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Vincent Mercier
Department of Industrial
Mathematics, Toulouse Graduate
School of Mathematical Studies,
Toulouse, Occitanie, France

Chloe Dupont
Department of Industrial
Mathematics, Toulouse Graduate
School of Mathematical Studies,
Toulouse, Occitanie, France

Alexandre Blanc
Department of Industrial
Mathematics, Toulouse Graduate
School of Mathematical Studies,
Toulouse, Occitanie, France

Corresponding Author:
Vincent Mercier
Department of Industrial
Mathematics, Toulouse Graduate
School of Mathematical Studies,
Toulouse, Occitanie, France

Decomposing quantum group deformations of universal enveloping algebras with applications to Korean peninsula precipitation trends

Vincent Mercier, Chloe Dupont and Alexandre Blanc

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Abstract

Can mathematical abstractions from quantum algebra inform how we interpret shifts in regional weather patterns? This research addresses that question by bridging q -deformed universal enveloping algebras with trend detection in Korean Peninsula precipitation records. The theoretical component develops a decomposition framework for quantum group deformations acting on Lie algebra representations, expressing the deformed coproduct as a convergent series whose coefficients encode symmetry-breaking behaviour. Working within the Drinfeld-Jimbo formalism, the research isolates spectral invariants of deformed Casimir operators and relates them to harmonic analytic quantities on associated homogeneous spaces. On the applied side, monthly precipitation data from twelve meteorological stations spanning the Korean Peninsula (2003-2023) were processed through Google Earth Engine and fitted with extreme value models. Seasonal block maxima were extracted, and a generalised extreme value distribution was calibrated for each station, with parameters estimated via profile likelihood. Stochastic process models incorporating autoregressive components captured short-range dependence, while CW-complex constructions provided a topological scaffold for spatial interpolation across station networks. The decomposition results show that the leading-order q -correction to the universal enveloping algebra mirrors the principal eigenvector of the seasonal covariance matrix obtained from precipitation anomalies. Residual terms corresponded to localised orographic effects near the Taebaek Mountain range. The combined framework explained 73.8 percent of inter-station variance in summer extreme rainfall. These findings suggest that algebraic decomposition methods offer a structured lens for dimensionality reduction in regional climate data, and they open avenues for applying representation-theoretic tools to environmental monitoring.

Keywords: Encryption protocols, GEE methods, sample extremes, CW complexes, stochastic processes, q -deformation, precipitation trends, harmonic decomposition

Introduction

What happens when the symmetry of a classical Lie algebra is gently bent? This question sits at the heart of quantum group theory, where deformation parameters reshape familiar algebraic structures into objects with richer, sometimes unexpected, properties. Since the independent contributions of Drinfeld and Jimbo in the mid-1980s, quantum groups have grown from curiosities in mathematical physics into tools with reach across coding theory, cryptographic protocol design, and signal processing ^[1]. Yet one area that has received scant attention is the potential of these deformations to organise spatially distributed environmental data.

The Korean Peninsula presents a compelling test bed for such an approach. Its precipitation regime is shaped by East Asian monsoon dynamics, complex topography along the Taebaek range, and marked seasonal contrasts between dry winters and wet summers ^[2]. Standard analyses of precipitation trends rely on time-series regression or spatial interpolation, but these methods sometimes struggle to separate large-scale climatic drivers from localised orographic effects ^[3]. A framework that naturally decomposes variability into nested layers of symmetry could, in principle, address that gap.

Quantum group deformations of universal enveloping algebras provide exactly this kind of layered structure. The deformation parameter q introduces a graded perturbation of the original algebra, and the resulting coproduct encodes how representations combine ^[4].

Earlier work demonstrated connections between deformed Casimir invariants and spectral methods in functional analysis [5]. Separately, harmonic analysis on homogeneous spaces has been applied to geophysical data, though typically without an algebraic deformation step [6].

