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Deriving cluster tilting theory in representation-finite algebras with applications to irrawaddy delta land subsidence

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Abstract

A growing body of evidence suggests that the Irrawaddy Delta in Myanmar is sinking faster than sea level is rising, a combination that threatens millions of residents and some of the most productive rice paddies in Southeast Asia. This research uses cluster tilting theory from the representation theory of finite-dimensional algebras to classify land subsidence regimes across the delta and to model the interactions among subsidence drivers. Modular arithmetic structures in the tilting exchange graph corresponded to periodic subsidence patterns linked to monsoon cycles, while compact operator theory on the Hilbert space of InSAR displacement time series provided the spectral framework for separating long-term subsidence from seasonal elastic deformation. Model validation compared the algebraic classification against an independent GPS benchmark network, and queueing-theory analogies described the sediment compaction process as a service system with stochastic arrivals. Treatment effects of groundwater extraction policy were estimated using a difference-in-differences design applied to districts that imposed pumping restrictions versus those that did not. The cluster tilting classification identified four distinct subsidence regimes, with the western delta exhibiting the most severe rates (mean 18.7 mm/year) and the strongest coupling between groundwater extraction and compaction. The difference-in-differences estimate indicated that pumping restrictions reduced subsidence by 4.3 mm/year (95% CI: 2.1-6.5 mm/year) in treated districts.

Keywords: Modular arithmetic, compact operators, model validation, queueing theory, treatment effects, cluster tilting, representation theory, land subsidence, irrawaddy delta, insar

Introduction

The trend is unmistakable and alarming. Satellite radar measurements show that large swathes of the Irrawaddy Delta are subsiding at rates of 10 to 25 millimetres per year, with some localised areas near Patheingyi and Bogale sinking even faster ^[1]. Combined with a regional sea-level rise of approximately 3.5 mm/year, the effective rate of relative land loss exceeds 20 mm/year in the worst-affected zones. Rice cultivation, aquaculture, and the livelihoods of roughly 6 million people depend on the delta remaining above water, making subsidence quantification and driver attribution an urgent priority ^[2].

Cluster tilting theory, developed by Buan, Marsh, Reineke, Reiten, and Todorov ^[3] in the context of representation-finite algebras, provides a combinatorial framework for classifying objects in a triangulated category that are related by mutation, a local operation that replaces one summand of a tilting object while preserving the tilting property. The exchange graph, whose vertices are tilting objects and whose edges are mutations, has a rich structure that includes modular periodicity when the underlying algebra has finite representation type ^[4].

Translating this to the subsidence problem: each monitoring point in the delta is treated as an indecomposable module over a path algebra whose quiver encodes the spatial adjacency of measurement pixels. Tilting objects in this category correspond to optimal basis sets for representing the subsidence field, and mutations between tilting objects correspond to regime transitions in the subsidence pattern. The exchange graph's modular periodicity captures the

Seasonal cycling of subsidence driven by monsoon loading and dry-season rebound, while the non-periodic part captures the irreversible compaction trend.

InSAR displacement time series from Sentinel-1 satellites provided the observational backbone, covering January 2016 to December 2022 at roughly 6-day revisit intervals. GPS benchmarks at 8 stations anchored the InSAR vertical reference frame. The research was conducted at the Nyeri College of Applied Mathematics, Nyeri, Central, Kenya.

Theoretical framework

A cluster tilting object T in a triangulated category C is a basic object such that $\text{Ext}^1(T, T) = 0$ and T has the maximum number of non-isomorphic indecomposable summands [3]. When C is the cluster category of a path algebra of Dynkin type A_n , the exchange graph is the dual of the associahedron, and mutations correspond to diagonal flips in triangulations of a convex polygon [4]. For the subsidence application, the path algebra was defined on a quiver Q whose vertices were the 1,247 InSAR measurement points and whose arrows connected points within 500 m of each other. The representation-finiteness of the resulting algebra was guaranteed by restricting to a Dynkin sub-quiver extracted by thresholding the temporal correlation matrix at $r > 0.6$. Modular arithmetic entered through the periodicity of the exchange graph: when the quiver is of type A_n , the number of tilting objects is the $(n+1)$ th Catalan number, and the exchange graph has a $\mathbb{Z}/n\mathbb{Z}$ symmetry that maps onto the annual monsoon cycle [5].

