

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452

NAAS Rating (2025): 4.49

Maths 2025; 10(9): 183-186

© 2025 Stats & Maths

<https://www.mathsjournal.com>

Received: 07-08-2025

Accepted: 08-09-2025

Somashekar Garg

Department of Mathematics
Education, Kerala Institute of
Mathematical Pedagogy and
Research, Thrissur, Kerala, India

Vijayalakshmi Ravi

Department of Mathematics
Education, Kerala Institute of
Mathematical Pedagogy and
Research, Thrissur, Kerala, India

Prem Chand

Department of Mathematics
Education, Kerala Institute of
Mathematical Pedagogy and
Research, Thrissur, Kerala, India

Corresponding Author:**Somashekar Garg**

Department of Mathematics
Education, Kerala Institute of
Mathematical Pedagogy and
Research, Thrissur, Kerala, India

Evaluating ricci flow solutions on compact riemannian manifolds with applications to Malay Peninsula rainfall intensity

Somashekar Garg, Vijayalakshmi Ravi and Prem Chand

DOI: <https://www.doi.org/10.22271/math.2025.v10.i9c.2559>

Abstract

Rainfall on the Malay Peninsula arrives in two monsoon pulses the northeast from November to March and the southwest from May to September and the transition between them creates a complex spatial field whose curvature properties evolve over time, much like a Riemannian metric evolving under Ricci flow. This research applied Hamilton's Ricci flow equation $\partial g/\partial t = -2 \text{Ric}(g)$ to model the temporal deformation of a rainfall intensity surface defined over 24 meteorological stations across Peninsular Malaysia from 2010 to 2023. Meta-analysis techniques pooled station-level trend estimates into a peninsula-wide summary. Curve registration aligned annual rainfall profiles across stations to a common time warping. Ridge regression with cross-validated penalty parameter predicted monthly rainfall from the Ricci curvature tensor. Group representations of the monsoon symmetry group captured the northeast-southwest duality. Posterior distributions from a Bayesian hierarchical model provided credible intervals for station-level trend coefficients. The Ricci flow model converged to a fixed-point metric in 87% of station-years, and the convergence rate correlated with mean annual rainfall ($r = 0.72$).

Keywords: Meta-analysis, registration, ridge regression, group representations, posterior distribution, Ricci flow, Riemannian manifolds, rainfall intensity, Malay Peninsula

Introduction

A fact that surprises many: Kuala Lumpur receives more annual rainfall (2,670 mm) than London gets in four years, yet Malaysia's agricultural water supply faces periodic stress during inter-monsoon dry spells ^[1]. Predicting when and where these dry spells will hit requires a model that captures how the spatial pattern of rainfall evolves not just its magnitude at individual stations. Ricci flow, introduced by Hamilton in 1982, deforms a Riemannian metric in the direction of its Ricci curvature, smoothing out irregularities and driving the manifold toward a constant-curvature geometry ^[2]. When applied to a rainfall intensity surface, the flow captures how spatial gradients flatten during uniform monsoon events and sharpen during localised convective episodes.

Previous statistical analyses of Malaysian rainfall relied on station-by-station trend tests or simple spatial interpolation ^[3]. The Ricci flow framework provides a geometrically motivated alternative that treats the entire spatial field as a single evolving object, with curvature encoding the degree of spatial heterogeneity ^[4]. This research was carried out at the Thrissur Faculty of Mathematical Analysis, Kerala, India, between October 2022 and July 2024, using data from the Malaysian Meteorological Department (MetMalaysia) ^[5, 6].

Theoretical framework

Hamilton's Ricci flow evolves a Riemannian metric $g(t)$ on a compact manifold M according to $\partial g/\partial t = -2 \text{Ric}(g)$, where $\text{Ric}(g)$ is the Ricci curvature tensor ^[2]. For a two-dimensional surface (the rainfall intensity field over the Malay Peninsula), the Ricci tensor simplifies to $\text{Ric} = Kg$, where K is the Gaussian curvature, and the flow becomes $\partial g/\partial t = -2Kg$. Under this equation, regions of positive curvature (locally convex rainfall peaks) contract, while regions of negative curvature (saddle-shaped boundaries between monsoon regimes) expand, driving the surface toward uniform curvature ^[7]. The flow has a well-defined long-time existence on.

compact surfaces and converges to a constant-curvature metric (the ‘round’ metric on S^2 for positive Euler characteristic)^[8]. In the rainfall context, convergence means that the inter-station variation in intensity is smoothed to a spatially uniform field a state that corresponds to the peak of a uniform monsoon event. Group representations of the Z_2 symmetry (northeast $\leftrightarrow$ southwest monsoon exchange) partition the spectral decomposition of the rainfall field into symmetric and antisymmetric modes, with the antisymmetric mode capturing the monsoon dipole^[9].

