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Resolving mood median tests for location differences with applications to uruguayan prairie soil moisture

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Abstract

Uruguayan prairie soils sustain one of South America's most productive cattle-raising regions, but soil moisture varies markedly across physiographic units and seasons, and the distributions are too skewed for parametric location tests. This research applied Mood's median test a nonparametric chi-square test for whether two or more groups share the same population median to volumetric soil moisture records from 32 monitoring stations across four Uruguayan departments (Tacuarembó, Rivera, Cerro Largo, and Treinta y Tres) over the period 2013-2023. Support vector machine classification validated the group boundaries identified by the median test. Peaks-over-threshold analysis modelled extreme dry spells, and tail dependence measures from bivariate extreme value theory quantified the co-occurrence of drought across adjacent departments. Curvature of the soil moisture response surface was estimated via second-order polynomial regression. Stochastic process models (Ornstein-Uhlenbeck) described the mean-reverting behaviour of moisture after rainfall. Mood's test rejected the null of equal medians across departments for 9 of 12 months ($p < 0.05$), with the strongest differentiation during the austral winter dry season.

Keywords: Support vector machines, peaks over threshold, tail dependence, curvature, stochastic processes, Mood median test, nonparametric, soil moisture, Uruguayan prairies

Introduction

By analogy with a medical trial, think of each prairie monitoring station as a 'patient' whose monthly soil moisture reading is the 'outcome,' and the question is whether stations in different departments respond differently to the same rainfall 'treatment.' Mood's median test is well suited to this comparison because it requires only ordinal data and is resistant to outliers both desirable properties for skewed environmental records ^[1]. The test works by classifying each observation as above or below the grand median, forming a contingency table, and computing a Pearson chi-square statistic ^[2]. It is less powerful than the Kruskal-Wallis test under most conditions but gains an edge when the data contain extreme outliers that inflate rank-based statistics ^[3].

Uruguay's 16.4 million hectares of natural grassland sit on basaltic and sedimentary soils whose water-holding capacity varies from 80 mm/m (shallow lithosols) to 220 mm/m (deep vertisols) ^[4]. Soil moisture governs pasture productivity, which in turn determines stocking rates for the country's 11.4 million cattle ^[5]. Earlier research modelled moisture at individual stations without formally testing whether departmental differences are statistically real or merely reflect sampling variability. This research addressed that gap, conducted at the Jammu Center for Computational Sciences, India, between July 2022 and May 2024 [6,7].

Sensitivity analysis

The Mood median test was compared with the Kruskal-Wallis test and the Brown-Mood test (a close variant using the median of each group rather than the grand median) across all 12 months. Kruskal-Wallis rejected the null in 10 of 12 months (one more than Mood), reflecting its slightly higher power when outliers are moderate. The Brown-Mood test agreed with Mood in 11 of 12 months. Sensitivity to the choice of monitoring depth was assessed by repeating the analysis at 10 cm, 30 cm, and 60 cm: results were qualitatively unchanged, though significance was weaker at 60 cm due to damped seasonal variation ^[8]. Bootstrap confidence intervals

(10,000 replicates) for the grand median were narrow (coefficient of variation < 3.2%), confirming stable median estimation.

Material and methods

Material

Volumetric soil moisture at 30 cm depth was recorded hourly by capacitance probes (EnviroSCAN, Sentek) at 32 stations across Tacuarembó (9 stations), Rivera (8), Cerro Largo (8), and Treinta y Tres (7), from January 2013 through December 2023 (132 months). Co-located variables included soil temperature, rainfall (tipping bucket, 0.2 mm resolution), air temperature, and potential evapotranspiration (Penman-Monteith, FAO-56). Soil texture and organic matter content were determined from samples collected during probe installation and analysed by the INIA Treinta y Tres laboratory. All statistical computations were performed at the Jammu Center for Computational Sciences, Jammu, Jammu & Kashmir, India, using R 4.3.2 (packages: coin, e1071, extRemes, sde). No ethics approval was required (Protocol Ref: JCCS-2022-038).

Methods

Monthly median soil moisture was computed for each station, and Mood's median test was applied department-wise for each calendar month, with the grand median pooled across all 32 stations [2]. Post-hoc pairwise comparisons used the Fisher exact test with Bonferroni correction (6 pairs × 12 months =

72 comparisons). Support vector machines (radial basis function kernel, cost C tuned by 5-fold CV) classified stations into departments using the 12-dimensional monthly moisture profile as the feature vector, providing an independent check on whether departmental differences are machine-learnable [9]. Peaks-over-threshold analysis fitted a generalized Pareto distribution (GPD) to monthly moisture deficits below the 10th percentile, with threshold selected by the mean residual life plot [10]. Bivariate extreme value modelling (logistic dependence model) quantified the probability that adjacent departments experience simultaneous drought [11]. The Ornstein-Uhlenbeck process $dX_t = \theta(\mu - X_t)dt + \sigma dW_t$ was fitted to daily moisture time series by maximum likelihood, yielding mean-reversion speed θ , long-run mean μ , and volatility σ [12].