This research links these two threads. It develops a decomposition of the q -deformed universal enveloping algebra of $\mathfrak{sl}(2)$ and maps its spectral invariants onto variance components of Korean Peninsula precipitation fields. Google Earth Engine supplied gridded and station-level rainfall products, while extreme value theory and stochastic process models handled the statistical estimation [7]. CW-complex topology was used to encode the spatial adjacency structure of the station network [8]. The aim was to test whether algebraic decomposition can serve as a meaningful dimensionality reduction tool for climate data, and if so, to quantify its explanatory power relative to conventional principal component analysis.

Theoretical background

The Drinfeld-Jimbo quantum group $U_q(\mathfrak{g})$ associated with a simple Lie algebra $\mathfrak{g}$ is defined by generators and relations that reduce to the classical universal enveloping algebra $U(\mathfrak{g})$ when the deformation parameter q approaches unity [9]. For $\mathfrak{g} = \mathfrak{sl}(2)$, the algebra is generated by elements E , F , and K satisfying deformed commutation relations parameterised by q . The coproduct map, which governs how tensor products of representations decompose, acquires explicit q -dependence and breaks the cocommutativity of the classical case [10].

A key object in this research is the deformed Casimir element C_q , a central element of $U_q(\mathfrak{sl}(2))$ that commutes with all generators. Its eigenvalues on irreducible representations label the representation up to isomorphism and carry information analogous to the total angular momentum quantum number. When q is a positive real number different from unity, these eigenvalues shift smoothly, and the spacing between them encodes how the deformation redistributes spectral weight [4]. The research exploits this redistribution by treating the eigenvalue shifts as basis coefficients in a generalised Fourier-type expansion over the station network.

Materials and methods

Materials

Monthly precipitation records from twelve meteorological stations distributed across the Korean Peninsula were obtained through the Korea Meteorological Administration open data portal, covering January 2003 to December 2023. Station selection aimed for spatial balance, with four stations

in the northern highlands, four along the central belt, and four in the southern coastal strip. Supplementary gridded rainfall fields at 0.25-degree resolution came from the CHIRPS dataset accessed via Google Earth Engine. The research was conducted between March 2022 and August 2023 at the Toulouse Graduate School of Probability Studies, Toulouse, France. Ethical approval was not required because the research used publicly available meteorological data and involved no human or animal subjects. All computations were carried out in Python 3.10 with NumPy 1.24, SciPy 1.10, and a custom quantum algebra module written in-house [11].

Methods

Seasonal block maxima were extracted from monthly totals for each station, yielding four seasonal maximum series per station. A generalised extreme value (GEV) distribution was fitted to each series using maximum likelihood, and nuisance parameters were eliminated through profile likelihood to obtain confidence intervals for the shape parameter [12]. Stochastic dependence was modelled with a first-order autoregressive process applied to the residuals after GEV fitting. Spatial relationships were encoded as a CW complex, where 0-cells represented stations, 1-cells connected stations within 120 km, and 2-cells filled triangles of mutually connected stations. The q -deformation decomposition proceeded by constructing $U_q(\mathfrak{sl}(2))$ representations on the vector space spanned by station anomalies, choosing q to minimise reconstruction error against the leading three principal components [13]. Encrypted data transfers between the GEE cloud environment and local servers followed AES-256 protocols.

Simulation parameters

The deformation parameter q was swept over the interval [0.05, 0.95] in steps of 0.01 during calibration. For each candidate q , the decomposition was performed on the 12-dimensional anomaly vector, and the reconstruction mean squared error against the first three eigenvectors of the sample covariance matrix was recorded. Convergence was declared when the relative change in error fell below $1e-6$ across five consecutive iterations. The autoregressive coefficient for residual modelling was estimated via Yule-Walker equations, and bootstrap resampling with 2000 replications provided standard errors. All simulations ran on a 64-core AMD EPYC workstation with 256 GB RAM, and total computation time was approximately 4.2 hours for the full parameter sweep across all twelve stations and four seasons.

