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An EOQ model with exponentially increasing demand for deteriorating items under two levels of storage

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Abstract

The present paper deals with a single period inventory model for a deteriorating item which is stored in two different warehouses. Here the demand is assumed to increase exponentially with time. The results obtained are shown to agree the classical model. The effect of deterioration cannot be ignored in many inventory systems; deterioration is defined as decay, damage or spoilage such that the items cannot be used for the original purpose. Food items, photographic films, drugs and pharmaceuticals, chemicals, electronic components and radio active substances are some examples of items in which sufficient deterioration may occur during the normal storage period of the units and consequently and this loss must be taken into account while analysing the inventory systems.

Keywords: Inventory model, exponentially increasing demand, deterioration, own warehouse, rented warehouse

Introduction

The term inventory includes both the manufactured products that are available for sale and the raw materials utilized in the manufacturing process. Inventory is among the most valuable assets possessed by a business, as inventory turnover is one of the principal contributors to revenue generation and, consequently, shareholder returns. Inventory is classified as a current asset on the balance sheet of the company. Raw materials, finished goods, and work-in-progress constitute the three categories of inventory.

Inventory models for deteriorating items subject to exogenous demands have been studied by various authors in different periods. Most of the authors have assumed a constant rate of deterioration and also constant demand. These include Shah and Jaiswal (1977), Aggarwal and Goel (1984), Nahimias (1991), Aggarwal (1978), Aggarwal and Veena Jain (2001) ^[1, 2, 3]. Decay in radio active elements, spoilage in food grain storage, Pilferages form on hand inventory etc., are continuous in time and they are approximately proportional to the on-hand inventory. Efforts in analysing mathematical models of inventory in which a constant or variable proportion of the on-hand inventory gets deteriorated per unit of time have been studied. Ghare and Schrader (1963) and Emmons (1968) considered the case of a constant fraction of the on-hand inventory gets deteriorated per unit of time. This work has been extended by Covert and Philip (1973), who developed EOQ model for units with a variable rate of deterioration of items in the inventory. In all these models, shortages are not allowed and the demand is assumed to be deterministic. Shah (1977) has generalised the work of Ghare and Schrader (1963) and Covert and Philip (1973) have developed an inventory model by considering general deteriorating function with shortages. Two probabilistic models for deteriorating items with general deteriorating function are developed by Shah and Jaiswal (1977). Teng and Chang (2005) discussed an EOQ model for deteriorating items where demand depends on stock and price ^[4, 5, 6, 10, 11, 12].

We now develop a single period inventory model with exponentially increasing demand for deteriorating items under L_2 - system like the one discussed by Hartley (1976), Sarma & Sastry (1988) ^[7, 9]. Whenever initial on-hand inventory exceeds the storage capacity of the own warehouse (OW), the excess stock will be kept in a rented warehouse (RW). The goods kept in

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RW are prone to a lower rate of deterioration than the goods kept on OW. This is one of the reasons for a costly holding cost in RW.

We now study the effect of stock deterioration in both warehouses on the policy variables when the demand occurs with exponentially increasing during the period and determine the optimal order level.

Notations and Assumptions of the Model

We adopt the following notations in carrying out the mathematical model of the inventory system.

- **T:** Length of one inventory cycle
- **t_w :** Time at which the consumption from OW starts
- **t_1 :** Time at which the Shortages start
- **Q:** Order quantity
- **$Q_R(t)$, $Q_O(t)$:** On-hand inventory at time 't' in RW & OW respectively
- **C_3 :** Order cost per order
- **H, F:** Unit cost of holding/ unit time in OW & RW respectively, $F > H$
- **π :** Shortage cost per unit per unit time
- **C:** Unit cost of item
- **f(t):** Demand rate as a function of time
- **θ_1, θ_2 :** Rates of deterioration per unit time in OW & RW respectively, such that $\theta_2 < \theta_1$
- **Q(t):** Inventory level at time 't'

We assume that

1. Lead time is negligible,
2. Replenishment is instantaneous,
3. A deteriorating item is neither repaired nor replaced during given cycle,
4. Shortages are allowed and fully backlogged,
5. There is no extra fixed cost associated with the use of RW,
6. The stock kept in OW is used only after exhausting the stock kept in RW,
7. A constant fraction of the on-hand inventory gets deteriorated per unit time.

Model description and analysis

Let $Q(t)$ be the inventory level at any time t ($0 \leq t \leq T$) and demand rate $r(t)$ assumed to be deterministic and is increasing exponentially with time 't'.

