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Daniel Kim
Department of Algebra and
Geometry, Calgary Laboratory
of Numerical Sciences, Calgary,
Alberta, Canada

Rachel Bergeron
Department of Algebra and
Geometry, Calgary Laboratory
of Numerical Sciences, Calgary,
Alberta, Canada

Constructing green function estimates for Schrodinger operators with applications to atlas mountain erosion models

Daniel Kim and Rachel Bergeron

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Abstract

The interplay between quantum mechanical operators and classical erosion processes seems unlikely at first, yet both involve elliptic differential operators whose Green functions encode spatial propagation characteristics. This research constructs pointwise estimates for the Green function of Schrodinger operators with bounded measurable potentials and maps these estimates onto erosion rate predictions across the Atlas Mountains of Morocco. Survival analysis models for bedrock exposure events at 35 monitoring sites provided the geomorphological framework, with erosion treated as a time-to-event process. Linear programming optimized the mapping between operator spectral parameters and topographic covariates. Ramsey-theoretic bounds on the minimum number of monitoring sites needed for reliable spectral estimation guided the sampling design. The breakdown point of the Green function estimator, measuring its resistance to outlier contamination, was computed at 0.31 for the bounded potential class. Coding theory principles structured the error-correcting scheme used when interpolating between sparse monitoring stations. Green function decay estimates of the form $G(x,y) \leq C * \exp(-\alpha * |x-y|)$ with $\alpha = 0.47$ per kilometer correlated with observed erosion rates at $r = 0.83$ ($p < 0.001$). Survival models showed median bedrock exposure times ranging from 12.7 to 41.3 years across elevation zones. Linear programming solutions identified three spectral parameters that, combined, predicted 71.8% of erosion rate variance. The Ramsey bound indicated that 28 stations sufficed for the spectral estimation at the achieved precision.

Keywords: Survival analysis, linear programming, Ramsey theory, breakdown point, coding theory, Green function, Schrodinger operator, Atlas Mountains, erosion modeling, spectral estimates

Introduction

Consider the contrast between a quantum particle tunneling through a potential barrier and rainwater dissolving limestone on a mountainside. Both processes depend on properties of elliptic operators: the Schrodinger operator in one case, the diffusion-reaction operator in the other ^[1]. The Green function, which inverts the operator and describes how a point source propagates through the medium, is the shared mathematical object. If pointwise estimates for the Green function can be constructed accurately, they may carry information about spatial propagation rates in either physical setting.

The Atlas Mountains extend roughly 2,500 kilometers across Morocco, Algeria, and Tunisia, with peaks reaching 4,167 meters at Jbel Toubkal ^[2]. Erosion in this semi-arid mountain system is driven by a combination of rainfall events, freeze-thaw cycles, and tectonic uplift, producing spatially variable denudation rates that have been measured at scattered sites using cosmogenic nuclide dating and sediment yield studies ^[3]. The sparse and irregular distribution of monitoring data creates an interpolation challenge that conventional geostatistical methods handle through spatial covariance modeling ^[4].

Green function estimates for Schrodinger operators with bounded potentials have been developed extensively in the mathematical physics literature ^[5]. Key results include Gaussian upper bounds, exponential decay away from the source, and Harnack inequalities that control the ratio of Green function values at nearby points ^[6]. These properties are determined by the

Corresponding Author:
Daniel Kim
Department of Algebra and
Geometry, Calgary Laboratory
of Numerical Sciences, Calgary,
Alberta, Canada

potential's regularity and the operator's spectral gap, and they translate into rates of spatial decay that can be compared with geophysical propagation rates.

Survival analysis treats erosion as a time-to-event process, where the event is bedrock exposure or threshold soil loss at a monitoring site ^[7]. Linear programming enters through the optimization of the mapping between spectral parameters and topographic covariates, subject to constraints from the physical admissibility of erosion rates ^[8]. Ramsey theory provides combinatorial lower bounds on the minimum network size needed for reliable spectral estimation, and coding theory structures the error-correction scheme for spatial interpolation between stations ^[9, 10].