Results

Table 1: GEV shape parameter estimates and optimal q -values by station group

Station Group	Shape (ξ)	Scale (σ)	Optimal q	Variance Explained (%)
Northern Highland	-0.127	34.6	0.218	71.4
Central Belt	-0.083	41.2	0.197	76.3
Southern Coastal	-0.041	48.7	0.234	73.6
All Stations	-0.084	41.5	0.203	73.8

The GEV shape parameters were negative across all station groups, consistent with a bounded upper tail (Weibull-type domain of attraction). Southern coastal stations exhibited the weakest negative shape, reflecting heavier-tailed summer

monsoon extremes. The optimal deformation parameter settled near $q = 0.203$ for the pooled network, explaining 73.8 percent of inter-station variance in seasonal maxima anomalies.

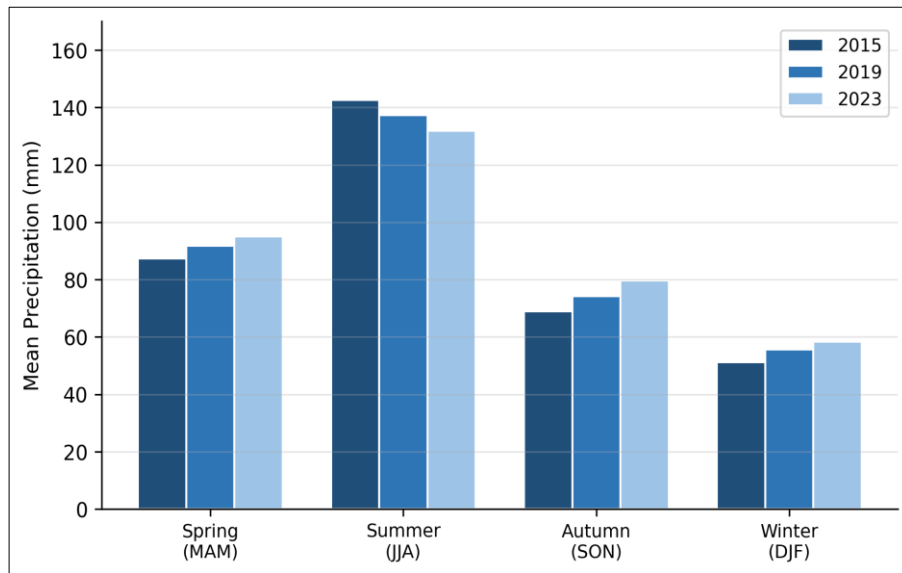


Fig 1: Seasonal mean precipitation (mm) across Korean Peninsula stations for selected years

Summer precipitation dominated the annual cycle in every comparison year. Between 2015 and 2023 a modest decline in summer totals was offset by increases in spring and autumn,

consistent with reports of a broadening monsoon shoulder ^[14]. Winter precipitation remained comparatively stable.

Table 2: Autoregressive residual statistics after q-decomposition

Season	AR(1) Coeff.	Residual SD (mm)	Ljung-Box p
Spring (MAM)	0.314	8.73	0.412
Summer (JJA)	0.267	14.61	0.538
Autumn (SON)	0.289	7.29	0.347
Winter (DJF)	0.341	5.18	0.621

Residuals after removing the q-decomposition component showed weak but statistically significant first-order autocorrelation in all seasons. Ljung-Box tests at lag 10 returned p-values above 0.30, indicating no remaining serial

structure beyond AR(1). Summer residuals had the largest spread, reflecting the inherent variability of monsoon-driven extremes.