Further let $f(t) = ae^{\alpha t}$, $0 \leq \alpha \leq 1$, $a > 0$

The inventory is depleting due to the simultaneous occurrence of demand and the deterioration.

Situation in RW

Let $Q_R(t)$ denote the on-hand inventory at time t in RW. Then the differential equation that describes the state of inventory is given by

$$Q'_R(t) + \theta_2 Q_R(t) = -ae^{\alpha t}, \quad 0 \leq t \leq t_w \quad (1)$$

The solution of (1) with boundary condition at $t = 0$, $Q_R(t) = Z$ is given by

$$Q_R(t) = Ze^{-\theta_2 t} + \frac{a}{\alpha + \theta_2} (e^{-\theta_2 t} - e^{\alpha t}), \quad 0 \leq t \leq t_w \quad (2)$$

At $t = t_w$, $Q_R(t_w) = 0$ hence (2) becomes

$$\frac{a}{\alpha + \theta_2} (e^{-\theta_2 t_w} - e^{\alpha t_w}) + Ze^{-\theta_2 t_w} = 0$$

so that

$$t_w = \frac{1}{\alpha + \theta_2} \log \left[1 + \frac{Z(\alpha + \theta_2)}{a} \right] \quad (3)$$

Stock loss due to deterioration in the interval $(0, t_w)$ is given by

$$D(t_w) = Z - \int_0^{t_w} ae^{\alpha t} dt - Q_R(t) \quad (4)$$

Again (2) gives

$$Z = e^{\theta_2 t} \left[Q_R(t) - \frac{a}{\alpha + \theta_2} (e^{-\theta_2 t} - e^{\alpha t}) \right] \quad (5)$$

Using (5) in (4), we get

$$D(t_w) = e^{\theta_2 t} \left[Q_R(t) - \frac{a}{\alpha + \theta_2} (e^{-\theta_2 t} - e^{\alpha t}) \right] - \int_0^{t_w} ae^{\alpha t} dt - Q_R(t) \quad (6)$$

Loss of stock due to deterioration till time t_w is given by

$$D(t_w) = (e^{\theta_2 t_w} - 1)Q_R(t_w) - \frac{a}{\alpha + \theta_2} (1 - e^{(\alpha + \theta_2)t_w}) - \frac{a}{\alpha} (e^{\alpha t_w} - 1) \quad (7)$$

Since $Q_R(t_w) = 0$,

$$D(t_w) = \frac{a}{\alpha + \theta_2} (e^{(\alpha + \theta_2)t_w} - 1) - \frac{a}{\alpha} (e^{\alpha t_w} - 1) \quad (8)$$

and

$$Z = \frac{a}{\alpha + \theta_2} (e^{(\alpha + \theta_2)t_w} - 1) \quad (9)$$

Thus

$$Q_R(t) = \frac{a}{\alpha + \theta_2} (e^{-\theta_2 t} - e^{\alpha t}) + e^{-\theta_2 t} \left(\frac{a}{\alpha + \theta_2} \right) (e^{(\alpha + \theta_2)t_w} - 1) \quad (10)$$

Situation in OW

During the interval $(0, t_1)$ the stock kept in OW is subjected to both deterioration and consumption. The inventory level $Q_O(t)$ at time 't' of the system in OW satisfies the equation

$$Q_O'(t) + \theta_1 Q_O(t) = -ae^{\alpha t}, \quad 0 \leq t \leq t_1 \quad (11)$$

Also

$$\frac{dQ(t)}{dt} = -ae^{\alpha t}, \quad t_1 \leq t \leq T \quad (12)$$

The solution of (11) with the boundary condition at $t = t_1$, $Q_O(t) = W$ is given by

$$Q_O(t) = We^{-\theta_1 t} + \frac{a}{\alpha + \theta_1} (e^{-\theta_1 t} - e^{\alpha t}), \quad 0 \leq t \leq t_1 \quad (13)$$

The solution of (12) with boundary condition at $t = t_1$, $Q(t_1) = 0$ is given by

$$Q(t) = \frac{a}{\alpha} \left(e^{\alpha t_1} - e^{\alpha t} \right), \quad t_1 \leq t \leq T \quad (14)$$

At $t = t_1$, $Q_0(t_1) = 0$ hence (13) becomes

$$\frac{a}{\alpha + \theta_1} (e^{-\theta_1 t_1} - e^{\alpha t_1}) + W e^{-\theta_1 t_1} = 0$$

so that

$$t_1 = \frac{1}{\alpha + \theta_1} \log \left[1 + \frac{W(\alpha + \theta_1)}{a} \right] \quad (15)$$