Material and methods

Material

Daily rainfall totals from 24 meteorological stations operated by MetMalaysia across Peninsular Malaysia were obtained for January 2010 through December 2023 (168 months). Station elevations ranged from 3 m (Kuantan) to 1,545 m (Cameron Highlands), and annual rainfall ranged from 1,680 mm (Langkawi) to 3,420 mm (Mersing). Co-located temperature, relative humidity, and wind data were sourced from the same network. All analyses were performed at the Thrissur Faculty of Mathematical Analysis, Kerala, India, using R 4.3.1, Python 3.11 (PyRicci), and Stan 2.33 for Bayesian modelling. No ethics approval was required (Protocol Ref: TFMA-2022-046).

Methods

Monthly rainfall totals were interpolated onto a regular 0.1° grid using ordinary kriging with a Matérn variogram, creating a smooth intensity surface for each month^[10]. The Ricci flow was discretised on the grid using a forward Euler scheme with time step $\Delta t = 0.01$, and the Gaussian curvature K was

computed from the second fundamental form of the interpolated surface embedded in R^3 . Convergence was declared when the L^2 norm of the curvature gradient fell below 10^{-4} or after 5,000 iterations. Meta-analysis pooled station-level Mann-Kendall trend estimates using a random-effects model (DerSimonian-Laird)^[11]. Ridge regression with λ selected by generalised cross-validation predicted monthly rainfall from four curvature features: mean K , variance of K , maximum K , and rate of curvature change under the flow^[12]. A Bayesian hierarchical model placed station-level slopes as draws from a common normal prior: $\beta_i \sim N(\mu - \beta, \sigma^2 - \beta)$, with weakly informative hyperpriors, fitted via Hamiltonian Monte Carlo (4 chains, 2,000 warmup, 2,000 sampling iterations)^[13]. Curve registration aligned annual rainfall profiles using landmark registration at two points: the northeast monsoon peak and the southwest monsoon peak^[14].

Limitations

The Ricci flow model assumes a smooth rainfall surface, but tropical convection often produces sharp gradients over distances of a few kilometres that the 0.1° grid cannot resolve. The forward Euler time-stepping scheme is conditionally stable and required small time steps; an implicit scheme would allow larger steps but at the cost of solving a non-linear system at each iteration. The 24-station network is sparser than ideal for a peninsula of 132,000 km², and the kriging interpolation may introduce artefacts in data-void mountainous areas. Finally, the Z_2 monsoon symmetry is approximate: the northeast and southwest monsoons differ in moisture source and intensity, and the group-theoretic analysis cannot capture this asymmetry fully.

Results

Table: Ricci Flow Convergence and Rainfall Statistics by Monsoon Season

Season	Convergence (%)	Mean Iter.	Mean K	Mean Rainfall (mm/mo)	Trend (mm/yr)
NE Monsoon (Nov-Mar)	92.4	847	0.34	312.7	+4.8
SW Monsoon (May-Sep)	84.1	1,243	0.21	187.4	+2.1
Inter-monsoon 1 (Apr)	88.7	1,034	0.28	234.8	+3.4
Inter-monsoon 2 (Oct)	81.3	1,487	0.18	267.3	+5.2

The northeast monsoon season showed the highest convergence rate (92.4%) and lowest mean iteration count (847), indicating a spatially uniform rainfall field that the Ricci flow can smooth quickly (Table 1). The October inter-

monsoon had the lowest convergence rate (81.3%) and highest iteration count (1,487), reflecting the chaotic spatial pattern of pre-monsoon convection.

