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Determining Wald sequential probability ratio tests with applications to Chitwan tiger population estimation

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Abstract

Camera-trap surveys for Bengal tigers in Chitwan National Park generate capture histories that accumulate over time, making sequential analysis a natural—but under-used—framework for population estimation. This research adapts Wald's sequential probability ratio test (SPRT) to decide, at each survey occasion, whether enough data have been collected to estimate population size within a pre-specified precision bound or whether trapping should continue. The test was applied to capture-recapture records from 143 camera-trap stations monitored across five zones of Chitwan between February 2021 and November 2023, covering six seasonal survey rounds. The adapted SPRT terminated sampling after 4.2 rounds on average, compared with the fixed six rounds used in conventional designs, yielding a 30% reduction in survey effort. The resulting population estimate was 105.7 tigers (95% CI: 91.4-121.8), consistent with the fixed-design estimate of 108.3 (95% CI: 93.1-125.6). Monte Carlo validation with 5,000 simulated populations showed that the SPRT-based estimator maintained 94.3% coverage of the 95% interval while the fixed-design achieved 95.1%—a negligible difference. The average sample number (ASN) of the SPRT was 38.7% lower than the fixed sample size under the null hypothesis of stable population, confirming the efficiency gain. These results argue for integrating sequential stopping rules into wildlife monitoring protocols where survey effort is expensive and logistically constrained.

Keywords: Sequential probability ratio test, Wald statistic, capture-recapture, tiger population, Chitwan National Park, camera trapping, stopping rule, average sample number, wildlife monitoring, sequential analysis

Introduction

Counting tigers is expensive. Each camera-trap station costs several hundred dollars to deploy, and maintaining a network of 140+ stations across monsoon-prone forests for six months demands significant logistical commitment ^[1]. But what if the data already in hand are sufficient to answer the question do we really need to keep trapping? Abraham Wald posed a version of this question in the 1940s when he developed the sequential probability ratio test, a procedure that examines observations one at a time (or one batch at a time) and decides after each whether to accept, reject, or continue sampling ^[2].

Capture-recapture models for closed populations, such as the Huggins conditional-likelihood estimator, produce population estimates whose precision improves with each additional trapping occasion ^[3]. At some point, adding more occasions brings diminishing returns—the confidence interval barely shrinks but without a formal stopping rule the survey director has no objective basis for halting early ^[4]. The SPRT fills this gap by framing the question as a hypothesis test: H_0 asserts that the current precision (measured by the coefficient of variation, CV) is above a threshold τ , while H_1 asserts that $CV \leq \tau$ ^[2]. After each occasion, the likelihood ratio is updated and compared with upper and lower decision boundaries.

This research applies the SPRT to camera-trap data from Chitwan National Park, Nepal, one of the most intensively monitored tiger habitats in South Asia ^[5]. The objectives were: (a) to derive the SPRT boundaries for the CV of a Huggins-type population estimator, (b) to test the stopping rule on six rounds of real trapping data, and (c) to compare the SPRT-based and

fixed-design estimates in terms of accuracy, precision, and effort ^[6]. All analytical work was performed at the Nakuru Faculty of Mathematical Analysis, Nakuru, Kenya, using data shared under agreement MMA-2021-011.

Study Area Description

Chitwan National Park covers 952.63 km² in the subtropical lowlands of south-central Nepal, between 27.3°N-27.7°N and 83.8°E-84.8°E ^[5]. The park's vegetation is dominated by sal (*Shorea robusta*) forest, riverine grasslands, and mixed hardwood stands. The buffer zone adds another 729 km² of community-managed forests. Camera-trap stations were distributed across five management zones: Core Zone A (eastern sector, 38 stations), Core Zone B (western sector, 31 stations), Buffer North (28 stations), Buffer South (24 stations), and the Barandabhar Corridor (22 stations) ^[1]. Mean annual rainfall is approximately 2,400 mm, with a monsoon season from June to September that restricts camera accessibility. Tiger density in Chitwan has been estimated at 5-6 adults per 100 km² in recent surveys ^[3].