Stock loss due to deterioration in the interval $(0, t_1)$ is given by

$$D(t_1) = W - \int_0^{t_1} a e^{\alpha t} dt - Q_0(t) \quad (16)$$

Again (3.3.13) gives

$$W = e^{\theta_1 t} \left[Q_0(t) - \frac{a}{\alpha + \theta_1} (e^{-\theta_1 t} - e^{\alpha t}) \right] \quad (17)$$

Using (17) in (16), we get

$$\begin{aligned} D(t_1) &= e^{\theta_1 t} \left[Q_0(t) - \frac{a}{\alpha + \theta_1} (e^{-\theta_1 t} - e^{\alpha t}) \right] - \int_0^{t_1} a e^{\alpha t} dt - Q_0(t) \\ D(t_1) &= (e^{\theta_1 t} - 1) Q_0(t) - \frac{a}{\alpha + \theta_1} (1 - e^{(\alpha + \theta_1)t}) - \frac{a}{\alpha} (e^{\alpha t_1} - 1) \end{aligned} \quad (18)$$

Loss of stock due to deterioration within cycle T till time t_1 is

$$\therefore D(t_1) = (e^{\theta_1 t_1} - 1) Q_0(t_1) - \frac{a}{\alpha + \theta_1} (1 - e^{(\alpha + \theta_1)t_1}) - \frac{a}{\alpha} (e^{\alpha t_1} - 1) \quad (19)$$

Since $Q_0(t_1) = 0$,

$$D(t_1) = \frac{a}{\alpha + \theta_1} (e^{(\alpha + \theta_1)t_1} - 1) - \frac{a}{\alpha} (e^{\alpha t_1} - 1) \quad (20)$$

Backlogged demand i.e., $B(t_1)$ is given by

$$B(t_1) = \int_{t_1}^T a e^{\alpha t} dt \quad B(t_1) = \frac{a}{\alpha} (e^{\alpha T} - e^{\alpha t_1}) \quad (21)$$

Now

$$Q = D(t_w) + D(t_1) + \int_0^T a e^{\alpha t} dt \quad Q = \frac{a}{\alpha + \theta_2} (e^{(\alpha + \theta_2)t_w} - 1) + \frac{a}{\alpha + \theta_1} (e^{(\alpha + \theta_1)t_1} - 1) + \frac{a}{\alpha} (e^{\alpha T} - e^{\alpha t_1}) \quad (22)$$

And

$$W = \frac{a}{\alpha + \theta_1} (e^{(\alpha + \theta_1)t_1} - 1) \quad (23)$$

Thus

$$Q_o(t) = \frac{a}{\alpha + \theta_1} (e^{-\theta_1 t} - e^{\alpha t}) + e^{-\theta_1 t} \left(\frac{a}{\alpha + \theta_1} \right) (e^{(\alpha + \theta_1)t_1} - 1) \quad (24)$$

The total cost per unit time $C(T, t_w, t_1)$, is the sum of the ordering cost, inventory holding cost and shortage cost, i.e.,

$$C(T, t_w, t_1) = \frac{C_3}{T} + \frac{CQ}{T} + \frac{F}{T} \int_0^{t_w} Q_R(t) dt + \frac{H}{T} \int_0^{t_1} Q_o(t) dt - \frac{\pi}{T} \int_{t_1}^T Q(t) dt \quad (25)$$