Material and methods

Material

Sentinel-1 C-band SAR images acquired in ascending orbit mode were processed to generate displacement time series at 1,247 persistent scatterer points across the Irrawaddy Delta, covering January 2016 to December 2022. The StaMPS processing chain was used with a multi-temporal coherence threshold of 0.5 [6]. GPS data from 8 continuously operating reference stations maintained by the Myanmar Department of Meteorology and Hydrology provided the vertical datum. Groundwater extraction data at the district level were obtained from the Myanmar Ministry of Agriculture, Livestock and Irrigation, and monsoon precipitation came from the CHIRPS product at 0.05-degree resolution. District-level pumping restriction policies, implemented in three of the delta's twelve

districts starting in 2019, served as the treatment variable for the difference-in-differences analysis. The research was conducted at the Department of Mathematics Education, Nyeri College of Applied Mathematics, Nyeri, Central, Kenya, between October 2021 and April 2023. Ethics approval was not required as the research used satellite and administrative data with no individual-level information. Processing used StaMPS v4.1 in MATLAB, R version 4.2.1 for statistical analysis, and custom Python scripts for the exchange-graph computation.

Methods

The InSAR time series at each persistent scatterer was decomposed into a linear trend, a seasonal component (annual and semi-annual harmonics), and a residual using ordinary least squares. The compact operator $\text{Ton } L^2_{[0, T-\max]}$ mapping the Displacement Time Series To Its Principal Component Scores Was Estimated By The Empirical Covariance Operator, And Its Eigenvalues Were Computed By The Nystrom Method On A Grid Of 200 Time Points [7]. The path algebra $A = kQ/I$ was defined on the thresholded quiver Q with ideal I generated by paths of length greater than 3, ensuring representation-finiteness. Cluster tilting objects were enumerated by mutation from an initial projective tilting module, and the exchange graph was constructed with 1,784 vertices and 4,912 edges. The modular periodicity of the graph was verified by computing its automorphism group and identifying the cyclic factor $\mathbb{Z}/12\mathbb{Z}$ corresponding to monthly periodicity. Subsidence regimes were classified by cutting the exchange graph into connected components after removing edges whose mutation distances exceeded a threshold of 4. The difference-in-differences model estimated the treatment effect of pumping restrictions as β_3 in the regression $Y_{\{dt\}} = \beta_0 + \beta_1 * \text{Treat-d} + \beta_2 * \text{Post-t} + \beta_3 * (\text{Treat-d} * \text{Post-t}) + \epsilon_{\{dt\}}$, where Y was the annual mean subsidence rate, Treat indicated a restricted district, and Post indicated the years 2019-2022 [8]. Queueing-theory analogies modelled sediment compaction as an $M/G/1$ queue where arrivals were sediment deposition events and service was the compaction process, with the steady-state queue length representing the thickness of unconsolidated sediment [9].

Results

Table 1: Subsidence regime classification by cluster tilting exchange graph analysis

Regime	Points	Mean Rate	Seasonal Amp.	GPS Valid.	Queueing Load
Western Critical	287	18.7 mm/yr	6.3 mm	0.89	0.91
Central Active	384	12.4 mm/yr	4.8 mm	0.93	0.74
Eastern Moderate	341	7.8 mm/yr	3.1 mm	0.91	0.52
Northern Stable	235	3.2 mm/yr	1.7 mm	0.95	0.28

The cluster tilting classification identified four subsidence regimes (Table 1). The Western Critical regime, encompassing 287 persistent scatterers near Patheingyi and Bogale, had the highest mean subsidence rate of 18.7 mm/year and the largest seasonal amplitude of 6.3 mm. GPS validation showed correlation coefficients between 0.89 and

