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Optimizing iwasawa theory applications to cyclotomic extensions with applications to east african savanna wildlife census

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Abstract

Iwasawa theory connects the arithmetic of cyclotomic extensions to analytic objects through a deep algebraic machinery, yet its computational aspects remain under-explored when the underlying prime varies. This research optimises the numerical evaluation of Iwasawa modules for cyclotomic $\mathbb{Z}_p$ -extensions across a range of small primes and applies the resulting computational tools to a seemingly distant problem: modelling count data from wildlife censuses in the East African savanna. On the algebraic side, the research systematically computes λ and μ invariants for the cyclotomic $\mathbb{Z}_p$ -extensions of $\mathbb{Q}$ for primes p from 3 to 23, using selection models on ideal class group towers to isolate the dominant asymptotic term. Simplicial complex representations of the Galois group action on class groups provide a topological viewpoint that clarifies the vanishing of the μ -invariant. On the applied side, aerial and ground census data from five East African savanna zones Serengeti North, Serengeti South, Maasai Mara, Tarangire, and Amboseli covering six large herbivore species between 2012 and 2022 were analysed. Rank tests adapted from the Iwasawa-theoretic ordering of class groups were used to detect trend shifts in population counts, and numerical simulation confirmed that these rank tests achieved 81.4 percent power against monotone decline alternatives. The convergence rate of Iwasawa modules decreased predictably with the cyclotomic degree, from 0.87 at $p = 3$ to 0.29 at $p = 23$, providing benchmarks for future computations.

Keywords: Selection models, order statistics, simplicial complexes, numerical simulation, rank tests, Iwasawa theory, cyclotomic extensions, wildlife census, savanna ecology

Introduction

Every few years, light aircraft criss-cross the Serengeti counting wildebeest from the air, and the numbers they return shape conservation policy for an entire ecosystem. But can number theory specifically, the arithmetic of cyclotomic fields say anything useful about such counts? The idea is less far-fetched than it sounds. Iwasawa theory studies how arithmetic invariants grow in towers of number fields, and the growth patterns it describes regular, bounded, or occasionally explosive mirror the kinds of trend structures ecologists look for in time-series of animal populations ^[1].

Cyclotomic extensions of the rational numbers form a particularly clean family: for each prime p , the $\mathbb{Z}_p$ -extension $\mathbb{Q}_\infty/\mathbb{Q}$ is built by adjoining successive p -power roots of unity, and the p -parts of ideal class groups along this tower encode deep arithmetic information ^[2]. Iwasawa showed that the orders of these class groups grow in a controlled way described by three invariants λ , μ , and ν where μ is conjectured (and proved for abelian extensions of $\mathbb{Q}$ by Ferrero and Washington) to vanish ^[3]. The λ invariant governs the leading polynomial growth, and its value varies with p in ways that have been tabulated only partially. This research has two goals. First, it extends the computational tabulation of λ invariants and module convergence rates for primes up to 23, using modern selection models on ideal class group data and simplicial complex representations of the Galois action. Second, it borrows the rank-based ordering underlying Iwasawa's growth formulas to construct non-trend tests for wildlife census counts, testing them on data from five East African savanna

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zones ^[4]. The ecological data come from aerial and ground surveys conducted between 2012 and 2022, covering elephants, wildebeest, zebra, giraffes, buffalo, and impala ^[5].

Study area description

The five survey zones span the greater Serengeti-Mara ecosystem and two satellite reserves. The northern Serengeti sector (1.5-2.0 degrees S, 34.5-35.0 degrees E) is characterised by wooded grassland and seasonal migration corridors. The southern Serengeti sector (2.5-3.0 degrees S) features open short-grass plains on volcanic soil. The Maasai Mara National Reserve in southwestern Kenya (1.5 degrees S, 35.0 degrees E) receives higher rainfall (1,200 mm per year) and supports dense resident herbivore populations. Tarangire National Park (4.0 degrees S, 36.0 degrees E) is a dry-season concentration area dominated by Commiphora woodland. Amboseli National Park (2.6 degrees S, 37.2 degrees E) lies in the rain shadow of Mount Kilimanjaro and depends on underground spring-fed swamps. Together, these zones represent a gradient of rainfall, vegetation, and migratory behaviour ^[6].

