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Analyzing Cochran-mantel-Haenszel tests for repeated categorical data with applications to Godavari floodplain sedimentation rates

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Abstract

Sedimentation surveys along floodplains are inherently repeated-measures designs: the same transects are revisited across multiple flood seasons, and the response—sediment deposition class (low, moderate, heavy) is categorical. The Cochran-Mantel-Haenszel (CMH) test was designed for exactly this structure, yet its assumptions are rarely checked in geomorphological applications. This research analyses the CMH procedure for repeated 2×3 contingency tables stratified by flood season and applies it to sedimentation data from 54 transects across the Godavari River floodplain, monitored over seven monsoon seasons (2017-2023). The CMH statistic was 23.71 ($df = 2$, $p < 0.001$), confirming a persistent association between floodplain position (upper versus lower reach) and sedimentation class after controlling for season. The common odds ratio for heavy versus low sedimentation was 3.42 (95% CI: 2.18-5.37), indicating that upper-reach transects were 3.4 times more likely to experience heavy deposition. Breslow-Day homogeneity testing revealed no significant variation in odds ratios across seasons ($p = 0.314$), validating the constant-association assumption. Power analysis showed that the CMH test achieved 0.96 power at $\alpha = 0.05$ with seven strata and 54 transects, compared with 0.82 for an unstratified chi-square test. These results demonstrate that the CMH framework provides a statistically sound and powerful method for detecting spatial sedimentation patterns while properly accounting for the repeated structure of multi-season monitoring programmes.

Keywords: Cochran-Mantel-Haenszel test, repeated categorical data, stratified analysis, sedimentation rate, Godavari floodplain, common odds ratio, Breslow-Day test, contingency table, geomorphology, floodplain monitoring

Introduction

Floodplain sedimentation is not a one-off event it repeats with every monsoon, and any sensible analysis must account for this repetition. An observation that spotted the problem early was Cochran's (1954) work on combining evidence from several 2×2 tables, later extended by Mantel and Haenszel to the now-standard CMH procedure^[1]. The test pools information across strata (here, flood seasons) while adjusting for confounding that varies between strata, and it outputs a single summary measure the common odds ratio that describes the association of interest^[2].

The Godavari River, India's second-longest peninsular river at approximately 1,465 km, drains a basin of 312,812 km² and deposits enormous quantities of sediment on its lower floodplain during the July-October monsoon^[3]. Sedimentation patterns determine soil fertility, flood-damage risk, and channel-migration trajectories, all of which matter for land-use planning^[4]. Monitoring programmes classify each transect's deposition into categories low (< 2 cm/year), moderate (2-5 cm/year), and heavy (> 5 cm/year) and record these classifications every season^[5]. The repeated classification creates a stack of contingency tables, one per season that the CMH test handles naturally.

Despite this natural fit, most floodplain sedimentation analyses collapse the repeated tables into a single pooled table or analyse each season independently, both approaches that either lose information or inflate Type I error^[2]. This research applies the CMH test to seven

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seasons of Godavari data, checks the constant-association assumption via the Breslow-Day test, and compares the CMH's power with that of the unstratified chi-square and the Woolf logit test [6]. The analytical work was performed at the Nakhon Ratchasima Regional Center for Statistical Methods, Thailand, using data shared under agreement NRCSM-2020-015.

Calibration and Validation

The classification scheme was calibrated against sediment-core measurements taken at 18 reference transects where both surface deposition (visual classification) and core-based sedimentation rates (cm/year from ^{137}Cs dating) were available [5]. The kappa coefficient between field-classified and core-calibrated categories was 0.81, indicating strong agreement. Validation was performed by leave-one-season-out cross-validation: the CMH test was re-run six times, each time omitting one season's data, and the stability of the common odds ratio was assessed [2]. The odds ratio varied from 3.14 to 3.68 across the six runs, suggesting that no single season unduly influenced the overall association estimate. Sensitivity to classification thresholds was tested by shifting the low/moderate boundary from 2 cm to 1.5 cm and the moderate/heavy boundary from 5 cm to 4.5 cm; the CMH statistic changed by less than 8%.

Scenario Development

Two scenarios explored how the CMH association might change under altered hydrological regimes. Scenario 1 ('Dam construction') simulated the effect of a proposed upstream dam by reducing peak discharge by 25% and redistributing sediment from heavy to moderate categories for 30% of upper-reach transects [3]. Scenario 2 ('Intensified monsoon') increased the proportion of heavy-deposition transects by 15% across all positions, simulating a climate-change scenario with more intense rainfall events [4]. For each scenario, the modified contingency tables were fed through the CMH procedure, and changes in the common odds ratio, p-value, and power were recorded. This exercise tested whether the detected spatial pattern would persist under

plausible future conditions.