$$\begin{aligned} C(T, t_w, t_1) &= \frac{C_3}{T} + \frac{C}{T} \left[\frac{a}{\alpha + \theta_2} (e^{(\alpha + \theta_2)t_w} - 1) + \frac{a}{\alpha + \theta_1} (e^{(\alpha + \theta_1)t_1} - 1) + \frac{a}{\alpha} (e^{\alpha T} - e^{\alpha t_1}) \right] dt \\ &+ \frac{F}{T} \int_0^{t_w} \left[\frac{a}{\alpha + \theta_2} (e^{-\theta_2 t} - e^{\alpha t}) + e^{-\theta_2 t} \left(\frac{a}{\alpha + \theta_2} \right) (e^{(\alpha + \theta_2)t_w} - 1) \right] dt \\ &+ \frac{H}{T} \int_0^{t_1} \left[\frac{a}{\alpha + \theta_1} (e^{-\theta_1 t} - e^{\alpha t}) + e^{-\theta_1 t} \left(\frac{a}{\alpha + \theta_1} \right) (e^{(\alpha + \theta_1)t_1} - 1) \right] dt \\ &+ \frac{\pi a}{T \alpha} \int_{t_1}^T (e^{\alpha t} - e^{\alpha t_1}) dt \\ C(T, t_w, t_1) &= \frac{C_3}{T} + \frac{a}{T \alpha} \left(C + \frac{\pi}{\alpha} \right) (e^{\alpha T} - e^{\alpha t_1}) + \frac{Ca}{T(\alpha + \theta_2)} (e^{(\alpha + \theta_2)t_w} - 1) + \frac{Ca}{T(\alpha + \theta_1)} (e^{(\alpha + \theta_1)t_1} - 1) \\ &- \frac{Fa}{T \alpha (\alpha + \theta_2)} (e^{\alpha t_w} - 1) + \frac{Fa}{T \theta_2 (\alpha + \theta_2)} (e^{(\alpha + \theta_2)t_w} - e^{\alpha t_w}) - \frac{Ha}{T \alpha (\alpha + \theta_1)} (e^{\alpha t_1} - 1) \\ &+ \frac{Ha}{T \theta_1 (\alpha + \theta_1)} (e^{(\alpha + \theta_1)t_1} - e^{\alpha t_1}) - \frac{\pi a}{\alpha} e^{\alpha t_1} + \frac{\pi a}{T \alpha} t_1 e^{\alpha t_1} \end{aligned} \quad (26)$$

For $F = H$ in the above equation, the model reduces to

$$C(T, t_w, t_1) = \frac{A}{T} + \frac{a}{T\alpha} \left(C + \frac{\pi}{\alpha} \right) (e^{\alpha T} - e^{\alpha t_1}) + \frac{Ca}{T(\alpha + \theta)} (e^{(\alpha + \theta)t_1} - 1) + \frac{\pi a}{T\alpha} t_1 e^{\alpha t_1} \\ - \frac{Ha}{T\alpha(\alpha + \theta)} (e^{\alpha t_1} - 1) + \frac{Ha}{T\theta(\alpha + \theta)} (e^{(\alpha + \theta)t_1} - e^{\alpha t_1}) - \frac{\pi a}{\alpha} e^{\alpha t_1}$$

This is same as in Aggarwal and Veena Jain (2001).

Our objective is to minimise the cost function $C(T, t_w, t_1)$ which is a differentiable function.

$$\text{Let } t_w = \beta t_1 \text{ and } t_1 = \gamma T, \text{ where } 0 < \beta < 1 \text{ and } 0 < \gamma < 1 \quad (27)$$

This assumption is meaningful since the units kept in RW will be consumed first as the holding cost of the RW is higher than the holding cost of OW. So, the length of time used by RW is less than the length of time used by OW. Obviously, the total length of the cycle (T) is more than the length of the time t_1 .

From this assumption the cost function which is a three variable now becomes a function of single variable, i.e. T and the cost function of the entire inventory system become

$$C(T) = \frac{C_3}{T} + \frac{a}{T\alpha} \left(C + \frac{\pi}{\alpha} \right) (e^{\alpha T} - e^{\alpha \gamma T}) + \frac{Ca}{T(\alpha + \theta_2)} (e^{(\alpha + \theta_2)\beta \gamma T} - 1) + \frac{Ca}{T(\alpha + \theta_1)} (e^{(\alpha + \theta_1)\gamma T} - 1) \\ - \frac{Fa}{T\alpha(\alpha + \theta_2)} (e^{\alpha \beta \gamma T} - 1) + \frac{Fa}{T\theta_2(\alpha + \theta_2)} (e^{(\alpha + \theta_2)\beta \gamma T} - e^{\alpha \beta \gamma T}) - \frac{Ha}{T\alpha(\alpha + \theta_1)} (e^{\alpha \gamma T} - 1) \\ + \frac{Ha}{T\theta_1(\alpha + \theta_1)} (e^{(\alpha + \theta_1)\gamma T} - e^{\alpha \gamma T}) - \frac{\pi a}{\alpha} e^{\alpha \gamma T} + \frac{\pi a \gamma}{\alpha} e^{\alpha \gamma T} \quad (28)$$

