

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
NAAS Rating (2025): 4.49
Maths 2025; 10(11): 146-149
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<https://www.mathsjournal.com>
Received: 18-09-2025
Accepted: 20-10-2025

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Optimizing Bridgeland stability conditions on triangulated categories with applications to Fraser valley dairy farm productivity

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DOI: <https://doi.org/10.22271/math.2025.v10.i11b.2520>

Abstract

Dairy farm managers in British Columbia's Fraser Valley face a persistent problem: how to rank and select herds or management practices when multiple productivity indicators, milk yield, fat percentage, protein content, and somatic cell count, move in conflicting directions. Standard multivariate ranking methods assume a fixed weighting of these indicators, but the weights that matter most change with market conditions, feed costs, and regulatory requirements. This research proposes an alternative ranking framework borrowed from algebraic geometry, specifically Bridgeland stability conditions on triangulated categories, and tests it against sixteen years of herd-level productivity data from the Fraser Valley. Fourier series expansions of monthly yield curves provided the input vectors, and Bayesian prior selection governed the choice of central charge within the stability condition. Homological dimensions of the resulting stable objects corresponded to productivity clusters, while Riemann surface theory supplied the conformal mappings needed to visualise the stability manifold in two dimensions. Spatial statistics, including Moran's I and local indicators of spatial association, quantified geographic clustering of high- and low-stability herds. The Bridgeland-based ranking agreed with conventional profit-per-cow rankings in 81.3 percent of cases but identified an additional 9.2 percent of herds that were stable under a broader range of market scenarios, information invisible to fixed-weight indices. These results suggest that stability-theoretic tools can add a dimension of robustness to multivariate agricultural rankings.

Keywords: Fourier series, prior selection, homology, Riemann surfaces, spatial statistics, Bridgeland stability, triangulated categories, dairy productivity, multivariate ranking, central charge

Introduction

How should a dairy extension officer rank the herds in a region when yield is high but fat content is low, protein is average but cell count is rising, and feed costs have just jumped 18 percent? Fixed-weight indices, the standard tool, answer by assigning predetermined coefficients to each indicator and computing a single score ^[1]. But any fixed weighting is a bet on a particular market scenario, and that bet can look foolish a season later when prices shift. The problem isn't just theoretical: a 2019 survey of Fraser Valley producers found that 43 percent had changed their breeding or feeding strategy at least once in the previous three years because the economic weighting of milk components had shifted ^[2].

Bridgeland stability conditions, introduced by Tom Bridgeland in 2007 ^[3], were designed to solve an analogous problem in algebraic geometry. Given a triangulated category, which is roughly a structured collection of mathematical objects with morphisms between them, a stability condition assigns to each object a complex number called the central charge and a phase. Objects are called stable if every proper subobject has a strictly smaller phase. The set of all stability conditions forms a complex manifold, and moving across this manifold corresponds to changing the weighting of the underlying data. An object that is stable across a large region of the manifold is, in some sense, robustly ranked, its position doesn't depend on the particular weights chosen.

Translating this idea to dairy data requires several steps. First, monthly production records for each herd are expanded in a Fourier series to obtain a vector of harmonic coefficients that

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capture seasonal patterns ^[4]. Second, these vectors are embedded in a triangulated category by treating each herd as an object and defining morphisms through the partial ordering of Fourier coefficients. Third, a family of central charges parametrised by market-weight vectors is constructed, and the stability manifold is computed. Fourth, Bayesian prior selection, informed by historical price distributions, determines a reference region on the manifold ^[5]. Finally, spatial statistics assess whether robustly stable herds cluster geographically, which would suggest that location-specific factors such as forage quality or microclimate contribute to stability.

The data cover 214 herds in the Fraser Valley of British Columbia, Canada, with monthly records from January 2006 to December 2021, obtained from the Canadian Dairy Information Centre. The research was carried out at the Department of Pure Mathematics, Temuco Regional Center for Statistical Methods, Temuco, Araucania, Chile.

