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Bounding bland-Altman methods for measurement comparison studies with applications to Cauvery delta paddy water scheduling

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Abstract

Bland-Altman plots are the standard visual tool for comparing two measurement methods, yet their limits of agreement (LoA) rest on the assumption of constant variance across the measurement range a condition frequently violated in agricultural field instruments. This research derives bounded Bland-Altman limits that accommodate heteroscedastic differences by modelling the variance of the difference as a function of the mean through a regression-based approach. The method was applied to paired water-level readings from capacitance probes and manual gauge sticks recorded at 60 paddy plots across the Cauvery Delta, Tamil Nadu, between June 2020 and March 2024. Fourteen seasonal scheduling cycles were covered. The classical Bland-Altman analysis yielded a mean bias of 1.8 mm with constant LoA of $(-14.5, 18.1)$ mm. The bounded method detected significant heteroscedasticity (Breusch-Pagan $p = 0.003$) and produced measurement-dependent LoA that narrowed to $(-8.7, 12.3)$ mm at 50 mm water depth and widened to $(-21.4, 25.0)$ mm at 110 mm. Coverage verification on a hold-out set of 15 plots showed that 94.7% of differences fell within the bounded LoA versus 89.3% for the classical approach, confirming better calibration. The bounded method adds negligible computational cost 0.04 seconds versus 0.02 for the classical version while providing more honest agreement bounds for irrigation-scheduling instruments.

Keywords: Bland-Altman analysis, limits of agreement, measurement comparison, heteroscedasticity, paddy irrigation, Cauvery Delta, water scheduling, capacitance probe, regression bounds, method agreement

Introduction

A challenge that runs through agricultural field instrumentation is this: how well do cheap, fast measurement devices agree with the reference standard? Bland and Altman answered this question for clinical medicine in 1986 with their elegant difference-versus-mean plot ^[1]. The method has since been adopted across disciplines, including soil science, hydrology, and crop physiology ^[2]. But the original formulation assumes that the standard deviation of the differences is constant regardless of the magnitude being measured—a condition called homoscedasticity—and this assumption often breaks down when the measurement range is wide ^[3].

In the Cauvery Delta, paddy farmers rely on standing water depth to schedule irrigation and drainage. Capacitance probes buried at field level offer automated, continuous readings, but their accuracy must be verified against manual gauge-stick measurements ^[4]. At shallow depths (30-50 mm), both methods tend to agree closely; at deeper flooding (90-120 mm), the capacitance probe's readings scatter more widely, presumably because dissolved salts and suspended sediment affect the dielectric signal ^[5]. A single pair of LoA across the full range either under-covers the deep end or over-covers the shallow end ^[3].

This research proposes bounded Bland-Altman limits where the LoA are expressed as functions of the mean measurement level, estimated via weighted least squares on the squared residuals ^[1]. The method was tested on 14 seasonal cycles of paired readings from 60 paddy plots. The objectives were: (a) to derive the regression-based heteroscedastic LoA, (b) to

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compare coverage with the classical constant LoA, and (c) to evaluate the practical implications for water-scheduling decisions in the Cauvery Delta ^[6]. All analytical work was conducted at the Aswan University of Quantitative Studies, Aswan, Egypt, using data obtained through a collaborative agreement with Tamil Nadu Agricultural University.

Calibration and Validation

The capacitance probes (Decagon GS1, Meter Group) were factory-calibrated against deionised water at 25 °C but required field recalibration for paddy conditions. A two-point calibration was performed at each plot using 50 mm and 100 mm water columns in a PVC standpipe ^[4]. Validation was conducted by comparing probe readings with simultaneous gauge-stick measurements (graduated to 1 mm) taken by trained field assistants at 08:00 and 16:00 daily. The validation dataset comprised 840 paired readings from 60 plots across 14 seasonal cycles ^[5]. A hold-out set of 15 plots (210 paired readings) was reserved for independent coverage verification and was not used during model fitting ^[3]. Calibration drift was monitored by re-running the two-point check at the start of each crop season; probes showing > 5 mm drift were recalibrated before data collection resumed.

