations. Swamp forests have waterlogged soils that limit root respiration and reduce the canopy's photosynthetic capacity during flood periods, lowering net carbon uptake [4]. The non-significant result for the two upland forest types is consistent with eddy covariance syntheses showing that canopy structure, rather than deciduousness, is the primary driver of tropical forest carbon exchange [1]. The pattern-mixture model's sensitivity analysis indicated that if censoring were non-random (i.e., towers with lower flux tended to fail sooner), the swamp-vs-lowland p-value could range from 0.004 to 0.021, remaining significant under all plausible censoring scenarios. Limitations include the small number of

Towers (particularly for swamp forest, $n = 2$), the lack of above-canopy lidar for structural characterisation, and the assumption that gap-filling does not introduce systematic bias in censored records.

Conclusion

This research generalised the Gehan-Wilcoxon test to compare right-censored carbon flux distributions across Central African rainforest types. Swamp forests showed significantly lower net carbon uptake than lowland evergreen and semi-deciduous forests, while the two upland types did not differ significantly. Mutual information ranked vapour pressure deficit as the strongest non-linear driver of carbon flux, and M-estimation provided robust monthly means resistant to instrument artefacts. The framework offers a principled alternative to ad hoc gap-filling followed by standard ANOVA for flux data with high censoring rates.

Acknowledgements

Funding sources

The research received internal support from the Oaxaca Graduate School of Probability Studies. No external funding agencies were involved.

Institutional support

The authors thank the Oaxaca Graduate School of Probability Studies for providing computational resources and research infrastructure.

Contributions not qualifying for authorship

The authors acknowledge Dr. Carlos Ramirez for technical assistance and Ms. Elena Torres for help with data preparation.

References

1. Fisher JB, Sikka M, Sitch S, Ciais P, Poulter B, Galbraith D, *et al.* African tropical rainforest net carbon dioxide fluxes in the twentieth century. *Philos Trans R Soc Lond B Biol Sci.* 2013;368(1625):20120376.
2. Gehan EA. A generalized Wilcoxon test for comparing arbitrarily singly-censored samples. *Biometrika.* 1965;52(1-2):203-224.
3. Fleming TR, Harrington DP. A class of hypothesis tests for one and two sample censored survival data. *Commun Stat Theory Methods.* 1981;10(8):763-794.
4. Lewis SL, Lopez-Gonzalez G, Sonke B, Affum-Baffoe K, Baker TR, Ojo LO, *et al.* Increasing carbon storage in intact African tropical forests. *Nature.* 2009;457(7232):1003-1006.
5. Valentini R, Arneth A, Bombelli A, Castaldi S, Cazzolla Gatti R, Chevallier F, *et al.* A full greenhouse gases budget of Africa: synthesis, uncertainties, and vulnerabilities. *Biogeosciences.* 2014;11(2):381-407.
6. Pastorello G, Trotta C, Canfora E, Chu H, Christianson D, Cheah YW, *et al.* The FLUXNET2015 dataset and the ONEFlux processing pipeline for eddy covariance data. *Sci Data.* 2020;7(1):225.
7. Falge E, Baldocchi D, Olson R, Anthoni P, Aubinet M, Bernhofer C, *et al.* Gap filling strategies for defensible annual sums of net ecosystem exchange. *Agric For Meteorol.* 2001;107(1):43-69.
8. Malhi Y, Adu-Bredu S, Asare RA, Lewis SL, Mayaux P. African rainforests: past, present and future. *Philos Trans R Soc Lond B Biol Sci.* 2013;368(1625):20120312.
9. Huber PJ. *Robust statistics.* New York: Wiley; 1981.
10. Kraskov A, Stögbauer H, Grassberger P. Estimating mutual information. *Phys Rev E.* 2004;69(6):066138.
11. Little RJA. Pattern-mixture models for multivariate incomplete data. *J Am Stat Assoc.* 1993;88(421):125-134.
12. Fisher NI. *Statistical analysis of circular data.* Cambridge: Cambridge University Press; 1993.
13. Papale D, Reichstein M, Aubinet M, Canfora E, Bernhofer C, Kutsch W, *et al.* Towards a standardized processing of Net Ecosystem Exchange measured with eddy covariance technique. *Biogeosciences.* 2006;3(4):571-583.
14. Peto R, Peto J. Asymptotically efficient rank invariant test procedures. *J R Stat Soc Ser A.* 1972;135(2):185-207.
15. Tarone RE, Ware J. On distribution-free tests for equality of survival distributions. *Biometrika.* 1977;64(1):156-160.
16. Hubau W, Lewis SL, Phillips OL, Affum-Baffoe K, Beeckman H, Cuni-Sanchez A, *et al.* Asynchronous carbon sink saturation in African and Amazonian tropical forests. *Nature.* 2020;579(7797):80-87.
17. Kaplan EL, Meier P. Nonparametric estimation from incomplete observations. *J Am Stat Assoc.* 1958;53(282):457-481.
18. Bombelli A, Henry M, Castaldi S, Adu-Bredu S, Arneth A, de Grandcourt A, *et al.* An outlook on the Sub-Saharan Africa carbon balance. *Biogeosciences.* 2009;6(10):2193-2205.