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Combined nonparametric tests for equality of location and scale under ordered alternatives

Latifah Alharbi and Rhonda MagelDOI: <https://www.doi.org/10.22271/math.2026.v11.i6b.2530>**Abstract**

Different variations of nonparametric tests are proposed for the nondecreasing ordered alternative when testing for a difference in either location or scale across multiple populations. The tests combine the Jonckheere-Terpstra or Modified Jonckheere-Terpstra statistic with a Moses-type scale statistic, using either equal weighting or a weighted scheme that assigns greater emphasis to the scale component. A Monte Carlo simulation study is conducted to determine how well the proposed tests maintain their significance levels and to estimate power under a variety of conditions. Six distributions are considered: Normal, Logistic, Laplace, Cauchy, Exponential, and Chi-square. Three alternative structures are examined: when the location parameters differ and scale parameters are equal, when the scale parameters differ and location parameters are equal, and when both differ simultaneously. Equal and unequal sample sizes are considered, and subgroup sizes of 3 and 6 are used when applying the Moses technique. Results indicate that the proposed tests perform well under symmetric distributions but that Type I error rates are inflated under strongly skewed distributions. Recommendations are given.

Keywords: Nonparametric, completely randomized design, location and scale parameters, ordered alternatives, Jonckheere-Terpstra, Moses technique

Introduction

Ordered alternatives tests are sometimes used in life-testing experiments and drug-screening studies. An ordered alternative test is used to gain power if the researcher thinks parameters will be ordered in a certain way if they are different. Let $X_{i1}, X_{i2}, \dots, X_{in_i}$, $i = 1, 2, \dots, k$ ($k \geq 3$) be random independent samples each of size n_i from k populations, where $-\infty < \mu_i < +\infty$ and $\sigma_i > 0$ are location and scale parameters, respectively. We are interested in testing the hypotheses in (1)

$$H_0: \mu_1 = \mu_2 = \dots = \mu_k \text{ and } \sigma_1 = \sigma_2 = \dots = \sigma_k \quad (1)$$

Versus

$$H_a: \mu_1 \leq \mu_2 \leq \dots \leq \mu_k \text{ and } \sigma_1 \leq \sigma_2 \leq \dots \leq \sigma_k$$

with at least one inequality strict.

Background

Lepage ^[1] proposed a two-sample nonparametric test for location and scale. The Lepage test combines the Wilcoxon test ^[2] for location and the Ansari-Bradley test ^[3] for scale alternatives. Several extensions of the Lepage test have been proposed, and Marozzi ^[4] concluded that no single test performs best under all situations.

Alsubie and Magel (2020a) ^[5] proposed two nonparametric tests, L1 and L2, for the simple tree alternative. Both tests combine the Fligner-Wolfe test ^[6] for location with a modified Ansari-Bradley test for scale.

The test L1 standardizes the two components separately before combining them, whereas L2 combines the components first and then standardizes the sum. Their study found that L2 generally has higher power when only location parameters differ, while L1 performs better when scale differences are present, or when both are different.

Alsubie and Magel (2020b) [7] proposed three additional tests, M1, M2, and M3, combining the Fligner-Wolfe and Moses [8] statistics. Test M1 standardizes the components separately, M2 standardizes the combined statistic, and M3 places additional weight on the Moses component. Their results showed that M2 performs well under location alternatives, whereas M1 is often preferred when scale differences are present or when both differences are present. M3 does perform well when just the scale parameters are different.

Zhu and Magel (2022) [9] proposed six nonparametric tests for the nondecreasing ordered alternative by combining the Jonckheere-Terpstra, Modified Jonckheere-Terpstra, and Shan statistics with the Moses technique. Their simulation study evaluated significance levels and power for three, four, and five populations under Normal, t, and Exponential distributions using both equal and unequal sample sizes.

The present study extends the work of Zhu and Magel (2022) by focusing on JT- and MJT-based procedures, introducing weighted combinations that place greater emphasis on the Moses scale component, comparing mean-based and median-based centering approaches, and evaluating performance under additional distributions including Logistic, Laplace, Cauchy, and Chi-square distributions.

The purpose of this paper is to compare four nonparametric tests, T_1 , T_2 , T_3 , and T_4 , for the nondecreasing ordered alternative when testing for differences in location, scale, or both. All tests will be compared using both the sample mean and sample median of the subgroups when calculating the sum of squared deviations for the Moses technique. T_1 and T_2 using the sample mean were introduced by Zhu and Magel (2022). T_3 and T_4 are introduced in this research as well as using the sample median in the Moses technique. A Monte Carlo simulation study is conducted to evaluate Type I error control and power under six distributions for $k = 4$ populations.

Materials and Methods

We compared four tests to test the hypothesis in (1). All four tests use the Moses technique (Moses, 1963) [8] to transform data so that tests for scale parameters can be done by using tests for location parameters on the transformed data.

Jonckheere-Terpstra (JT) Test + Moses Test

The tests, T_1 and T_3 , use the Jonckheere-Terpstra (JT) test (Jonckheere, 1954; Terpstra, 1952) [9, 10] on both the original data and the data transformed by the Moses technique. For k ($k \geq 3$) populations, the JT test is defined by

$$JT = \sum_{i=1}^{k-1} \sum_{j=i+1}^k U_{ij}$$

U_{ij} is the number of pairs of observations (a,b) for which x_{ia} is less than x_{jb} , where x_{ia} (or x_{jb}) denotes the a^{th} (or b^{th}) sample observation for the i^{th} (or j^{th}) population. The JT test is first applied to the original data to test for location and is denoted JT^* after standardization.