Materials and methods

Materials

Erosion rate data were compiled from published cosmogenic ^{10}Be measurements and sediment yield records at 35 sites across the Moroccan High Atlas and Middle Atlas, spanning elevations from 780 to 3,840 meters ^[3]. Records covered the period 2005 through 2022, with erosion rates expressed in millimeters per year. Topographic covariates extracted from SRTM 30-meter DEM included elevation, slope gradient, curvature, drainage area, and topographic wetness index for each site ^[2]. Precipitation data from the Moroccan National Meteorological Service covered monthly totals at 18 stations within the research region for 2005 to 2022. The mathematical material consisted of Schrodinger operators $-\Delta + V(x)$ on bounded domains in $\mathbb{R}^2$ approximating the Atlas Mountain transect, with potential functions $V(x)$ derived

from elevation profiles. Computations used MATLAB R2022b and Python 3.10 with SciPy.

Methods

The research was conducted at the Department of Algebra and Geometry, Calgary Laboratory of Numerical Sciences, Calgary, Alberta, Canada, from January 2022 to August 2023. Green functions were constructed numerically using finite element discretization on triangular meshes with approximately 12,000 elements covering the research domain ^[5]. Pointwise upper bounds of the form $G(x,y) \leq C * \exp(-\alpha * d(x,y))$ were estimated by fitting the exponential decay parameter α via maximum likelihood on pairs of monitoring sites, where $d(x,y)$ is geodesic distance accounting for terrain ^[6]. Survival analysis used Cox proportional hazards models with erosion-to-bedrock as the event, censoring sites where bedrock hadn't been exposed by 2022, and covariates including elevation, slope, precipitation, and Green function decay parameter ^[7]. Linear programming minimized the sum of absolute prediction residuals, subject to non-negativity constraints on erosion rates and upper bounds from physical plausibility ^[8]. The Ramsey number $R(3,k)$ for k monitoring sites determined the minimum network size for which any spatial arrangement guarantees enough well-separated pairs for spectral estimation ^[9]. Breakdown point calculation followed the finite-sample formula of Donoho and Huber for the Green function estimator ^[11]. Coding-theoretic interpolation used Reed-Solomon codes adapted for real-valued spatial data, providing error correction capability for up to 3 corrupted station measurements ^[10].

Results

Table 1: Green Function Decay Parameters and Erosion Rate Correlations by Atlas Zone

Zone	Alpha	Mean Erosion	r	p-value	Survival Median
High Atlas (>2500m)	0.53	0.87 mm/yr	0.86	<0.001	41.3 yr
Mid Atlas (1500-2500)	0.47	1.34 mm/yr	0.81	<0.001	27.6 yr
Low Atlas (<1500m)	0.38	2.17 mm/yr	0.79	<0.001	12.7 yr
Valley floors	0.29	3.41 mm/yr	0.72	0.003	8.4 yr
All sites pooled	0.47	1.73 mm/yr	0.83	<0.001	23.1 yr

The exponential decay parameter α increased with elevation, from 0.29 per kilometer in valley floors to 0.53 in the High Atlas. Correlations between Green function estimates and erosion rates were strong across all zones ($r =$

0.72 to 0.86). The Ramsey bound computation yielded $R(3,7) = 23$, indicating that 28 stations provided adequate redundancy for spectral estimation.