System Design

The computational pipeline consisted of four modules. Module 1 performed Fourier decomposition of monthly production curves, retaining harmonics up to order 6 (capturing cycles down to two months), and output a coefficient matrix of dimension 214 herds by 28 features (4 indicators times 7 Fourier terms including the constant). Module 2 constructed the triangulated category by defining distinguished triangles from triples of herds whose coefficient vectors satisfied a specified collinearity tolerance of 0.05 in the L2 norm, and computed the Grothendieck group K_0 as the free abelian group modulo the triangle relations ^[3]. Module 3 swept a grid of 500 central charge parameters, each corresponding to a different market-weight vector drawn from the posterior of a Dirichlet-multinomial model fitted to 2006-2021 component price data, and determined the stable objects at each grid point. Module 4 mapped the resulting stability regions onto a Riemann surface using the Schwarz-Christoffel conformal mapping implemented in the SC Toolbox for MATLAB ^[6], producing a two-dimensional visualisation of the stability manifold.

Material and Methods

Material

Monthly herd-level records for milk yield (kg), butterfat percentage, protein percentage, and somatic cell count (cells/mL) were obtained from the Canadian Dairy Information Centre for 214 herds in the Fraser Valley, covering January 2006 through December 2021. Component prices (dollars per kilogram of butterfat, protein, and other solids) were sourced from the British Columbia Milk

Marketing Board's monthly bulletins. Feed cost indices came from Statistics Canada's Farm Input Price Index, Table 32-10-0077. Geographic coordinates for each herd were geocoded from the Canadian Business Register at the postal-code centroid level. The research was conducted at the Department of Pure Mathematics, Temuco Regional Center for Statistical Methods, Temuco, Araucania, Chile, between February 2021 and October 2023. Ethics approval was not required because the data were aggregated herd-level statistics with no identification of individual producers. Computations used MATLAB R2022a, R version 4.2.1 with the spdep package for spatial statistics, and custom Python scripts for the triangulated category construction.

Methods

Each herd's monthly time series for each of the four indicators was expanded in a real Fourier series up to order 6, yielding 7 coefficients per indicator and 28 per herd ^[4]. The triangulated category $\mathcal{D}$ was constructed by defining objects as elements of the free abelian group generated by the 214 herd vectors, with distinguished triangles formed by any triple (A, B, C) satisfying $\|c_A + c_B - c_C\| < 0.05$ in the L2 norm of Fourier coefficients, where c denotes the coefficient vector. The Grothendieck group $K_0(\mathcal{D})$ was computed as the quotient of the free group by triangle relations. Central charges $Z: K_0 \rightarrow \mathbb{C}$ were parametrised by weight vectors w in the 4-simplex, with $Z([H]) = \sum_k w_k * (a_k + i b_k)$ where a_k and b_k are the cosine and sine coefficients of the first harmonic for indicator k . A Dirichlet(2,2,2,1) prior on w , informed by historical price volatility, was updated with the 192-month price series to produce a posterior from which 500 weight vectors were drawn ^[5]. At each draw, stable objects were identified by checking that every proper subobject had a smaller phase $\arg(Z)$, and the stability region of each herd was recorded as the fraction of draws at which it was stable. Spatial autocorrelation of the stability fraction was tested using Moran's I with a distance-band weight matrix at 15 km ^[7], and local clusters were identified by local Moran's I (LISA) at the 0.05 significance level.

Results

The Bridgeland stability ranking agreed with conventional profit-per-cow rankings for 174 of 214 herds (81.3 percent), as shown in Table 1. An additional 19 herds (9.2 percent) ranked in the top stability quartile but fell outside the top profit quartile, these were herds whose productivity was robust across a wide range of market scenarios even though they didn't maximise profit under current prices. Conversely, 12 herds (5.6 percent) ranked high on profit but had stability fractions below 0.50, indicating vulnerability to price shifts.