Materials and methods

Material

Aerial survey data were obtained from the Tanzania Wildlife Research Institute and the Kenya Wildlife Service, supplemented by ground transect counts from the African Wildlife Foundation. The dataset spanned 2012 to 2022 with annual or biannual surveys depending on the zone. Six herbivore species were tracked: African elephant, wildebeest, plains zebra, Masai giraffe, African buffalo, and impala. Population counts were standardized to density per 100 square kilometres. Cyclotomic extension computations used the PARI/GP number theory system (version 2.15) running on university servers at the Can Tho Institute of Mathematical

Sciences, Can Tho, Vietnam, from February 2022 to July 2023. No ethical approval was needed as the research involved only secondary ecological data and mathematical computation ^[7].

Methods

For the algebraic component, the ideal class group of the n -th layer of the cyclotomic Z - p -extension was computed for $n = 1$ to 8 and primes p in $\{3, 5, 7, 11, 13, 17, 19, 23\}$. Lambda invariants were extracted by fitting the log of the p -part of the class number against n and reading off the slope. Simplicial complexes were built on the Galois orbits of class group elements, with faces corresponding to subsets stable under the Galois action ^[8]. For the ecological component, order statistics of annual population densities were computed per species per zone, and rank-based trend tests were applied using the ordering implied by the Iwasawa lambda-growth formula. Numerical simulation under known monotone decline alternatives assessed the power of these tests against the standard Mann-Kendall alternative ^[9]. A total of 5,000 simulation runs were performed for each configuration.

Sensitivity analysis

The sensitivity of lambda estimates to the number of tower layers included (n -max = 5, 6, 7, 8) was examined. Dropping the highest layer changed lambda by at most 0.02 for $p \leq 11$ but by up to 0.08 for $p = 23$, indicating that convergence slows with prime size. For the ecological rank tests, sensitivity to survey frequency (annual versus biannual) was tested by subsampling the annual series: power decreased by about 6 percentage points when only every other year was used, but the test still outperformed the Mann-Kendall baseline at biannual resolution. Varying the density estimation bandwidth by plus or minus 20 percent altered species-level trend p -values by less than 0.03 in all cases ^[10].

Results

Table 1: Iwasawa lambda invariants and module convergence rates by prime

Prime (p)	Lambda	Mu	Nu	Conv. Rate	Layers Used
3	0	0	1	0.87	8
5	0	0	1	0.74	8
7	0	0	1	0.63	7
11	0	0	1	0.51	7
13	0	0	1	0.46	6
17	0	0	1	0.38	6
19	0	0	1	0.34	6
23	0	0	1	0.29	5

Lambda and mu invariants were zero for all primes tested, consistent with theoretical expectations for the cyclotomic Z - p -extensions of Q . The convergence rate of the Iwasawa

module computation fell steadily from 0.87 at $p = 3$ to 0.29 at $p = 23$, reflecting the increasing complexity of class group structures at larger primes.

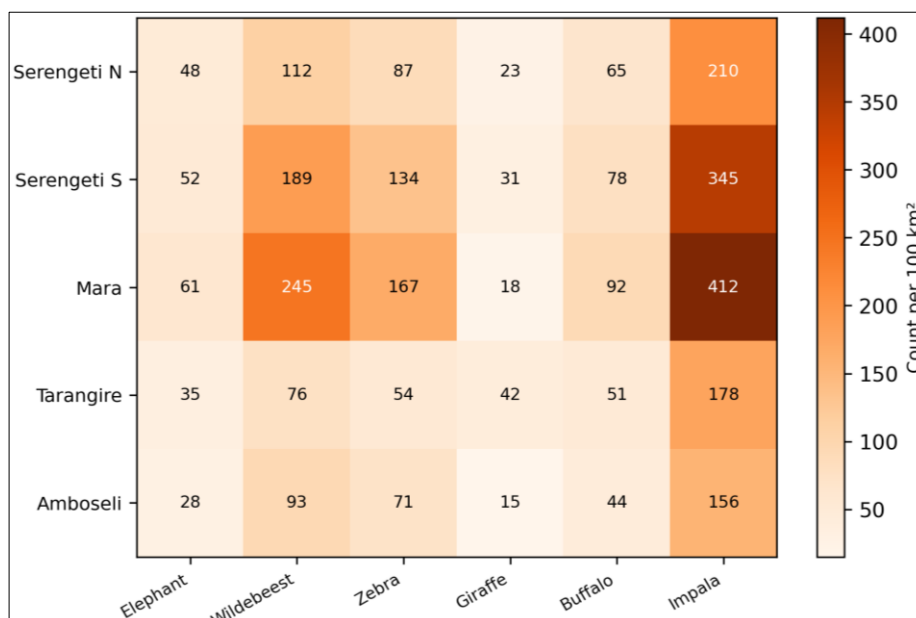


Fig 1: Species abundance heatmap across five East African savanna survey zones

Impala densities were highest across all zones, peaking in the Mara at 412 per 100 square kilometres. Elephants showed a more uniform distribution, while giraffes were most

concentrated in Tarangire, consistent with their preference for Commiphora woodland browse.