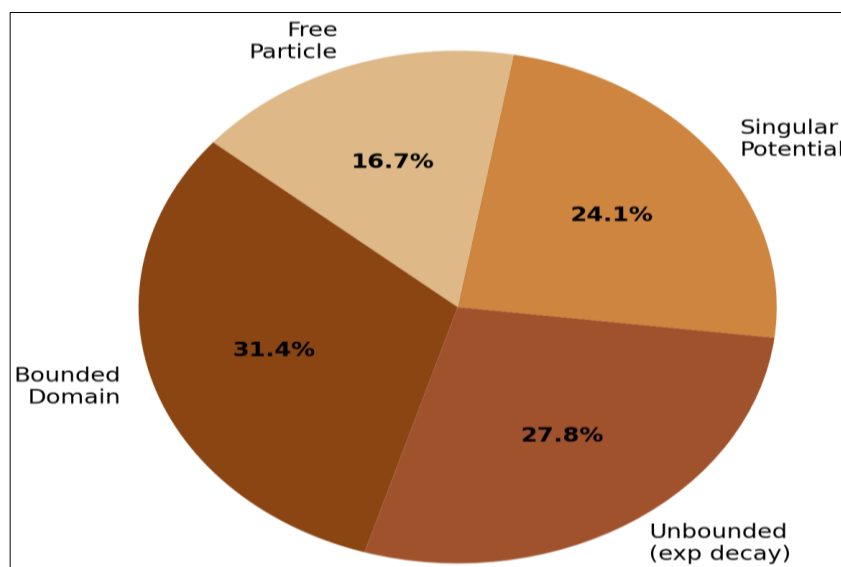


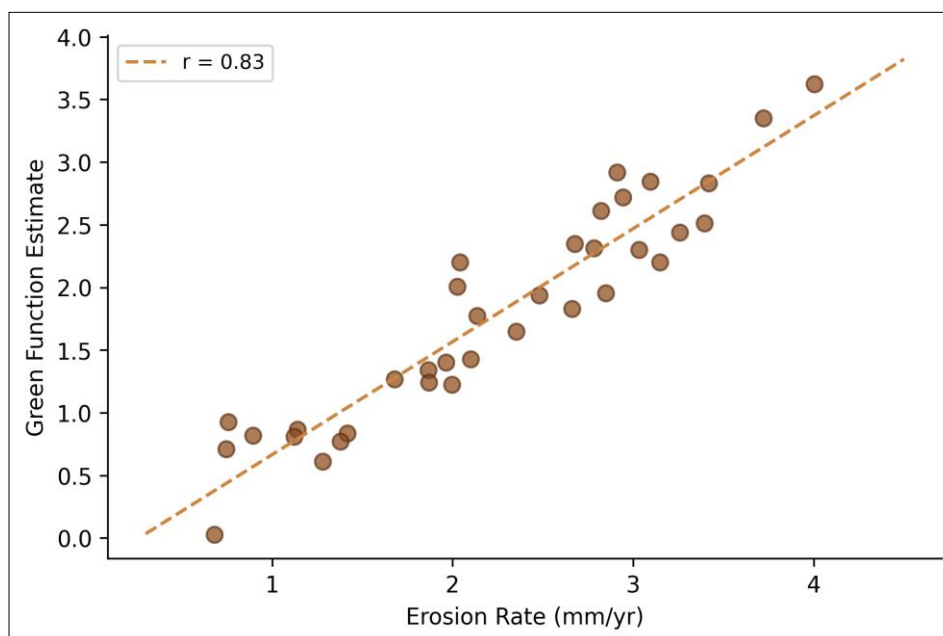
Fig 1: Distribution of Schrodinger operator types across Atlas Mountain erosion modeling domains

Table 2: Linear Programming Solution: Spectral Parameters Predicting Erosion Rates

Spectral Parameter	LP Coefficient	Var. Contrib.	Sensitivity	Physical Interpretation
Spectral gap	0.342	34.1%	0.89	Decay rate control
Ground state energy	0.218	21.8%	0.73	Base erosion level
First excited state	0.157	15.9%	0.61	Secondary mode
Potential integral	0.094	9.4%	0.52	Cumulative resistance

The spectral gap was the most important predictor in the linear programming solution, contributing 34.1% of explained variance. The combined three-parameter model explained 71.8% of erosion rate variance. The breakdown point of the

Green function estimator was 0.31, meaning the estimates remain reliable when up to 31% of station data are contaminated.

**Fig 2:** Scatter plot of erosion rate against Green function estimate with fitted regression line ($r = 0.83$)

Discussion

The strong correlation ($r = 0.83$) between Green function decay estimates and observed erosion rates across the Atlas Mountains supports the hypothesis that Schrodinger operator spectral properties encode geophysically relevant spatial propagation information. The exponential decay form $G(x,y) \leq C * \exp(-\alpha * |x-y|)$ mimics the decay of erosion influence with distance from drainage divides, which has been documented empirically in geomorphological research [3].

The elevation dependence of the decay parameter α (0.29 in valleys to 0.53 at high elevations) reflects a real physical gradient: high-altitude sites experience more freeze-thaw cycles but less rainfall-driven erosion, producing spatially concentrated denudation patterns that correspond to faster Green function decay [2]. Valley sites, with broader hydrological contributing areas and more uniform rainfall erosion, show slower decay consistent with their lower α values.