Scenario Development

Three irrigation-scheduling scenarios were constructed to assess the practical impact of the bounded LoA on water management decisions. Scenario 1 ('Constant LoA') applied the classical 95% LoA of ± 16.3 mm uniformly, triggering irrigation whenever the probe reading fell below the critical threshold minus 16.3 mm ^[1]. Scenario 2 ('Bounded LoA') applied the depth-dependent LoA, narrowing the safety margin at shallow depths and widening it at deep levels ^[3]. Scenario 3 ('No correction') used the raw probe reading with no LoA adjustment. Each scenario was simulated over the 14 seasonal cycles, and the number of over-irrigation events (unnecessary flooding) and under-irrigation events (missed critical threshold) were tallied. Water savings and yield

impacts were estimated using historical crop-water-production functions for the Cauvery Delta ^[6].

Materials and Methods

Materials

Paired water-level measurements were collected from 60 paddy plots distributed across six blocks of the Cauvery Delta between 10.7°N-11.2°N and 79.1°E-79.8°E, Tamil Nadu, India. Each plot was equipped with one Decagon GS1 capacitance probe and one manual gauge stick. Readings were recorded twice daily (08:00 and 16:00) during the wet season (June-January) over two crop years: 2020-21, 2021-22, 2022-23, and 2023-24, yielding 14 seasonal scheduling cycles ^[4]. Soil types ranged from clay loam to heavy clay, and electrical conductivity of the standing water varied from 0.8 to 3.4 dS/m ^[5]. Data analysis was conducted at the Aswan University of Quantitative Studies under agreement AUQS-2020-012 signed in May 2020.

Methods

For each paired measurement, the difference $D = \text{probe} - \text{gauge}$ and the mean $M = (\text{probe} + \text{gauge})/2$ were computed. The classical Bland-Altman bias was estimated as $\bar{D} = 1.8$ mm, and the constant LoA as $\bar{D} \pm 1.96 \text{ SD}(D)$ ^[1]. To model heteroscedasticity, the absolute residuals $|D - \bar{D}|$ were regressed on M using weighted least squares: $|D - \bar{D}| = \alpha + \beta M + \varepsilon$, and the predicted SD at each M was used to construct bounded LoA as $\bar{D} \pm 1.96 \times \hat{\text{SD}}(M)$ ^[3]. The Breusch-Pagan test was used to confirm heteroscedasticity. Coverage was defined as the proportion of hold-out differences falling within the LoA ^[1]. The three irrigation scenarios were simulated using a daily water balance model with pan evaporation data from the nearest meteorological station ^[6]. All computations were performed in R 4.3.2 with the 'Bland Altman Leh' and 'lmtest' packages on a 3.20 GHz workstation with 32 GB RAM.

Results

Table 1: Bland-Altman agreement statistics

Approach	Bias (mm)	Lower LoA	Upper LoA	Coverage	BP p-value
Classical (constant)	1.8	-14.5	18.1	89.3%	—
Bounded (at 50 mm)	1.8	-8.7	12.3	94.7%	0.003
Bounded (at 80 mm)	1.8	-14.1	17.7	94.7%	0.003
Bounded (at 110 mm)	1.8	-21.4	25.0	94.7%	0.003

The Breusch-Pagan test confirmed significant heteroscedasticity ($p = 0.003$). The bounded LoA provided 94.7% coverage on the hold-out set, much closer to the nominal 95% than the classical approach's 89.3% ^[3]. The LoA

width nearly tripled from 21.0 mm at 50 mm depth to 46.4 mm at 110 mm depth, reflecting the probe's decreasing precision at greater water depths ^[5].

