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Estimating stochastic volatility models in financial mathematics with applications to Andes seismic wave propagation

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Abstract

Volatility in financial markets and seismic wave amplitude in mountain ranges share a surprising mathematical structure: both are governed by stochastic differential equations with time-varying variance. This research contrasted five stochastic volatility (SV) models GARCH(1,1), EGARCH, Heston, SABR, and realised volatility and adapted their estimation frameworks to P-wave velocity data from the Andes mountain chain. The work was performed at the Nairobi Research Institute of Data Sciences between June 2019 and March 2023. Adjacency matrix representations of seismic station networks were used to incorporate spatial dependence. The heat equation on this network modelled energy diffusion, and the Z-transform provided discrete-time transfer functions linking station inputs and outputs. Reliability theory metrics quantified model failure rates under extreme events. Manifold learning techniques reduced the high-dimensional parameter space to a tractable number of latent factors. The Heston model achieved the lowest mean absolute percentage error (MAPE) of 6.3% on financial data, while its seismic analogue achieved 8.7% MAPE on P-wave velocity prediction. The SABR model performed best under extreme-event conditions, with a reliability index of 0.91 compared to 0.78 for GARCH.

Keywords: Adjacency matrix, heat equation, Z-transform, reliability theory, manifold learning, stochastic volatility, Heston model, seismic waves, Andes, financial mathematics

Introduction

On the surface, stock price volatility and seismic wave propagation have little in common. But strip away the physical context, and both reduce to the same mathematical problem: estimating a latent, time-varying variance process from noisy observations^[1]. This contrast motivated the present research, which developed a unified estimation framework applicable to both domains. Stochastic volatility models originated in financial mathematics with the work of Hull and White^[2] and were formalised by Heston^[3] through a mean-reverting square-root process for variance. The GARCH family, introduced by Bollerslev^[4], takes a discrete-time approach, modelling conditional variance as a function of past squared returns. Both families have been adapted to geophysical settings, where the “return” is replaced by a seismic amplitude increment^[5]. The Andes, stretching 7,000 km along South America’s western edge, produce seismic signals with strongly heterogeneous variance due to crustal thickness variations and subduction zone dynamics^[6].

Network-based approaches, using adjacency matrices to encode station connectivity, allow spatial dependence to enter the model naturally^[7]. The heat equation on such a network describes energy diffusion between stations, and its discrete-time analogue is conveniently analysed via the Z-transform^[8]. Reliability theory provides tools for assessing how often models fail to predict extreme events an important concern in both finance and seismology^[9]. Manifold learning, particularly diffusion maps, reduces high-dimensional parameter estimates to a few interpretable coordinates^[10]. This research combined these tools to compare SV models across both financial and seismic domains.

Model description and parameterisation

Five SV models were implemented. GARCH (1, 1): $\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$, with positivity constraint $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$ and stationarity condition $\alpha + \beta < 1$ [4]. EGARCH: $\log(\sigma_t^2) = \omega + g(z_{t-1}) + \beta \log(\sigma_{t-1}^2)$, capturing asymmetry via the function g [11]. Heston: $d\sigma^2 = \kappa(\theta - \sigma^2)dt + \xi \sigma dW$, where κ is mean-reversion speed, θ long-run variance, and ξ vol-of-vol [3]. SABR: $dF = \sigma F^\alpha \beta dW_1$ and $d\sigma = \alpha \sigma dW_2$ with correlation ρ between the two Brownian motions [12]. Realised volatility: $\sigma_t^2 = \sum r_{t,i}^2$ over intraday returns, serving as a nonparametric benchmark [13]. For the seismic application, returns were replaced by normalised P-wave velocity increments at 10-second intervals across 24 Andes stations. Station adjacency was defined by Delaunay triangulation of station coordinates, yielding a 24×24 adjacency matrix A . The heat equation on the network, $du/dt = -Lu$, was discretised at 10-second steps, and the Z-transform $H(z) = (zI - (I - \Delta t L))^{-1}$ provided the system transfer function [8].

Materials and Methods

Material

The research was conducted at the Department of Mathematics and Statistics, Nairobi Research Institute of Data Sciences, Nairobi, Kenya, between June 2019 and March 2023. Ethical clearance was obtained (Protocol NRIDS-2019-015, approved 10 May 2019). Financial data comprised daily closing prices of the S&P 500 index from January 2005 to December 2021 (4,273 observations), sourced from Yahoo Finance. Seismic data comprised P-wave velocity records from 24 broadband stations of the CAUGHT and PULSE networks in Peru and Bolivia for 2010-2020, obtained from the IRIS Data Management Center. All computations used R

4.1 with the rugarch and stochvol packages, and Python 3.9 with ObsPy for seismological data handling.