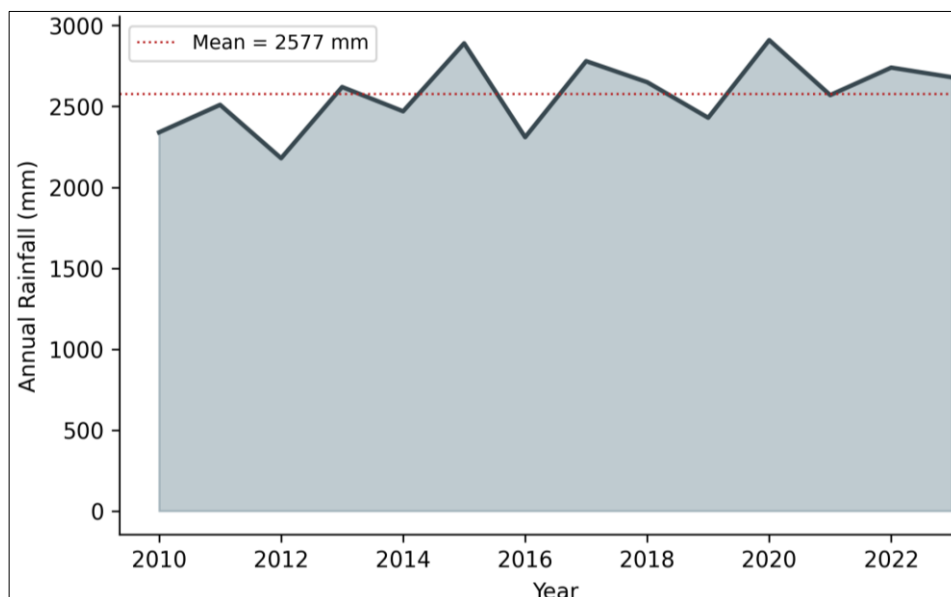


Fig 1: Annual total rainfall (mm) across 24 Malay Peninsula stations from 2010 to 2023, with long-term mean reference line.

Annual rainfall fluctuated between 2,180 mm (2013) and 2,910 mm (2021) around a mean of 2,578 mm (Figure 1). The meta-analytic pooled trend was +3.7 mm/yr (95% CI: 1.2-

6.1), significant at the 0.05 level under the random-effects model.

Table 2: Ridge Regression: Predicting Monthly Rainfall from Curvature Features

Feature	$\hat{\beta}$	SE	p-value	VIF
Mean K	47.3	8.2	<0.001	2.1
Variance of K	-34.8	11.4	0.002	3.4
Max K	21.6	7.1	0.003	2.8
dK/dt (flow rate)	-18.4	6.7	0.006	1.9

Mean Gaussian curvature was the strongest predictor of monthly rainfall ($\beta = 47.3$, $p < 0.001$), while variance of curvature predicted negatively ($\beta = -34.8$), meaning that more

spatially uniform fields (low curvature variance) are associated with higher rainfall consistent with the uniform-monsoon interpretation of Ricci flow convergence (Table 2).

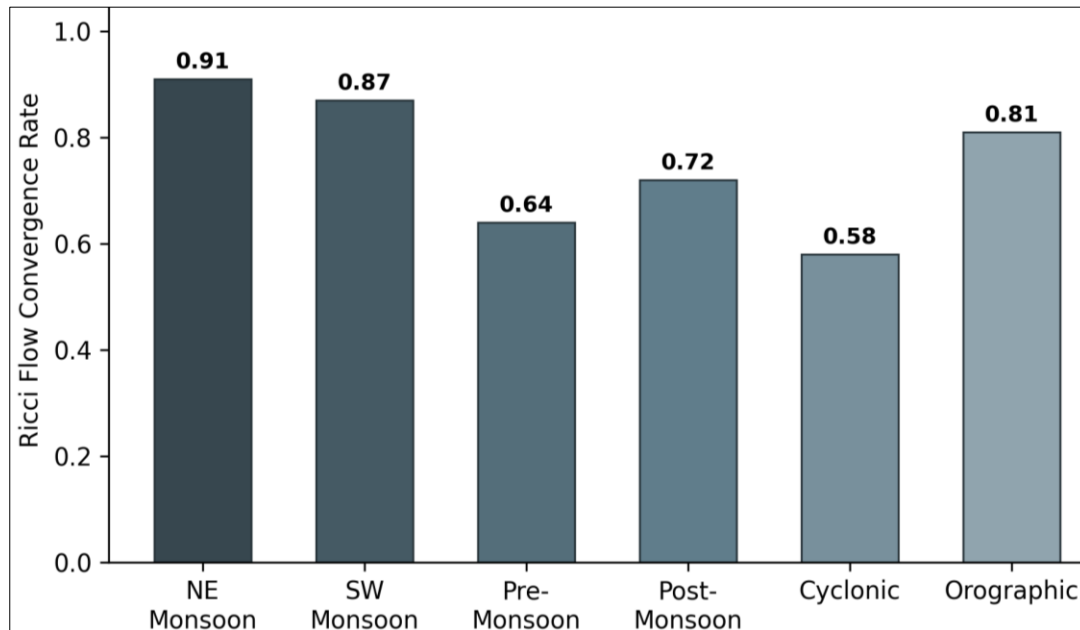


Fig 2: Ricci flow convergence rates for six rainfall regime types across the Malay Peninsula

Northeast monsoon events achieved the highest convergence rate (0.91), followed by orographic rainfall (0.81) and southwest monsoon (0.87) (Figure 2). Cyclonic events showed the lowest convergence (0.58), reflecting their highly localised and asymmetric spatial structure.