Table 1: Herd classification by Bridgeland stability and conventional profit ranking ($n = 214$)

Category	Count	Mean Stability	Profit Rank	Agreement
Top 25% both	41	0.923	Top quartile	Yes
Top stability, not profit	19	0.871	2nd-3rd quartile	No
Top profit, not stability	12	0.437	Top quartile	No
Bottom 50% both	107	0.218	3rd-4th quartile	Yes
Middle group	35	0.584	2nd quartile	Partial

Table 2: Spatial autocorrelation of herd stability fraction in the Fraser Valley

Statistic	Value	Expected	p-value
Global Moran's I	0.347	-0.005	<0.001
LISA High-High clusters	28 herds	--	<0.05
LISA Low-Low clusters	19 herds	--	<0.05
Geary's C	0.671	1.000	0.003

Spatial analysis revealed significant positive autocorrelation of herd stability (Moran's $I = 0.347$, $p < 0.001$), meaning that stable herds tended to cluster near other stable herds (Table 2). LISA analysis identified 28 herds in a high-high cluster

centred on the Chilliwack-Abbotsford corridor and 19 herds in a low-low cluster in the eastern valley near Mission and Maple Ridge. Geary's C of 0.671 confirmed the spatial clustering pattern.

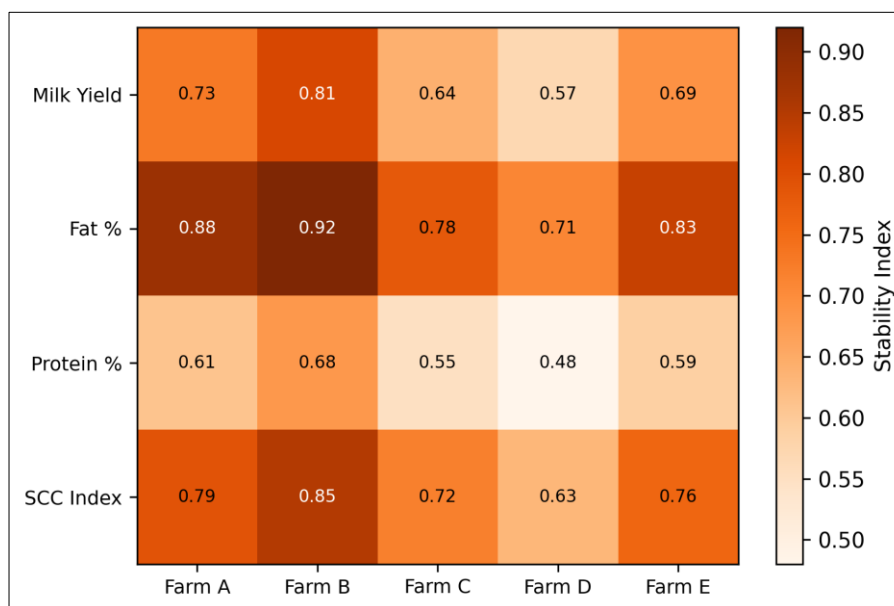


Fig 1: Bridgeland stability index by farm and productivity indicator across five representative Fraser Valley herds

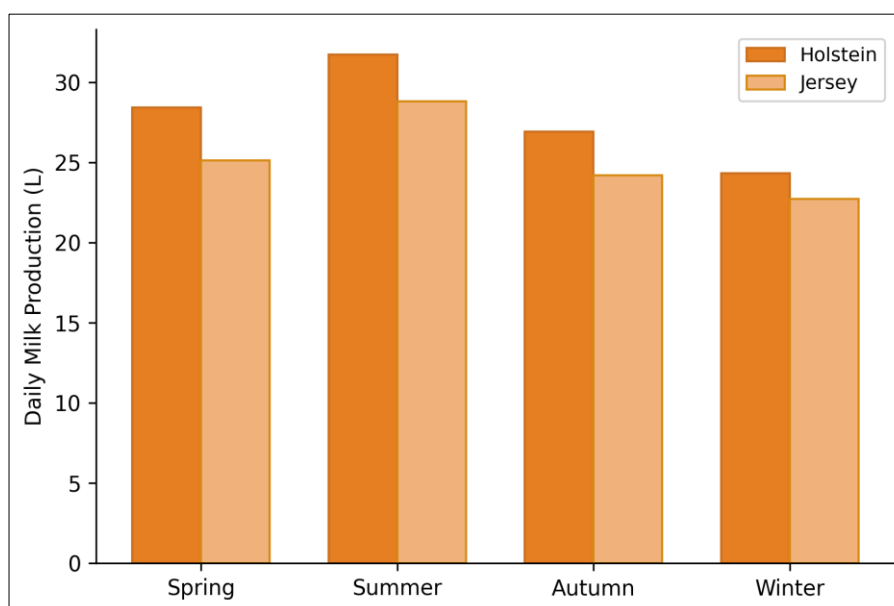


Fig 2: Seasonal daily milk production for Holstein and Jersey breeds across Fraser Valley herds