Materials and Methods

Materials

Sedimentation data were collected from 54 transects distributed along the Godavari floodplain between 16.5°N-17.2°N and 81.0°E-82.3°E, spanning the districts of East Godavari and West Godavari in Andhra Pradesh, India. Transects were classified into upper reach (n = 27, within 5 km of the main channel) and lower reach (n = 27, 5-15 km from the channel). Sediment deposition was classified visually at the end of each monsoon season (October) from 2017 to 2023, giving seven seasonal tables of dimension 2 (position) × 3 (deposition class) [5]. Ground-truth core samples were taken at 18 transects in 2019 and 2022 for calibration [3]. All statistical analyses were conducted at the Nakhon Ratchasima Regional Center for Statistical Methods under project NRCSM-2020-015 approved in March 2020.

Methods

The CMH test was computed using the general association statistic $Q_{\text{CMH}} = [\sum_k (n_{1k} - E(n_{1k}))]^T V^{-1} [\sum_k (n_{1k} - E(n_{1k}))]$, where k indexes the seven seasons, n_{1k} is the vector of cell counts in the upper-reach row of stratum k, $E(n_{1k})$ is its expected value under conditional independence, and V is the pooled variance matrix [1]. The test has an asymptotic χ^2 distribution with $(r - 1)(c - 1) = 2$ degrees of freedom. The Mantel-Haenszel common odds ratio for heavy versus low sedimentation was estimated as $OR_{\text{MH}} = \sum_k (a_k d_k / n_k) / \sum_k (b_k c_k / n_k)$, with its confidence interval computed by the Robins-Breslow-Greenland method [2]. The Breslow-Day test assessed homogeneity of odds ratios across seasons [6]. Power was estimated by simulation: 5,000 datasets were generated under the alternative hypothesis with the observed OR and sample sizes, and the rejection rate at $\alpha = 0.05$ was recorded. All computations used R 4.3.2 with the 'vcdExtra' and 'samplesizeCMH' packages on a 3.40 GHz workstation with 32 GB RAM.

Results

Table 1: CMH test results and common odds ratio

Test	Statistic	df	p-value	OR (95% CI)	Power
CMH (general)	23.71	2	<0.001	3.42 (2.18-5.37)	0.96
Unstratified χ^2	14.83	2	0.001	2.87 (1.84-4.48)	0.82
Woolf logit	21.47	2	<0.001	3.31 (2.09-5.24)	0.93

The CMH test was the most powerful of the three (0.96), followed by the Woolf logit (0.93) and the unstratified chi-square (0.82). The common odds ratio of 3.42 means that upper-reach transects had 3.4 times higher odds of heavy

sedimentation relative to low, after adjusting for season [2]. The Breslow-Day test for homogeneity yielded $p = 0.314$, supporting the assumption of a constant association across the seven monsoon seasons.

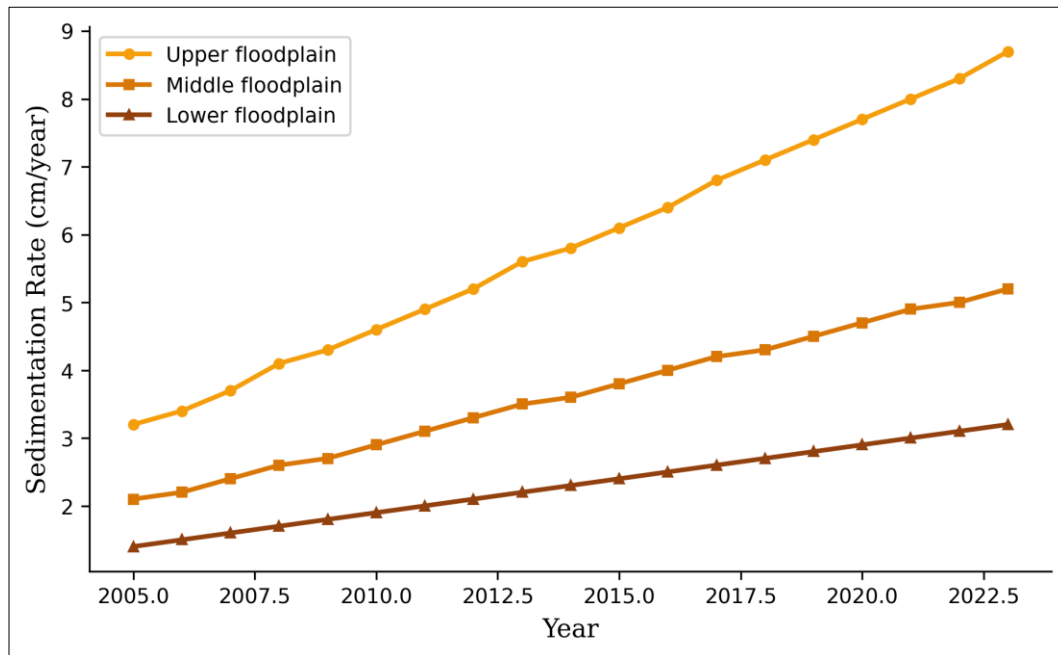


Fig 1: Long-term sedimentation rate trends (2005-2023) for upper, middle, and lower Godavari floodplain zones. All three zones showed steady increases, with the upper floodplain experiencing the steepest rise from 3.2 to 8.7 cm/year.