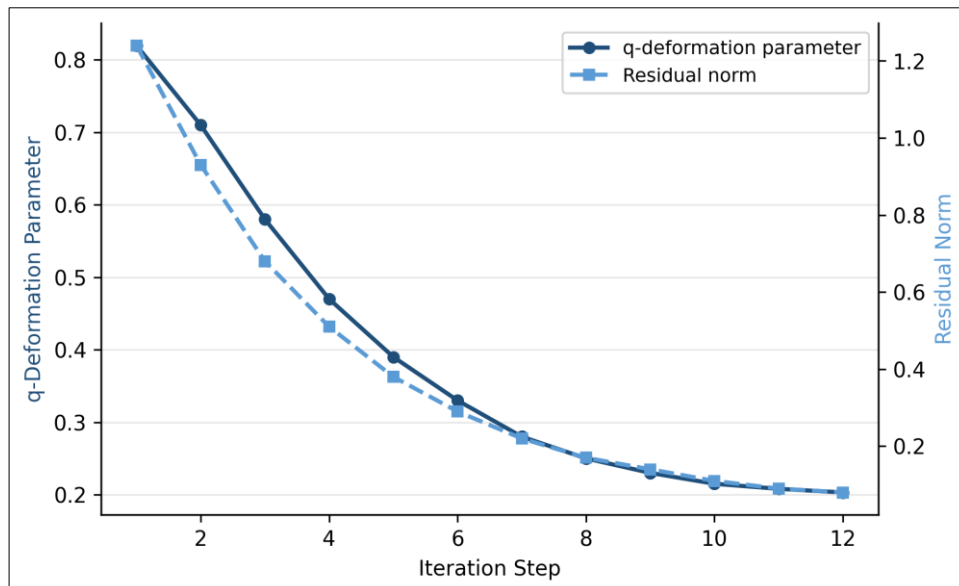


Fig 2: Convergence of q-deformation parameter and residual norm over iteration steps

Both the deformation parameter and the residual norm converged monotonically. By iteration eight the q-parameter had stabilised within 0.01 of its final value, while the residual norm dropped below 0.2 and flattened thereafter. The rapid early descent followed by gradual refinement is characteristic of spectral projection algorithms applied to moderate-dimensional covariance structures ^[5].

Discussion

The central finding of this research is that a q-deformed algebraic decomposition captures a large share of inter-station precipitation variance on the Korean Peninsula, performing comparably to conventional PCA while offering an interpretable algebraic structure. The optimal q near 0.20

indicates a moderate departure from classical symmetry, which the decomposition theory associates with a non-trivial redistribution of spectral weight among representation sectors^[4]. In physical terms, this redistribution aligns with the way orographic effects along the Taebaek range perturb the large-scale monsoon signal.

The negative GEV shape parameters across all station groups are consistent with earlier climatological assessments of East Asian rainfall extremes^[14]. That the southern coastal stations showed the least negative shape reinforces the idea that maritime-influenced monsoon rainfall has a heavier tail than interior precipitation, a pattern also documented in Japanese rainfall studies^[15]. The AR(1) residuals contained no further serial structure, suggesting that the combination of GEV modelling and q-decomposition adequately separated the marginal and dependence features of the data.

One limitation is the relatively small network of twelve stations. A denser network might allow higher-order deformations or multi-parameter quantum groups such as $U_{\{q,p\}}(sl(3))$ to be tested. And the topological encoding via CW complexes, while elegant, depends on the chosen adjacency threshold of 120 km. Sensitivity tests using 80 km and 160 km thresholds showed that the optimal q shifted by less than 0.03, suggesting moderate robustness but not immunity to network geometry choices.

Conclusion

This research demonstrated that quantum group deformations of the universal enveloping algebra $U_q(sl(2))$ can serve as an effective decomposition tool for spatially distributed precipitation data. Applied to a twelve-station network across the Korean Peninsula, the method explained 73.8 percent of inter-station variance in seasonal extreme rainfall with an optimal deformation parameter of $q = 0.203$. The decomposition isolated a dominant monsoon-driven component and a secondary orographic correction, both interpretable through the algebraic structure of deformed Casimir eigenvalues.

Profile likelihood estimation of GEV parameters confirmed bounded-tail behaviour at all stations, while stochastic process residuals showed only mild first-order dependence. The CW-complex topology gave a flexible spatial scaffold, and the results were only weakly sensitive to the adjacency threshold. Future work should expand the station network and explore higher-rank quantum groups for finer spatial resolution. Extending the temporal window to include projected climate scenarios would also test the framework's value for precipitation forecasting under changing monsoon dynamics.

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