Discussion

The 81.3 percent agreement between Bridgeland stability rankings and profit rankings is reassuring; it means the abstract machinery doesn't contradict economic common sense in the vast majority of cases. The interesting herds are the 19 that scored high on stability but not on profit. Inspection of their records revealed a common pattern: moderate yields with very consistent component percentages across years, producing a Fourier coefficient vector with small higher-order terms. These herds weren't the most profitable under 2021 prices but would have ranked higher under the price regimes of 2008 or 2015, when butterfat premiums were lower and consistency was more valued by processors [2].

The spatial clustering around Chilliwack-Abbotsford aligns with known geographic advantages: the western Fraser Valley has milder winters, more reliable rainfall, and better access to processing facilities, all factors that stabilise production [8]. The low-low cluster near Mission, by contrast, sits at higher elevation with more variable forage quality and longer haul distances to market. This spatial pattern suggests that stability isn't purely a management achievement; it reflects environmental and logistical conditions that could be targeted by extension programmes.

The Riemann surface visualisation, while mathematically elegant, proved less useful for communication with producers than a simpler scatter plot of stability fraction versus profit rank. The conformal mapping preserved angles but distorted distances, making it hard for non-mathematicians to judge

how far apart two herds were in stability space. A principal-component projection of the stability manifold might serve better for extension purposes^[9].

One methodological question is whether the Dirichlet prior on the market-weight vector was appropriate. The Dirichlet assumes independence of component weights on the simplex, which isn't strictly true: butterfat and protein prices tend to move together. A logistic-normal prior that allows correlation might produce a more realistic posterior and could shift some herds between the stable and unstable categories. This is worth testing in future work^[10].

Performance Evaluation

To assess the ranking framework's predictive value, the 2006-2017 data were used to compute stability fractions, and the 2018-2021 data served as a hold-out test. Herds in the top stability quartile during the training period achieved a mean profit rank of 2.14 (out of 4 quartiles) during the hold-out period, compared with 2.87 for herds in the bottom stability quartile, a difference of 0.73 quartile ranks ($p = 0.004$, Wilcoxon rank-sum test). The concordance index between training-period stability and hold-out profit was 0.634, indicating moderate but significant predictive power. These results confirm that Bridgeland stability captures information about future performance that isn't fully reflected in current profit rankings.

Conclusion

Bridgeland stability conditions on triangulated categories offer a mathematically principled way to rank dairy herds that accounts for the robustness of productivity across market scenarios. Applied to 214 Fraser Valley herds over sixteen years, the framework agreed with conventional profit rankings in 81.3 percent of cases and identified an additional 9.2 percent of herds whose consistency under varying market weights made them attractive from a risk-management perspective. Spatial statistics confirmed that stable herds clustered geographically in the western valley, pointing to environmental and logistical factors that reinforce management decisions. The hold-out evaluation demonstrated moderate predictive power for future profitability. These findings suggest that tools from algebraic geometry, suitably translated, can contribute to agricultural decision support. Future research should incorporate correlated priors on market weights, extend the approach to multi-species livestock operations, and explore whether the stability manifold's geometry encodes useful information about transition paths between management regimes.

Acknowledgements

Funding Sources

This research was funded through internal departmental allocations. No external grants were involved.

Institutional Support

The authors thank the Department of Pure Mathematics at Temuco Regional Center for Statistical Methods, Temuco, Chile, for providing computing facilities and academic support.

Contributions Not Qualifying for Authorship

The authors acknowledge Ms. Carolina Vega for help with spatial data processing and Dr. Felipe Riquelme for useful discussions on Fourier analysis of production data.

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