To minimise the cost function $C(T)$, the necessary condition is given by

$$\frac{dC(T)}{dT} = 0$$

This gives

$$\pi \gamma (1 - \gamma) T^2 = \left(C + \frac{\pi}{\alpha} \right) \left(T - \frac{1}{\alpha} \right) e^{\alpha (1 - \gamma) T} + \left(C + \frac{\pi}{\alpha} \right) \left(\frac{1}{\alpha} - \gamma T \right) + \frac{C}{(\alpha + \theta_2)} e^{-\alpha \gamma T} \\ - \left(C + \frac{F}{\theta_2} \right) e^{\{(\alpha + \theta_2)\beta \gamma - \alpha \gamma\} T} + \left(C + \frac{F}{\theta_2} \right) \beta \gamma T e^{\{(\alpha + \theta_2)\beta \gamma - \alpha \gamma\} T} - \frac{C}{(\alpha + \theta_1)} [e^{\{(\alpha + \theta_1)\gamma - \alpha \gamma\} T} - e^{\alpha \gamma T}] \\ + C \gamma T e^{\theta_1 \gamma T} + \frac{F}{\theta_2} \left(\frac{1}{\alpha} - \beta \gamma T \right) e^{\alpha \gamma (\beta - 1) T} - \frac{H}{\alpha(\alpha + \theta_1)} e^{-\alpha \gamma T} + \frac{H}{\theta_1} \left(\gamma T - \frac{1}{\alpha + \theta_1} \right) e^{\theta_1 \gamma T} \\ - \frac{H \gamma T}{\theta_1} + \frac{H}{\alpha \theta_1} \quad (29)$$

The optimal value of T says T^* can be obtained from equation (29) and optimal order quantity q^* can be obtained from (22). The sufficient condition for minimising $C(T)$ is

$$\frac{d^2 C(T)}{dT^2} > 0$$

Where

$$\begin{aligned} \frac{d^2 C(T)}{dT^2} = & \frac{2C_3}{T^3} + \frac{2C_3}{T^3\alpha} \left(C + \frac{\pi}{\alpha} \right) (e^{\alpha T} - e^{\alpha\gamma T}) - \frac{a}{T^2} \left(C + \frac{\pi}{\alpha} \right) (e^{\alpha T} - \gamma e^{\alpha\gamma T}) \\ & + \frac{2Ca}{T^3(\alpha + \theta_2)} (e^{(\alpha + \theta_2)\beta\gamma T} - 1) - \frac{2Ca\beta\gamma}{T^2} e^{(\alpha + \theta_2)\beta\gamma T} + \frac{Ca(\alpha + \theta_2)\beta^2\gamma^2}{T} e^{(\alpha + \theta_2)\beta\gamma T} \\ & + \frac{2Ca}{T^3(\alpha + \theta_1)} (e^{(\alpha + \theta_1)\gamma T} - 1) - \frac{2Ca\gamma}{T^2} e^{(\alpha + \theta_1)\gamma T} + \frac{Ca(\alpha + \theta_1)\gamma^2}{T} e^{(\alpha + \theta_1)\gamma T} - \frac{2Fa}{T^3\alpha\theta_2} e^{\alpha\beta\gamma T} \\ & + \frac{2Fa}{T^3\alpha(\alpha + \theta_2)} + \frac{2Fa}{T^3\theta_2(\alpha + \theta_2)} e^{(\alpha + \theta_2)\beta\gamma T} + \frac{2Fa\beta\gamma}{T^2(\alpha + \theta_2)} e^{\alpha\beta\gamma T} - \frac{Fa\alpha\beta^2\gamma^2}{T(\alpha + \theta_2)} e^{\alpha\beta\gamma T} \\ & - \frac{2Fa\beta\gamma}{T^2\theta_2} e^{(\alpha + \theta_2)\beta\gamma T} + \frac{2Fa\alpha\beta\gamma}{T^2\theta_2(\alpha + \theta_2)} e^{\alpha\beta\gamma T} + \frac{Fa(\alpha + \theta_2)\beta^2\gamma^2}{T\theta_2} e^{(\alpha + \theta_2)\beta\gamma T} - \frac{Fa\alpha^2\beta^2\gamma^2}{T\theta_2(\alpha + \theta_2)} e^{\alpha\beta\gamma T} \\ & - \frac{2Ha}{T^3\alpha\theta_1} e^{\alpha\gamma T} + \frac{2Ha}{T^3\alpha(\alpha + \theta_1)} + \frac{2Ha\gamma}{T^2\theta_1} e^{\alpha\gamma T} - \frac{Ha\alpha\gamma^2}{T(\alpha + \theta_1)} e^{\alpha\gamma T} + \frac{2Ha}{T^3\theta_1(\alpha + \theta_1)} e^{(\alpha + \theta_1)\gamma T} \\ & - \frac{2Ha\gamma}{T^2\theta_1(\alpha + \theta_1)} e^{(\alpha + \theta_1)\gamma T} + \frac{Ha(\alpha + \theta_1)\gamma^2}{T\theta_1} e^{(\alpha + \theta_1)\gamma T} - \frac{Ha\alpha^2\gamma^2}{T\theta_1(\alpha + \theta_1)} e^{\alpha\gamma T} - \pi a \alpha (1 - \gamma) \gamma^2 e^{\alpha\gamma T} \end{aligned} \quad (30)$$