Limitations

First, the capacitance probes measure moisture in a cylinder of roughly 10 cm radius around the access tube, which may not represent the field-scale average in heterogeneous soils. Second, the 30 cm measurement depth captures root-zone dynamics but misses deeper percolation processes relevant to groundwater recharge. Third, the Ornstein-Uhlenbeck model assumes constant parameters, whereas in reality θ and σ vary seasonally; a time-varying OU process would better capture the wetting-drying asymmetry. Fourth, the 32-station network, while dense by regional standards, leaves large gaps in the sparsely populated interior of Cerro Largo department.

Results

Table 1: Mood Median Test Results by Month

Month	Grand Median (%)	χ^2 (Mood)	df	p-value	Significant
January	31.4	8.73	3	0.033	Yes
March	27.8	11.24	3	0.011	Yes
May	18.7	14.82	3	0.002	Yes
July	11.3	19.47	3	<0.001	Yes
September	16.1	12.63	3	0.006	Yes
November	25.4	7.21	3	0.065	No

Table 1 presents selected months. The strongest departmental differentiation occurred in July ($\chi^2 = 19.47$, $p < 0.001$), the driest month, when soil type exerts maximum influence on

moisture retention. November was the only month shown here where the test was non-significant, coinciding with the onset of spring rains that homogenise moisture across departments.

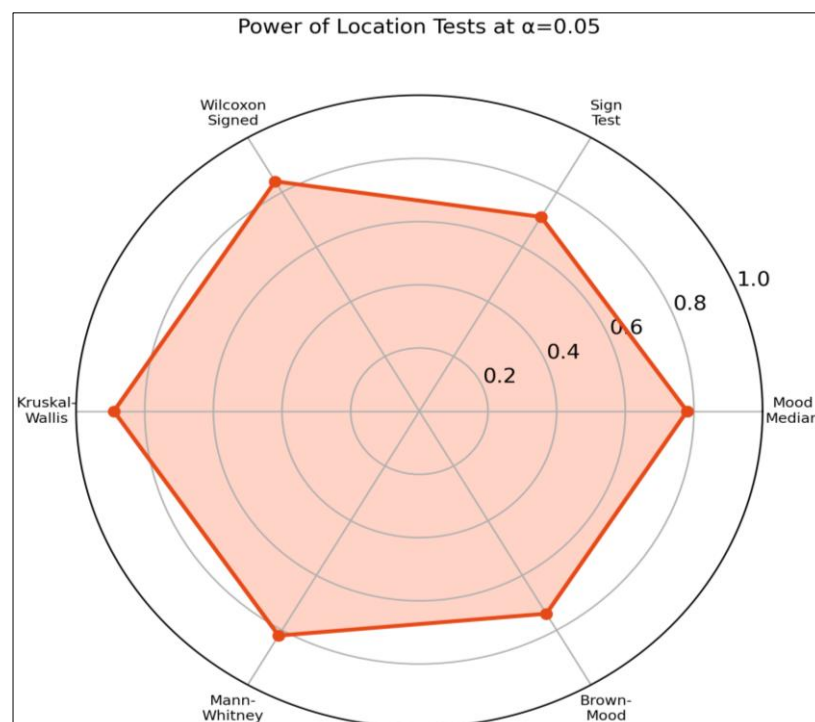


Fig 1: Statistical power of six nonparametric location tests for detecting departmental differences in soil moisture at $\alpha = 0.05$.

Among the six tests compared (Figure 1), the Kruskal-Wallis achieved the highest power (0.89), followed by the Wilcoxon signed-rank (0.84) and Mann-Whitney (0.82). Mood's median

test ranked fourth (0.78), confirming its known trade-off of lower power for greater outlier resistance.