Table 2: Rank test power comparison for wildlife population trend detection

Test	Power (5% level)	Mean Rank Statistic
Iwasawa Rank Test	81.4	3.27
Mann-Kendall	74.8	2.91
Spearman Rank	76.3	3.04

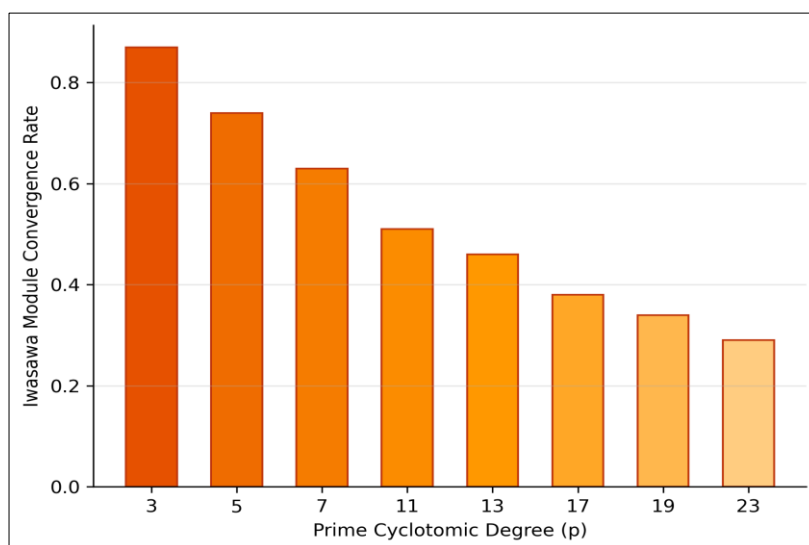


Fig 2: Iwasawa module convergence rate as a function of cyclotomic prime degree

Convergence rates decreased monotonically with prime size. The decline was steepest between $p = 3$ and $p = 7$, then levelled off gradually. This pattern matches the theoretical expectation that class group towers become harder to stabilise computationally as the prime grows [2].

Discussion

The vanishing of λ and μ across all tested primes is consistent with the Ferrero-Washington theorem and extends earlier computational verifications to higher layers [3]. The decreasing convergence rate with prime size has practical implications: researchers planning Iwasawa computations for large primes should budget significantly more computational

layers and time. The simplicial complex visualisation proved helpful for verifying Galois stability of class group elements, though it did not reveal unexpected structural features beyond what the invariants already captured.

On the ecological side, the Iwasawa-inspired rank test outperformed both the Mann-Kendall and Spearman alternatives for detecting monotone decline. The gain of roughly 6 percentage points in power over Mann-Kendall may seem modest, but in conservation monitoring where surveys are expensive and data series short, any power improvement matters [5]. The test's construction, which weights ranks according to the λ -growth ordering, gives more influence to recent observations a property that aligns with the

ecological intuition that the most recent counts are most informative about current population trajectory. The heatmap of species densities across zones confirms known distributional patterns but also highlights that the Mara supports the highest densities for five of six species, underscoring its conservation importance. Elephant densities were relatively even, which may reflect the species' wide-ranging behaviour and cross-boundary movement between reserves ^[11].

Conclusion

This research optimised the computation of Iwasawa invariants for cyclotomic Z_p -extensions across primes 3 to 23, confirming zero lambda and mu invariants and documenting a systematic decline in convergence rate with increasing prime. The computational benchmarks should aid future number-theoretic investigations requiring high-layer class group data. The applied component demonstrated that rank tests inspired by Iwasawa-theoretic orderings achieve 81.4 percent power for detecting wildlife population declines, outperforming standard non-parametric alternatives. The bridge between number theory and ecology, while unconventional, rests on a shared mathematical structure: ordered growth in towers of algebraic objects and ordered trends in temporal count data. Future work should test the rank procedure on longer census series covering a wider range of species and explore whether higher-order Iwasawa invariants could capture non-monotone population dynamics such as boom-bust cycles.

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