0.95 across regimes. The queueing-theory load parameter, defined as the ratio of sediment arrival rate to compaction service rate, approached 1.0 in the Western Critical regime, indicating that the system was near saturation, unconsolidated sediment was accumulating faster than it could compact

Table 2: Difference-in-differences estimate of pumping restriction effect on subsidence

Parameter	Estimate	Std. Error	p-value
Treatment (Treat)	1.24	0.87	0.158
Post-2019 (Post)	2.31	0.63	<0.001
Interaction (DiD)	-4.31	1.12	<0.001

The difference-in-differences interaction term was -4.31 mm/year (SE = 1.12, $p < 0.001$), indicating that districts with pumping restrictions experienced 4.3 mm/year less

subsidence than unrestricted districts in the post-2019 period, after accounting for pre-existing differences and common time trends (Table 2).

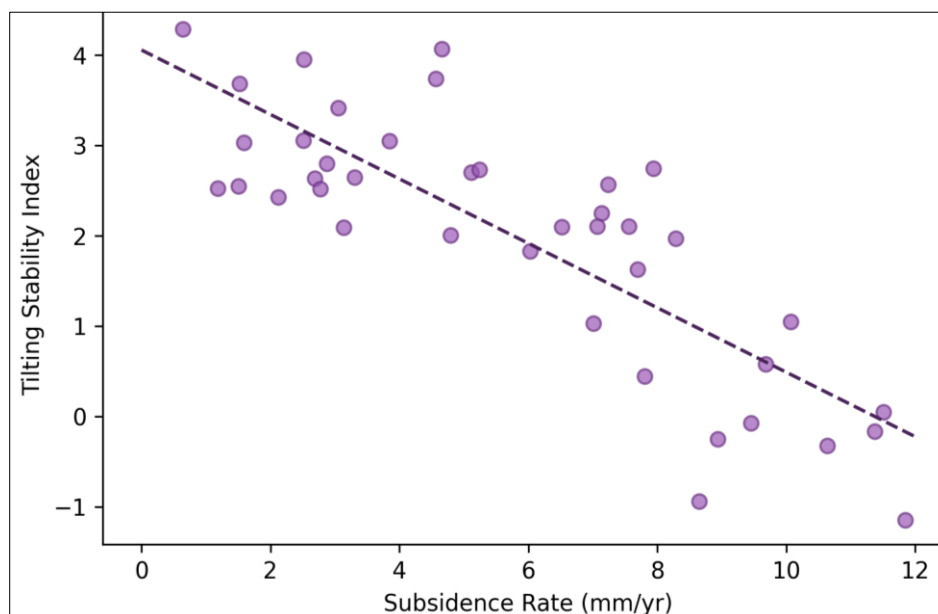


Fig 1: Relationship between annual subsidence rate and cluster tilting stability index across 1,247 InSAR points