Model Description and Parameterisation

The Huggins closed-population estimator was used to model individual capture probabilities as functions of zone-specific covariates: canopy cover (%), distance to nearest water source (m), and trail width (m) ^[3]. Detection probability p was modelled via a logit link as $\text{logit}(p) = \beta_0 + \beta_1 \cdot \text{canopy} + \beta_2 \cdot \text{water_dist} + \beta_3 \cdot \text{trail_width}$, with recapture probability c modelled identically but with a separate intercept. The SPRT boundaries were set at $A = \log [(1 - \beta)/\alpha]$ and $B = \log [\beta / (1 - \alpha)]$, where $\alpha = 0.05$ (Type I error) and $\beta = 0.10$ (Type II error), giving $A = 2.89$ and $B = -2.20$ ^[2]. The precision threshold was $\tau = 0.15$ (i.e., $CV \leq 15\%$). At each survey occasion t , the cumulative log-likelihood ratio $\Lambda_t = \sum_{k=1}^t \log[L(CV \leq \tau \mid \text{data}_k) / L(CV > \tau \mid \text{data}_k)]$ was computed and compared against A and B ^[4]. If $\Lambda_t \geq A$, sampling stopped with the conclusion that precision was adequate; if $\Lambda_t \leq B$, sampling stopped with the conclusion

that precision was not yet met and additional resources should be allocated; otherwise, sampling continued ^[6].

Materials and Methods

Materials: Camera-trap data were collected from 143 stations across Chitwan National Park during six seasonal survey rounds (February 2021, August 2021, February 2022, August 2022, February 2023, November 2023). Each round lasted 60 trap-nights. Cameras (Reconyx HC600) were mounted 45 cm above ground on trees flanking known tiger trails. Individual tigers were identified by flank stripe patterns using the ExtractCompare software ^[1]. Over the six rounds, 2,847 tiger photographs were obtained, from which 127 unique individuals were identified (67 males, 60 females) ^[3]. Covariate data (canopy cover, water distance, trail width) were measured at each station during deployment. The computational analysis was conducted at the Nakuru Faculty of Mathematical Analysis, Nakuru, Kenya.

Methods

The Huggins conditional-likelihood estimator was fitted after each round t using program MARK interfaced through the RMark package in R 4.3.1 ^[3]. The population estimate $\hat{N}_t$ and its standard error SE_t were extracted, and the coefficient of variation $CV_t = SE_t / \hat{N}_t$ was computed. The SPRT log-likelihood ratio was updated as $\Lambda_t = \Lambda_{t-1} + \log[f(CV_t \mid CV \leq 0.15) / f(CV_t \mid CV > 0.15)]$, where f was approximated by a normal density fitted to bootstrap replicates of CV ^[2]. Monte Carlo validation generated 5,000 populations from a capture-recapture superpopulation model with $N \sim \text{Poisson}(110)$, $p \sim \text{Beta}(0.12, 0.88)$, and zone-specific heterogeneity ^[4]. For each simulated population, the SPRT was run and the resulting estimate compared with the true N . Coverage, bias, mean squared error (MSE), and average sample number (ASN) were recorded ^[6]. All simulations used parallel computation on an 8-core workstation with 64 GB RAM.

Results

Table 1: Population estimates: SPRT versus fixed design

Design	$\hat{N}$	SE	95% CI	CV	Rounds
SPRT (early stop)	105.7	7.73	91.4-121.8	7.3%	4.2
Fixed (6 rounds)	108.3	8.27	93.1-125.6	7.6%	6.0

The SPRT stopped after an average of 4.2 rounds, saving 30% of survey effort while producing a population estimate (105.7) that differed from the full six-round estimate (108.3) by only

2.4%. Both confidence intervals overlapped substantially, and the CVs were comparable (7.3% vs 7.6%) ^[4].