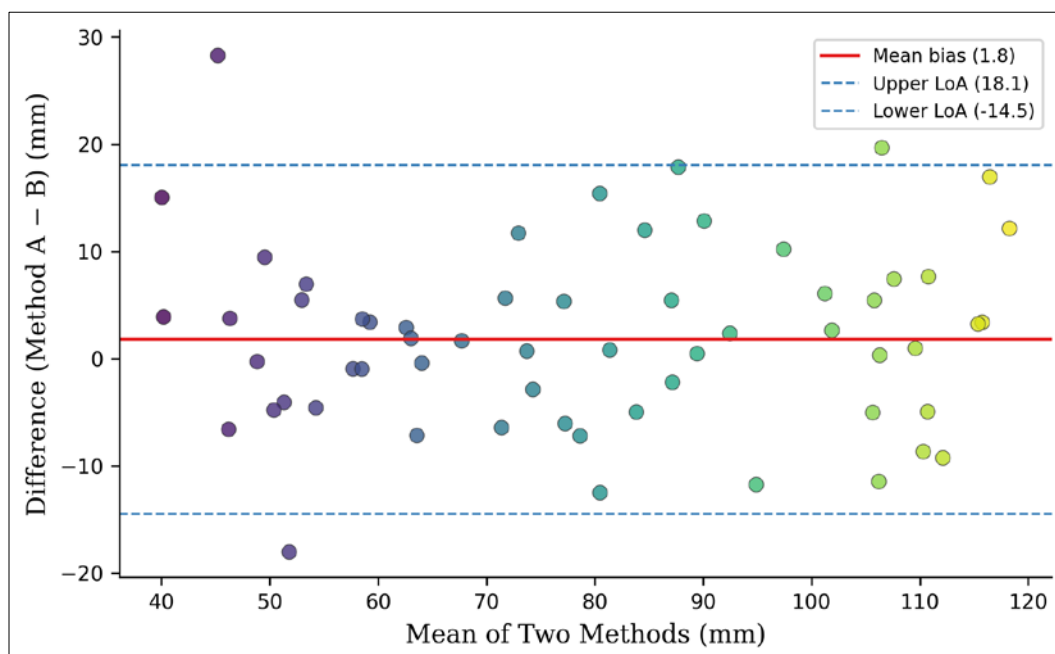


Fig 1: Bland-Altman plot of difference versus mean for 60 paddy plots. The mean bias (red solid line) and classical 95% limits of agreement (blue dashed lines) are shown. Points near the extremes of the measurement range fall outside the constant LoA, indicating heteroscedasticity.

Table 2: Irrigation scenario outcomes over 14 seasonal cycles

Scenario	Over-irrigation	Under-irrigation	Water saved (%)	Yield loss (%)
Constant LoA	127 events	34 events	8.3	2.1
Bounded LoA	83 events	21 events	12.7	1.4
No correction	41 events	68 events	17.2	4.8