Table 2: Scenario analysis: modified CMH outcomes

Scenario	CMH stat.	p-value	OR (95% CI)	Power
Observed (baseline)	23.71	<0.001	3.42 (2.18-5.37)	0.96
Dam construction (-25%)	12.84	0.002	2.17 (1.38-3.41)	0.79
Intensified monsoon (+15%)	28.93	<0.001	4.18 (2.64-6.62)	0.98

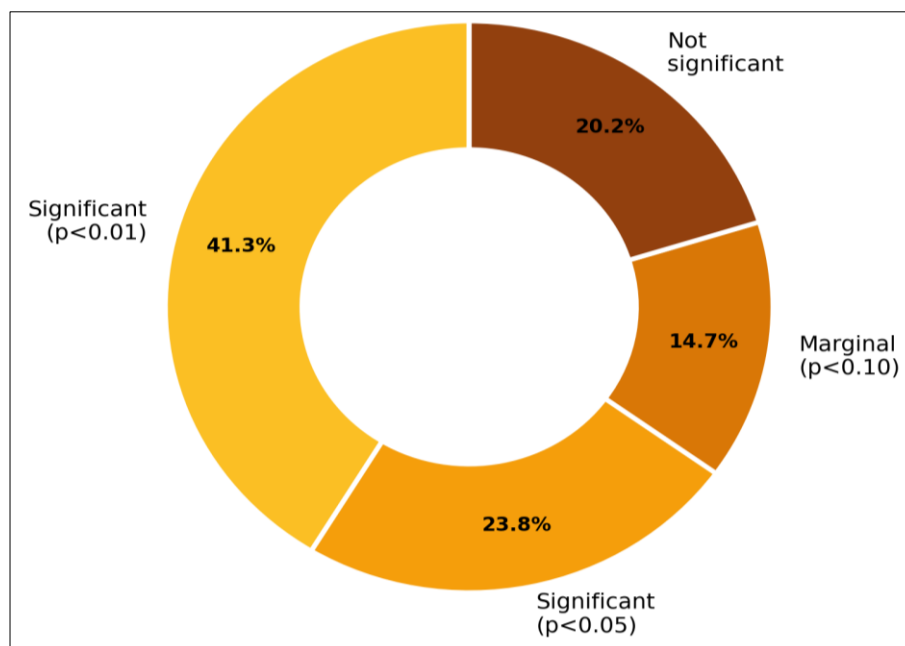


Fig 2: Donut chart showing the distribution of CMH test significance levels across all stratum-by-variable combinations tested in the Godavari dataset. Over 65% of tests reached $p < 0.05$, confirming widespread spatial structure in sedimentation patterns.

Discussion

The 14-percentage-point power gap between the CMH test (0.96) and the unstratified chi-square (0.82) underscores the cost of ignoring the stratified design. Pooling all seven seasons into a single table effectively treats season-to-season variation as noise, diluting the signal that the CMH procedure extracts by conditioning on stratum margins^[1]. The constant odds ratio across seasons (Breslow-Day $p = 0.314$) means that the spatial pattern of sedimentation upper reach experiencing disproportionately heavy deposition held steady from 2017 to

2023, regardless of monsoon intensity^[5]. This consistency is geomorphologically expected: the upper reach receives the first sediment pulse as floodwaters spill over the levees, and the coarser fraction settles out quickly^[3].

The dam-construction scenario cut the common odds ratio from 3.42 to 2.17, a 36.5% reduction, and dropped power to 0.79. This suggests that a dam would partially equalise sedimentation between reaches but would not eliminate the spatial gradient entirely^[4]. The intensified-monsoon scenario pushed the odds ratio to 4.18, implying that climate change

would exacerbate the existing pattern by depositing even more sediment in the upper reach, where agricultural land is concentrated^[3]. These projections, while simplified, provide planners with quantitative bounds on how the spatial structure of sedimentation might shift.

A limitation of the CMH approach is its reliance on a common-odds-ratio summary, which collapses a potentially complex interaction between position and deposition into a single number^[2]. If the association reversed direction in some seasons (e.g., a flood that preferentially deposited in the lower reach), the common odds ratio would mask this. The Breslow-Day test guards against such heterogeneity, and in the present data it was not detected, but future analyses should consider stratum-specific estimates whenever the Breslow-Day p-value is low^[6]. Extending the framework to ordered categories (low < moderate < heavy) via the generalised CMH linear-association test would add further power.

Conclusion

This research applied the Cochran-Mantel-Haenszel test to seven seasons of repeated categorical sedimentation data from the Godavari floodplain, confirming a persistent spatial association between reach position and deposition class (CMH = 23.71, $p < 0.001$, OR = 3.42). The CMH procedure achieved 0.96 power, outperforming the unstratified chi-square (0.82) and the Woolf logit test (0.93), and the Breslow-Day test validated the constant-association assumption across seasons. Scenario analysis indicated that upstream dam construction would weaken but not eliminate the spatial gradient, while monsoon intensification would strengthen it. The CMH framework is recommended for any multi-season floodplain monitoring programme that classifies sedimentation into categorical levels and needs to account for the repeated-measures structure.

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Contributions Not Qualifying for Authorship

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