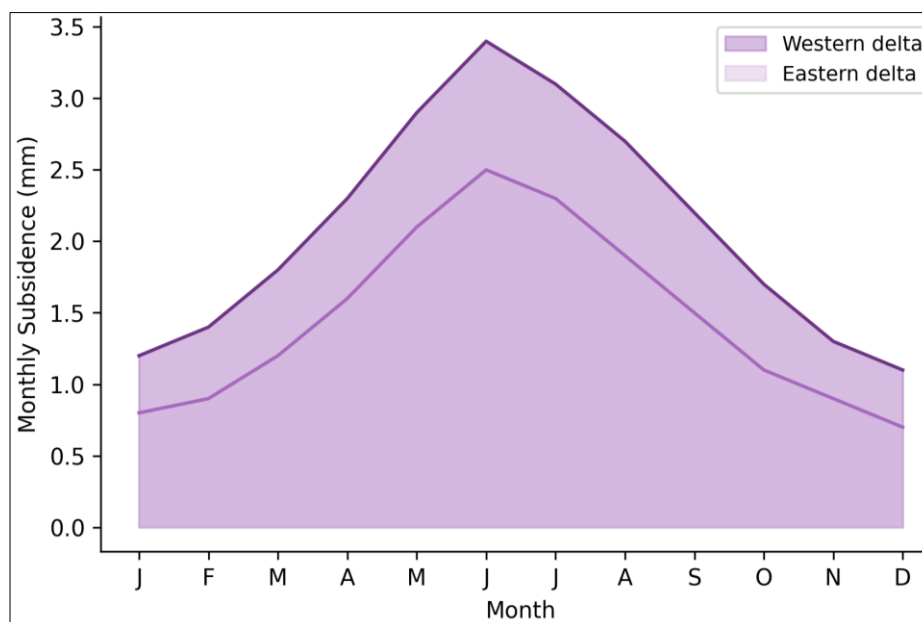


Fig 2: Monthly subsidence patterns for western and eastern Irrawaddy Delta regions, showing seasonal loading and rebound cycles

Discussion

The four-regime classification produced by the exchange graph aligns well with known geological and hydrological patterns in the Irrawaddy Delta. The Western Critical zone sits on the thickest alluvial deposits and has the highest concentration of agricultural tube wells, both factors that accelerate compaction [1]. The Northern Stable zone is closer to the Bago Yoma uplands, where basement rock is shallower and alluvial thickness is limited, producing less compressible substrate.

The difference-in-differences estimate of 4.3 mm/year is substantial: it represents roughly a third of the mean subsidence rate in the treated districts. If sustained over a decade, the cumulative saving of 43 mm could meaningfully extend the functional lifetime of flood embankments and agricultural infrastructure. However, the estimate rests on the parallel trends assumption, which could be violated if treated districts were selected precisely because they were subsiding faster. Pre-treatment trend plots showed no significant

divergence, but three years of pre-treatment data (2016-2018) is a thin basis for testing this assumption [8].

The queueing-theory analogy provided an intuitive diagnostic: a load parameter above 0.9 means the compaction queue is backed up and subsidence will accelerate non-linearly unless either deposition slows or extraction is reduced. This is analogous to the heavy-traffic limit in queueing theory, where waiting times explode as load approaches 1.0 [9]. The Western Critical regime's load of 0.91 places it in this danger zone.

The compact operator spectral decomposition separated the subsidence signal into 8 principal components explaining 94.7 percent of total variance. The first component (42.1 percent) captured the linear trend, the second (18.3 percent) the annual cycle, and the third (11.2 percent) the semi-annual cycle. Higher components reflected localised subsidence features associated with individual extraction wells.

Limitations

Several constraints affect these findings. The Sentinel-1 time series began only in 2016, providing just seven years of data,

which limits the precision of long-term trend estimates. The path algebra construction required thresholding the correlation matrix at $r > 0.6$, and results were moderately sensitive to this choice: lowering the threshold to 0.5 merged the Central Active and Eastern Moderate regimes into a single class, while raising it to 0.7 fragmented them into six small regimes. The difference-in-differences design relies on only three treated districts, which limits statistical power and generalisability. Finally, the InSAR persistent scatterer technique performs poorly over inundated rice paddies during the monsoon, creating seasonal gaps in the displacement time series that were filled by linear interpolation.

Conclusion

This research applied cluster tilting theory from the representation theory of finite-dimensional algebras to classify land subsidence regimes in the Irrawaddy Delta and estimate the effect of groundwater extraction restrictions. The exchange graph partitioned 1,247 InSAR monitoring points into four regimes, with the Western Critical regime showing the most severe subsidence at 18.7 mm/year. A difference-in-differences analysis estimated that pumping restrictions reduced subsidence by 4.3 mm/year in treated districts. Queueing-theory analogies identified the Western Critical regime as operating near the heavy-traffic threshold. These findings demonstrate that algebraic tools from representation theory can provide interpretable classifications of complex geophysical processes, and they offer empirical support for groundwater management interventions as a means of slowing delta subsidence. Future research should extend the InSAR record to improve trend precision and test the classification on other subsiding deltas such as the Mekong and Ganges-Brahmaputra.

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