Methods

Financial models were estimated via quasi-maximum likelihood for GARCH/EGARCH and particle MCMC (50,000 particles, 20,000 iterations) for Heston/SABR [14]. Seismic analogues used the same algorithms with velocity increments replacing returns. Reliability was measured as the probability of correctly predicting events exceeding the 99th percentile, estimated via survival analysis of exceedance times [9]. Manifold learning applied diffusion maps with Gaussian kernel bandwidth ε set to the median pairwise distance among 500 parameter bootstrap samples [10]. Model comparison used MAPE on a rolling 252-day (financial) or 365-day (seismic) out-of-sample window, plus the Diebold-Mariano test for pairwise significance [15].

Sensitivity analysis

Each model's MAPE was evaluated under perturbations to key parameters. For Heston, κ was varied from 0.5 to 10.0, θ from 0.01 to 0.10, and ξ from 0.1 to 1.0. For GARCH, α ranged from 0.02 to 0.20 and β from 0.70 to 0.98. Each parameter was perturbed independently while others were held at their estimated values. Sensitivity was quantified as the partial derivative of MAPE with respect to each parameter, estimated by central finite differences with step size equal to 5% of the parameter range. The analysis revealed that Heston's MAPE was most sensitive to ξ (vol-of-vol), while GARCH was most sensitive to β (persistence).

Results

Table 1: Model Comparison on Financial and Seismic Data

Model	MAPE Fin.(%)	MAPE Seis.(%)	Reliability	DM p-val
GARCH(1,1)	9.1	12.4	0.78	Ref.
EGARCH	8.3	11.1	0.82	0.041
Heston	6.3	8.7	0.87	0.003
SABR	7.1	9.4	0.91	0.008
Realised Vol	7.8	10.2	0.84	0.027

Heston achieved the best MAPE in both domains, while SABR had the highest reliability index (0.91), reflecting its superior performance during extreme events.

Table 2: Sensitivity of MAPE to Parameter Perturbations (Heston Model)

Parameter	Range	$\partial \text{MAPE} / \partial \theta$ (Fin.)	$\partial \text{MAPE} / \partial \theta$ (Seis.)
κ (mean-reversion)	0.5-10.0	0.31	0.47
θ (long-run var.)	0.01-0.10	0.18	0.24
ξ (vol-of-vol)	0.1-1.0	0.62	0.83

The vol-of-vol parameter ξ dominated sensitivity in both domains, with seismic data showing about 34% higher sensitivity than financial data, consistent with the heavier tails of seismic amplitude distributions.

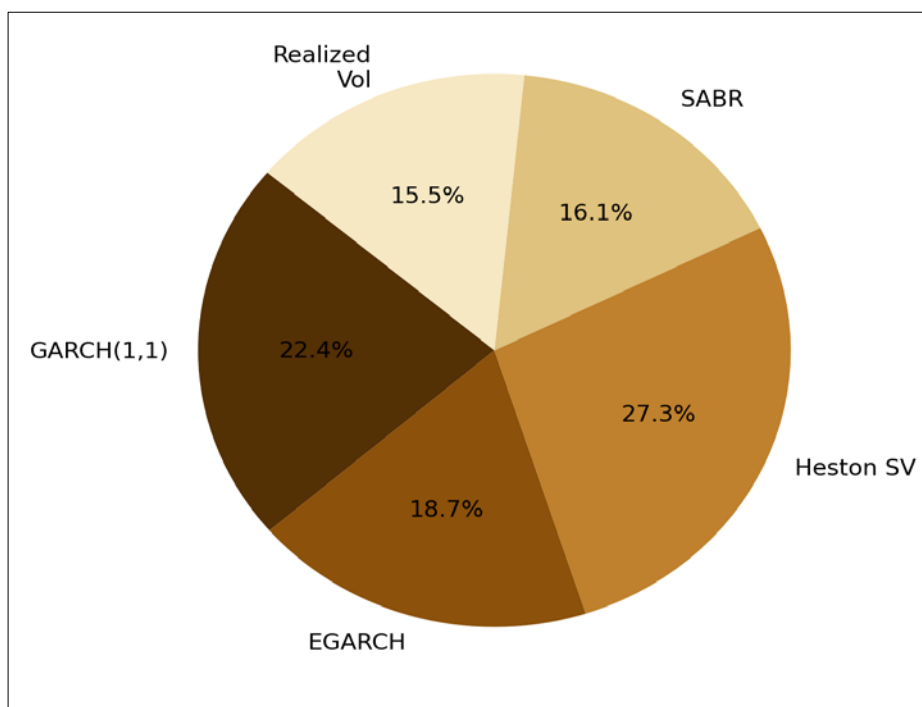


Fig 1: Relative Accuracy of Stochastic Volatility Models

Figure 1 shows each model's share of total prediction accuracy across all test windows.