Sensitivity Analysis of the Model:

As an illustration to the above model, we consider a hypothetical system with the following parameters:

$C_3 = 200$, $a = 50$, $\alpha = 0.1$, $\beta = 0.3$, $\gamma = 0.8$, $\theta_1 = 0.05$, $\theta_2 = 0.02$, $C = 15$, $F = 6$, $H = 4$, and $\pi = 25$.

The equation (29) generally does not yield to a closed form solution. However, one can find the approximate solution by using Taylor series expansion for exponential function and ignoring second and higher order terms $\alpha\theta_2$, $\alpha\theta_1$, $\alpha\beta$, $\alpha\beta\gamma$, etc., a quadratic equation in T will be obtained which can be solved to yield the approximate value of T and near optimum order quantity can be obtained from (22).

The following table shows the variations in the cycle length (T), cost and optimal order quantity for different values of ' α '.

Table 1: Variations in ' α '

α	t_w	t_1	T	q	Optimal Cost
0.1	0.21336	0.7112	0.889	80	496
0.2	0.20368	0.67893	0.84866	104	552
0.3	0.19464	0.64879	0.81098	127	627
0.4	0.18619	0.62062	0.77577	148	721
0.5	0.17828	0.59428	0.74284	167	836
0.6	0.17089	0.56962	0.71203	185	971
0.7	0.16396	0.54653	0.68317	201	1129
0.8	0.15747	0.52489	0.65611	216	1309
0.9	0.15138	0.50459	0.63074	230	1503

Observations

From the table 1, it is observed that for fixed values of deteriorating rates in OW and RW as the demand increases, optimum ordering quantity increases significantly and the associated cost is also increasing and the use of rented warehouse (RW) in time units is decreasing. This is quite expected result since the large amount of demand will empty the RW fastly.

The following table shows the variations in the cycle length (T), cost and optimal order quantity for different values of ' β '.

Table 2: Variations in β

β	t_w	t_1	T	Q	Optimal Cost
0.3	0.21336	0.7112	0.889	80	496
0.35	0.24606	0.70302	0.87878	84	501
0.4	0.27737	0.69343	0.86679	88	506
0.45	0.30718	0.68262	0.85328	92	512
0.5	0.33539	0.67078	0.83847	96	519
0.52	0.34621	0.66579	0.83224	97	522
0.54	0.35677	0.66069	0.82586	98	524
0.56	0.36706	0.65547	0.81934	100	527
0.58	0.37709	0.65016	0.81269	101	530
0.6	0.38685	0.64475	0.80593	102	534

Observations

It is observed from the above table that as β increases, the usage of rented warehouse increases in time units. However, the optimum ordering quantity and the associated cost marginally increases.

Table 3: Variations in γ

Γ	t_w	t_1	T	q	Optimal Cost
0.8	0.21336	0.7112	0.889	80	496
0.82	0.21822	0.72741	0.88709	80	480
0.84	0.22297	0.74325	0.88482	81	464
0.86	0.22761	0.7587	0.88221	81	449
0.88	0.23213	0.77376	0.87927	82	434
0.9	0.23652	0.78839	0.87589	83	420
0.92	0.24079	0.80262	0.87241	83	407
0.94	0.24492	0.81641	0.86852	84	394
0.96	0.24893	0.82977	0.86434	84	382

Observations

From the table 3, it can be seen that as γ increases there is a marginal increase in the usage of rented warehouse in time units and the marginal increase can be seen in the economic order quantity and there is considerable decrease in the inventory total cost.

Discussion

In this paper an economic order quantity for deteriorating items when demand increases exponentially with time under two levels of storage is presented. For mathematical simplification certain assumptions are made in equation (27), which is justifiable since the total cycle length is always greater than the length of time used by OW and RW.

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