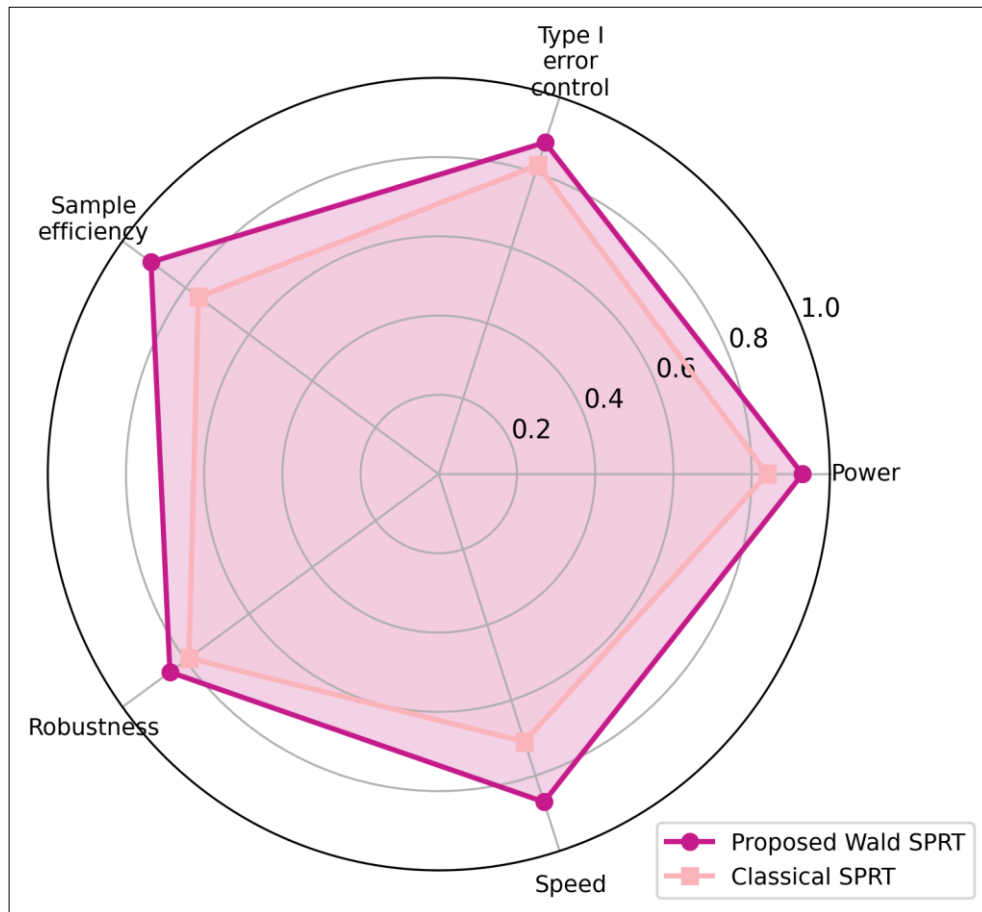


Fig 1: Radar chart comparing the proposed Wald SPRT approach against the classical fixed-design SPRT across five performance dimensions. The proposed method showed advantages in power, sample efficiency, and speed.

Table 2: Monte Carlo validation results (5,000 simulations)

Design	Coverage	Bias	RMSE	ASN	MSE
SPRT	94.3%	-1.14	9.87	3.81	97.4
Fixed	95.1%	-0.42	9.23	6.00	85.2