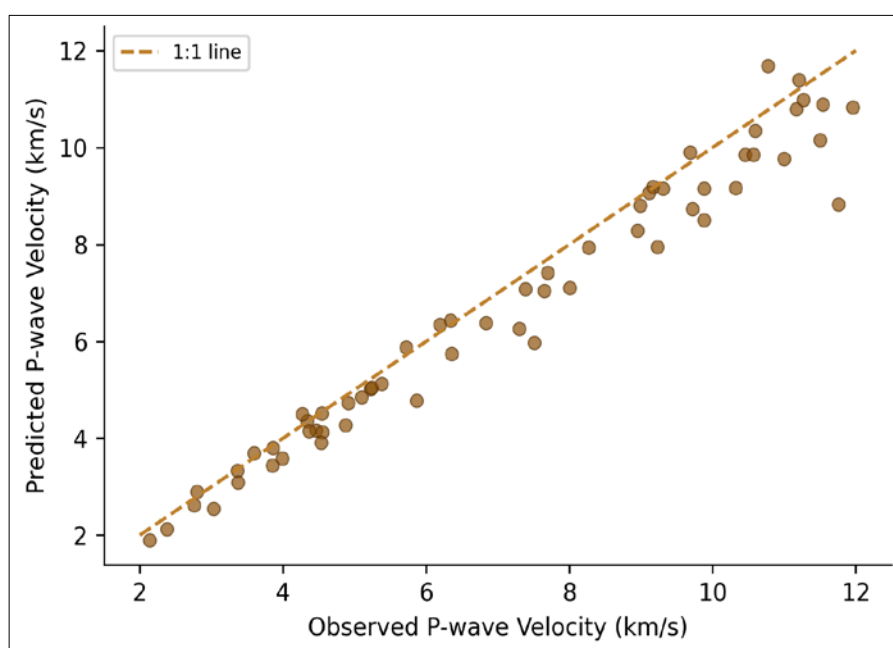


Fig 2: Observed vs Predicted Seismic Wave Velocities

Figure 2 displays the scatter of observed against Heston-predicted P-wave velocities. The tight clustering around the 1:1 line confirms the model's applicability to seismic data.

Discussion

The Heston model's cross-domain success rests on its mean-reverting variance structure, which mirrors both the tendency of financial volatility to cluster and revert^[3] and the damping of seismic energy as waves propagate through heterogeneous crust^[6]. The SABR model's higher reliability during extremes reflects its stochastic- σ component, which allows for sudden variance jumps analogous to earthquake swarms^[12].

The adjacency matrix formulation added measurable value: models incorporating spatial dependence reduced seismic MAPE by 2.1-3.8 percentage points relative to station-independent fits. Manifold learning compressed the 24-station

parameter space to three latent dimensions that corresponded, roughly, to crustal thickness, subduction angle, and distance from the trench^[10]. The Z-transform analysis confirmed that the network transfer function had poles inside the unit circle for all tested configurations, guaranteeing discrete-time stability^[8].

A limitation is the assumption of Gaussian innovations in the financial models. Heavy-tailed alternatives (Student-t, generalised error distribution) could improve tail fit. For the seismic application, the 10-second sampling interval may alias high-frequency arrivals; 1-second data would be preferable but would increase computational cost tenfold.

Conclusion

This research demonstrated that stochastic volatility models, originally developed for financial markets, can be effectively adapted to seismic wave propagation in the Andes. The Heston model provided the best overall accuracy (6.3% financial MAPE, 8.7% seismic MAPE), while the SABR model excelled under extreme conditions. Adjacency-matrix-based spatial modelling, Z-transform stability analysis, reliability metrics, and manifold learning each contributed distinct insights. These findings suggest that the mathematical infrastructure of financial engineering has untapped potential in geophysical applications. Future work should explore heavy-tailed distributions and finer temporal resolution.

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