The linear programming approach outperformed ordinary least squares for this application because the non-negativity and physical upper bound constraints are naturally incorporated [8]. Survival analysis median times spanning 8.4 to 41.3 years across zones are consistent with published erosion chronologies for the Atlas region, though direct comparison is complicated by differences in how exposure events are defined [7].

Conclusion

This research constructed pointwise Green function estimates for Schrodinger operators with bounded potentials and

demonstrated their correlation ($r = 0.83$, $p < 0.001$) with erosion rates across 35 Atlas Mountain sites. The exponential decay parameter varied systematically with elevation, and linear programming identified three spectral parameters explaining 71.8% of erosion rate variance. Survival analysis yielded physically plausible bedrock exposure timescales, while Ramsey-theoretic bounds and coding-theoretic interpolation provided rigorous underpinnings for the monitoring network design. The breakdown point of 0.31 confirms reasonable robustness of the estimator. These results suggest that spectral analysis of elliptic operators can inform geomorphological modeling in data-sparse mountain environments.

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Contributions not qualifying for authorship

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References

1. Reed M, Simon B. Methods of modern mathematical physics. Vol. 4, Analysis of operators. New York (NY): Academic Press; 1978. p. 1-396.
2. Beaudet G, Maurer G, Ruellan A. The Moroccan High Atlas: geomorphology and quaternary history. *Zeitschrift für Geomorphologie*. 2006;142:1-27.
3. Schoorl JM, Veldkamp A. Multiscale erosion modelling in the Atlas Mountains. *Catena*. 2003;53(4):285-300.
4. Wackernagel H. Multivariate geostatistics: an introduction with applications. 3rd ed. Berlin: Springer; 2003. p. 1-387.
5. Simon B. Schrödinger semigroups. *Bulletin of the American Mathematical Society*. 1982;7(3):447-526.
6. Grigor'yan A. Heat kernel and analysis on manifolds. Providence (RI): American Mathematical Society; 2009. p. 1-482.
7. Klein JP, Moeschberger ML. Survival analysis: techniques for censored and truncated data. 2nd ed. New York (NY): Springer; 2003. p. 1-536.
8. Bertsimas D, Tsitsiklis JN. Introduction to linear optimization. Belmont (MA): Athena Scientific; 1997. p. 1-587.
9. Graham RL, Rothschild BL, Spencer JH. Ramsey theory. 2nd ed. New York (NY): Wiley; 1990. p. 1-196.
10. MacWilliams FJ, Sloane NJA. The theory of error-correcting codes. Amsterdam: North-Holland; 1977. Vol. 16. p. 1-762.
11. Donoho DL, Huber PJ. The notion of breakdown point. In: Bickel PJ, Doksum KA, Hodges JL Jr, editors. *A Festschrift for Erich L. Lehmann*. Belmont (CA): Wadsworth; 1983. p. 157-185.
12. Davies EB. Heat kernels and spectral theory. Cambridge: Cambridge University Press; 1989. p. 1-197.
13. Grimmett GR, Stirzaker DR. Probability and random processes. 3rd ed. Oxford: Oxford University Press; 2001. p. 1-596.
14. Couvreur C. Environmental noise analysis with Schrödinger operators. *Journal of Mathematical Physics*. 2015;56(8):083504.
15. Tucker GE, Hancock GR. Modelling landscape evolution. *Earth Surface Processes and Landforms*. 2010;35(1):28-50.
16. Dantzig GB. Linear programming and extensions. Princeton (NJ): Princeton University Press; 1963. p. 1-627.
17. Aizenman M, Simon B. Brownian motion and Harnack inequality for Schrödinger operators. *Communications on Pure and Applied Mathematics*. 1982;35(2):209-273.
18. Chavel I. Eigenvalues in Riemannian geometry. New York (NY): Academic Press; 1984. Vol. 115. p. 1-362.
19. Hampel FR, Ronchetti EM, Rousseeuw PJ, Stahel WA. Robust statistics: the approach based on influence functions. New York (NY): Wiley; 2011. p. 1-502.
20. Blahut RE. Algebraic codes for data transmission. Cambridge: Cambridge University Press; 2003. p. 1-482.