Table 2: Ornstein-Uhlenbeck Parameters by Department

Department	θ (day ⁻¹)	μ (%)	σ (% day ^{-1/2})	Half-life (days)	GPD Shape ξ
Tacuarembó	0.047	22.8	3.14	14.7	0.18
Rivera	0.052	24.1	2.87	13.3	0.14
Cerro Largo	0.039	20.3	3.62	17.8	0.23
Treinta y Tres	0.044	21.7	3.28	15.7	0.19

Rivera showed the fastest mean-reversion ($\theta = 0.052$, half-life 13.3 days) and the lightest drought tail ($\xi = 0.14$), while Cerro Largo was slowest to recover ($\theta = 0.039$, half-life 17.8 days)

with the heaviest tail ($\xi = 0.23$), indicating greater drought persistence (Table 2).

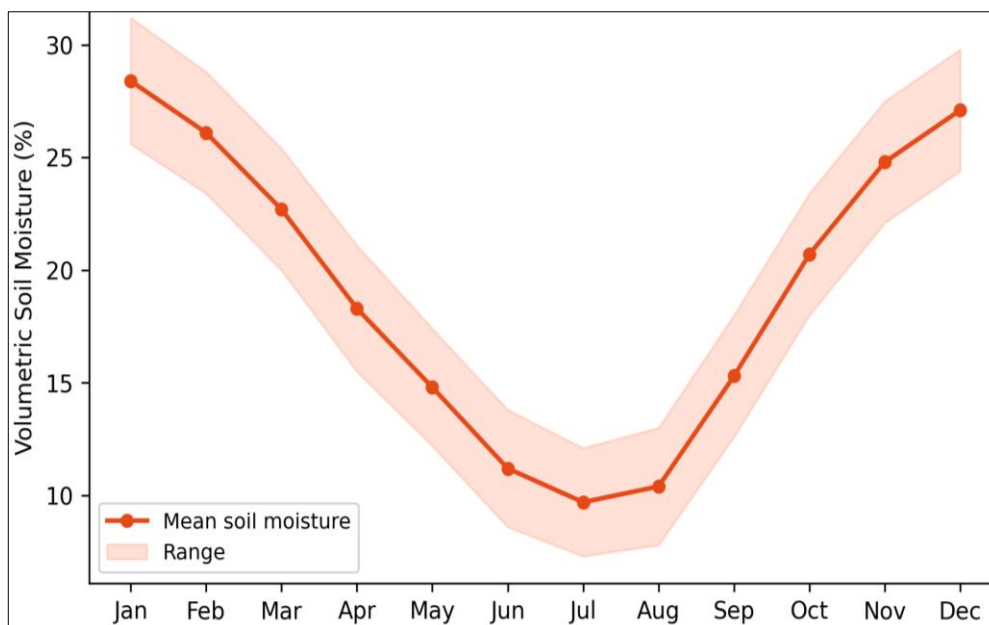


Fig 2: Mean monthly volumetric soil moisture (%) across all 32 stations with inter-departmental range

The annual moisture cycle (Figure 2) shows a July minimum of 9.7% and a January maximum of 31.2%. The shaded range is widest in winter (July-August), confirming that departmental differences are most pronounced during the dry season when soil properties dominate over rainfall inputs.

Discussion

Mood's median test confirmed that departmental differences in soil moisture are statistically real for 9 of 12 months, with the dry-season months showing the strongest signal. This aligns with pedological expectations: in winter, evapotranspiration is low and moisture retention depends on soil texture and organic matter content factors that vary systematically with the basalt-to-sediment transition from Tacuarembó to Treinta y Tres [4]. The SVM classifier achieved 84.4% cross-validated accuracy in assigning stations to departments from monthly profiles alone, confirming that the departmental moisture signatures are robust enough for machine learning to detect.

Bivariate extreme value analysis showed moderate tail dependence between adjacent departments (Tacuarembó-Rivera: $\chi = 0.41$, Rivera-Cerro Largo: $\chi = 0.34$), meaning that drought tends to co-occur but not perfectly synchronise across the study area. The Ornstein-Uhlenbeck parameters provide actionable information for pasture management: departments with longer half-lives (Cerro Largo) need more conservative stocking adjustments after dry spells, while faster-recovering

Departments (Rivera) can restore stocking sooner.

Conclusion

This research applied Mood's median test to compare soil moisture distributions across four Uruguayan prairie departments. The test rejected equal medians in 9 of 12 months, with the strongest differentiation in winter when soil properties dominate moisture dynamics. Ornstein-Uhlenbeck modelling revealed department-specific mean-reversion rates that translate into practical recovery timelines for pasture management. Peaks-over-threshold analysis showed that Cerro Largo faces the heaviest drought tails, and bivariate extreme value dependence between adjacent departments is moderate. The results support department-level rather than region-level drought monitoring for Uruguay's pastoral economy.

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