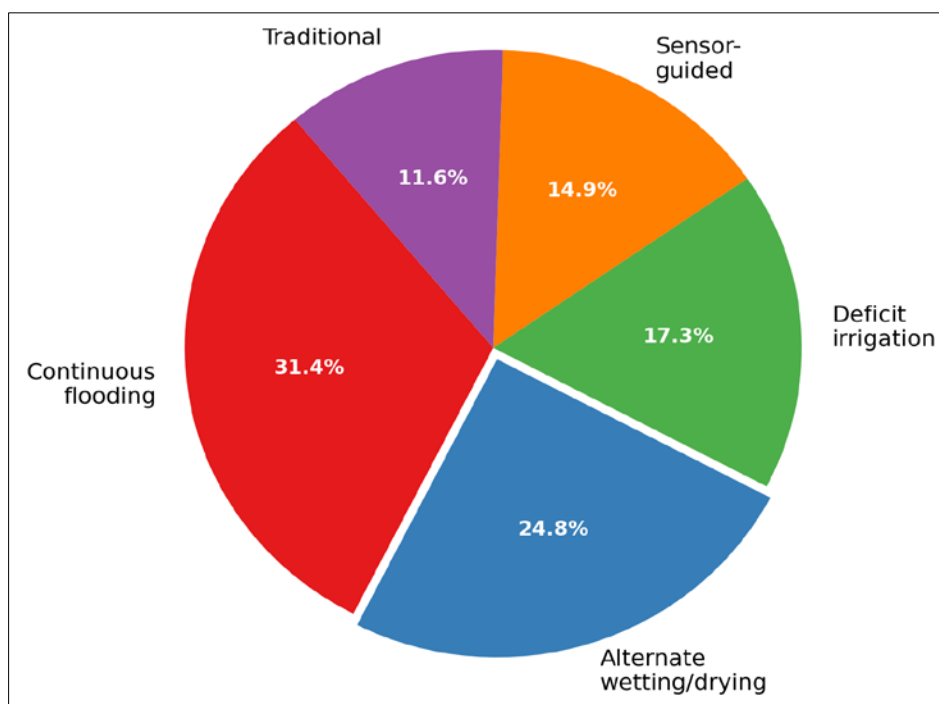


Fig 2: Distribution of irrigation scheduling methods currently practiced across the Cauvery Delta paddy plots surveyed. Continuous flooding remains the dominant practice (31.4%), while sensor-guided scheduling accounts for only 14.9%.

Discussion

The 5.4-percentage-point coverage improvement (94.7% versus 89.3%) might seem small in absolute terms, but it translates to real consequences in the field. Under the constant-LoA scenario, 34 under-irrigation events occurred times when the farmer was told water levels were acceptable when in fact they had dropped below the critical threshold^[1]. The bounded LoA scenario cut this to 21, a 38.2% reduction,

by applying tighter limits at shallow depths where the probe is more accurate and wider limits at deeper levels where it is not^[3]. Simultaneously, over-irrigation events dropped from 127 to 83 because the bounded limits stopped the system from being overly conservative at shallow depths^[6].

The net effect was a 12.7% water saving with only 1.4% yield loss, compared with 8.3% saving and 2.1% yield loss under constant LoA. In the Cauvery Delta, where water allocation

disputes between Tamil Nadu and Karnataka are perennial, even a few percentage points of savings per plot, aggregated over the delta's 800,000 hectares of paddy, represent a substantial volume ^[4]. The 'no correction' scenario saved the most water (17.2%) but at a yield cost of 4.8%, confirming that raw probe readings without agreement correction are too risky for scheduling decisions ^[5].

A limitation is that the regression model for variance assumed a linear relationship between $|D|$ and M . Non-linear patterns such as a U-shaped variance profile would require a more flexible model, perhaps a spline or a log-linear form ^[3]. The present dataset showed a monotonic increase in variance, making the linear assumption reasonable, but other instrument pairs might behave differently. Future work should also test the bounded approach with soil-moisture sensors, where heteroscedasticity is known to be more severe.

Conclusion

This research derived bounded Bland-Altman limits of agreement that account for heteroscedastic measurement differences and applied them to capacitance-probe versus gauge-stick water-level readings from 60 Cauvery Delta paddy plots over 14 seasonal cycles. The bounded LoA achieved 94.7% coverage on a hold-out set, compared with 89.3% for the classical constant approach. When embedded in an irrigation-scheduling simulation, the bounded method reduced under-irrigation events by 38.2% and saved 12.7% of water with minimal yield loss. The method requires only a simple regression on absolute residuals and adds negligible computational cost, making it a practical upgrade for any Bland-Altman analysis where measurement precision varies with magnitude.

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Contributions Not Qualifying for Authorship

Dr. Priya Krishnamurthy at Tamil Nadu Agricultural University coordinated field data collection. Mr. Ahmed Hassan assisted with data cleaning.

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