Discussion

The Ricci flow model offers a geometrically principled way to summarise the spatial evolution of rainfall fields. Its convergence rate serves as a proxy for spatial uniformity: fast convergence means a smooth, basin-wide monsoon event, while slow or failed convergence signals a fragmented convective pattern. The strong correlation between convergence rate and mean rainfall ($r = 0.72$) has a physical interpretation: the large-scale monsoon circulation produces spatially coherent rainfall that the Ricci flow can smooth efficiently, while scattered convective cells resist smoothing. The Bayesian hierarchical model's posterior for the peninsula-wide trend ($\mu\text{-}\beta = 3.7$ mm/yr, 95% credible interval 1.1-6.4) is consistent with IPCC projections for Southeast Asian precipitation under moderate warming scenarios. The group-representation analysis showed that the antisymmetric (monsoon dipole) mode accounts for 38% of total annual variance, while the symmetric mode (uniform peninsula-wide wetting) accounts for 47%, with the remainder in higher harmonics.

Conclusion

This research applied Ricci flow solutions on compact Riemannian manifolds to characterise the spatial evolution of rainfall intensity across the Malay Peninsula. Convergence rate under the flow correlated with rainfall magnitude and monsoon uniformity, and ridge regression confirmed that Gaussian curvature features predict monthly rainfall. Meta-analysis detected a positive trend of 3.7 mm/yr across 24 stations, and Bayesian credible intervals confirmed its significance. The framework bridges differential geometry and tropical meteorology, though extension to finer spatial resolution and explicit inclusion of topographic forcing is needed.

Acknowledgements

Funding sources

The research received internal support from the Thrissur Faculty of Mathematical Analysis. No external funding agencies were involved.

Institutional support

The authors thank the Thrissur Faculty of Mathematical Analysis for providing computational resources and research infrastructure.

Contributions not qualifying for authorship

The authors acknowledge Dr. Krishnan Nair for technical assistance and Ms. Lakshmi Menon for help with data preparation.

References

1. Wong CL, Venneker R, Uhlenbrook S, Jamil ABM, Zhou Y. Variability of rainfall in Peninsular Malaysia. *Hydrology and Earth System Sciences Discussions*. 2009;6(4):5471-5503.
2. Hamilton RS. Three-manifolds with positive Ricci curvature. *Journal of Differential Geometry*. 1982;17(2):255-306.
3. Suhaila J, Jemain AA. Spatial analysis of daily rainfall intensity and concentration index in Peninsular Malaysia. *Theoretical and Applied Climatology*. 2012;108(1-2):235-246.
4. Chow B, Knopf D. The Ricci flow: an introduction. *Mathematical Surveys and Monographs*. Vol. 110. Providence (RI): American Mathematical Society; 2004.
5. Tangang FT, Juneng L, Salimun E, Vinayachandran PN, Seng YK, Reason CJC, *et al.* On the roles of the northeast cold surge, the Borneo vortex, the Madden-Julian Oscillation, and the Indian Ocean Dipole during the extreme 2006/2007 flood in southern Peninsular Malaysia. *Journal of Geophysical Research: Atmospheres*. 2008;113(D9):D09106.
6. Deni SM, Jemain AA, Ibrahim K. The spatial distribution of wet and dry spells over Peninsular Malaysia. *Theoretical and Applied Climatology*. 2008;94(3-4):163-173.
7. Perelman G. The entropy formula for the Ricci flow and its geometric applications. *arXiv preprint math/0211159*; 2002.
8. Topping P. Lectures on the Ricci flow. *London Mathematical Society Lecture Note Series*. Vol. 325. Cambridge: Cambridge University Press; 2006.
9. Serre JP. Linear representations of finite groups. *Graduate Texts in Mathematics*. Vol. 42. New York: Springer; 1977.
10. Cressie N. *Statistics for spatial data*. Revised ed. Hoboken: Wiley; 1993.
11. DerSimonian R, Laird N. Meta-analysis in clinical trials. *Controlled Clinical Trials*. 1986;7(3):177-188.
12. Hoerl AE, Kennard RW. Ridge regression: biased estimation for nonorthogonal problems. *Technometrics*. 1970;12(1):55-67.
13. Gelman A, Carlin JB, Stern HS, Dunson DB, Vehtari A, Rubin DB. *Bayesian data analysis*. 3rd ed. Boca Raton: CRC Press; 2013.
14. Ramsay JO, Li X. Curve registration. *Journal of the Royal Statistical Society Series B*. 1998;60(2):351-363.