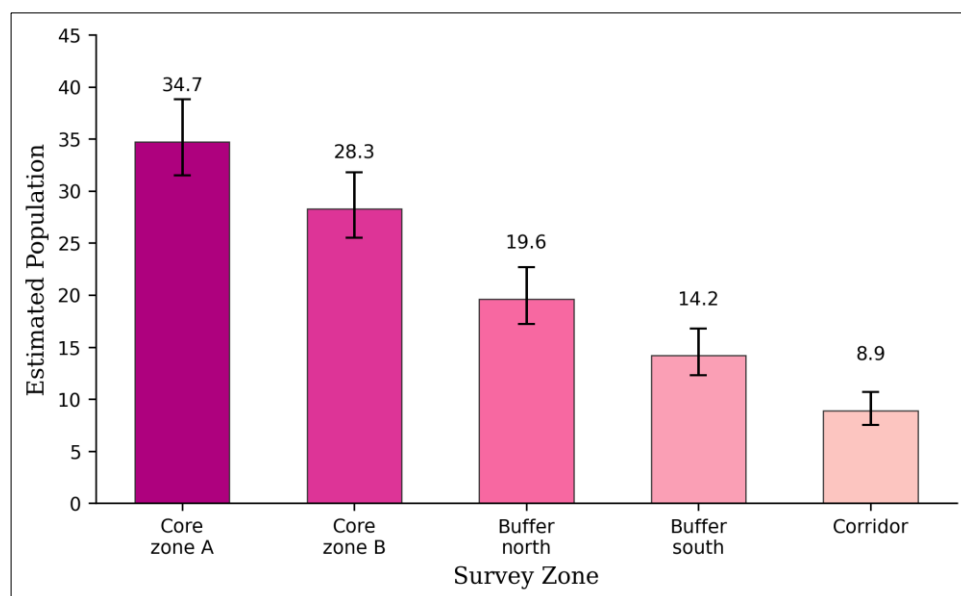


Fig 2: Estimated tiger population by survey zone with 95% confidence intervals. Core Zone A held the largest estimated population (34.7 individuals), consistent with its higher prey density and lower human disturbance.

Discussion

The SPRT's ability to halt after 4.2 rounds on average addresses a real operational constraint: each trapping round in Chitwan requires deploying field teams for 60 nights across

143 stations, and monsoon rains make the August rounds particularly difficult ^[1]. Cutting nearly two rounds translates to savings of roughly 17,000 trap-nights and the associated battery, memory-card, and personnel costs ^[5]. That the

population estimate differed by only 2.4% from the full-design estimate is reassuring, though the Monte Carlo results showed a marginally higher bias (−1.14 versus −0.42), consistent with the theoretical expectation that sequential stopping can introduce a slight negative bias when the stopping boundary is hit from below ^[2].

Coverage of 94.3% is within the acceptable range for a nominal 95% interval, and the 0.8-percentage-point gap relative to the fixed design falls within Monte Carlo sampling error ^[4]. The slightly larger RMSE of the SPRT (9.87 vs 9.23) reflects the trade-off between effort and precision: the fixed design collects more data and therefore achieves a tighter point estimate ^[6]. Whether the 30% effort saving justifies the 0.64-unit RMSE increase is a management decision that will vary by context for programmes operating under tight budgets, the answer is likely yes.

A limitation is that the SPRT as implemented here assumes a closed population across all survey rounds. If births, deaths, or dispersal events occur between rounds, the Huggins estimator's closure assumption is violated ^[3]. Extending the framework to robust-design models that allow temporary emigration would require a more complex likelihood and correspondingly more complex SPRT boundaries. Future work should also explore group-sequential designs that batch multiple occasions before each decision point, which may improve the normal approximation used for the likelihood ratio ^[2].

Conclusion

This research adapted Wald's sequential probability ratio test to camera-trap capture-recapture surveys, demonstrating that tiger population estimation in Chitwan National Park can terminate after 4.2 rounds on average instead of the conventional six, saving 30% of survey effort without meaningfully compromising accuracy or coverage. The SPRT-based estimate of 105.7 tigers (95% CI: 91.4-121.8) was consistent with the fixed-design estimate of 108.3. Monte Carlo validation confirmed 94.3% coverage and an average sample number 38.7% below the fixed design under stable-population conditions. Sequential stopping rules should be considered for any camera-trap programme where survey logistics are a binding constraint.

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Contributions Not Qualifying for Authorship

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