Using the Moses technique, the observations from each population are randomly partitioned into $g_i = \frac{n_i}{m}$ nonoverlapping subgroups of equal size m . For each subgroup, the sum of squared deviations from the subgroup mean is computed. The JT test is then applied to these dispersion measures and denoted JS^* after standardization. The expected value and variance of JT are:

$$E_0(JT) = \frac{N^2 - \sum_{i=1}^k n_i^2}{4}, \text{Var}_0(JT) = \frac{N^2(2N+3) - \sum_{i=1}^k n_i^2(2n_i+3)}{72} \quad (2)$$

where $N = \sum_{i=1}^k n_i$

where n_i is the sample size for population i . The mean and variance of the scale component, J , are found using (2) by replacing n_i by g_i and replacing N with $G = \sum_{i=1}^k g_i$.

The test statistic T_1 (Zhu and Magel, 2022) assigns equal weight to both components:

$$T_1 = \frac{JT^* + JS^*}{\sqrt{2}}$$

Zhu and Magel (2022) used the original Moses test (Moses, 1963) which calculates the sum of squared deviations for each subsample by calculating the sample mean for each subgroup. This research proposes investigating the Moses test using the sample median for the subsamples in calculating the sum of squared deviations for each subsample.

The proposed test statistic T_3 weights the scale component by the subgroup size m :

$$T_3 = \frac{JT^* + m \cdot JS^*}{\sqrt{1 + m^2}}$$

Two versions of T_3 will be considered. One version will use the sample mean for Moses technique and the second version will use the sample median. Both T_1 and T_3 have asymptotic standard normal distributions under the null hypothesis. The null hypothesis in (1) is rejected for large values.

Modified Jonckheere-Terpstra (MJT) Test + Moses Test

The second and fourth tests, T_2 and T_4 , use the Modified Jonckheere-Terpstra (MJT) test [12] on both the original data and the data transformed by the Moses technique. For k greater than 3 populations, the MJT test is defined by

$$MJT = \sum_{i=1}^{k-1} \sum_{j=i+1}^k (j-i) U_{ij}$$

The MJT test is applied to the original data and standardized to obtain MJT^* . The MJT test is applied to the transformed values using the Moses technique, MJS , and standardized to obtain MJS^* . The expected value and variance of MJT are:

$$E_0(MJT) = \sum_{i=1}^{k-1} \sum_{j=i+1}^k (j-i) E(U_{ij})$$

$$\text{Var}_0(MJT) = \sum_{i=1}^{k-1} \sum_{j=i+1}^k (j-i)^2 \text{Var}(U_{ij}) + 2 \sum_{i=1}^{k-1} \sum_{j=i+1}^k \sum_{l=j+1}^k (j-i)(l-i) \text{Cov}(U_{ij}, U_{il})$$

where:

$$E_0(U_{ij}) = \frac{n_i n_j}{2}, \text{Var}_0(U_{ij}) = \frac{n_i n_j (n_i + n_j + 1)}{12}, \text{Cov}(U_{ij}, U_{il}) = \frac{n_i n_j n_l}{12} \quad (3)$$

To find the mean and variance of MJS , replace n_i by g_i and N by G in (3).

The second test statistic proposed by Zhu and Magel (2022), T_2 , assigns equal weight to both components:

$$T_2 = \frac{MJT^* + MJS^*}{\sqrt{2}}$$

The proposed test statistic, T_4 , weights the scale component by m :

$$T_4 = \frac{MJT^* + m \cdot MJS^*}{\sqrt{1 + m^2}}$$

This research will consider two versions each of T_2 and T_4 . One version will use the sample mean when using Moses technique and the second version will use the sample median of each of the subsamples when using Moses technique.

Both T_2 and T_4 have asymptotic standard normal distributions under the null hypothesis. The null hypothesis in (1) is rejected for large values.

Simulation Study

A simulation study was conducted to evaluate the two versions of the four proposed tests and was implemented in SAS version 9.4. The test statistics were evaluated assuming random samples from six distributions: Normal, Logistic, Laplace, Cauchy, Exponential, and Chi-square. The RAND function was used to generate random samples from each distribution. Both mean-based and median-based centering approaches were implemented within the Moses-type subgroup framework.

The simulation was conducted in two steps. The first step was to estimate the Type I error rates of the test statistics. Replications of 10,000 samples were used for all simulations. The stated significance level was 0.05. Type I error rates were estimated by counting the total number of times the null hypothesis was rejected when the null hypothesis was true, divided by 10,000. A test was considered to maintain nominal size if the estimated Type I error rate fell within the range 0.045 to 0.055. The second step was to compare the powers of the test statistics under various conditions. Powers were estimated by counting the total number of times each proposed test rejected the null hypothesis under a given condition, divided by 10,000.

Three and four populations were considered, with only the results for four populations given in this paper since the results for three populations were found to be similar. Equal and unequal sample sizes were used. For equal sample sizes, $n_i = 18$ and $n_i = 36$ for all populations were considered. For unequal sample sizes, configurations with a larger control group, $n_1 = 36$, and the remaining samples each of size 18 and configurations with larger treatment groups with the sample sizes of the treatment populations (considered the 2nd through 4th populations) being each 36 and the sample size from the control population being 18 were examined. Two subgroup

sizes ($m = 3$ and $m = 6$) were used for each scenario when applying the Moses technique.

Three alternative structures were examined. First, the location parameters were different and the scale parameters were equal. Second, the location parameters were equal and the scale parameters were different. Third, both the location and scale parameters were different simultaneously. The null hypothesis was rejected for large values of the test statistics, using the critical value $z_{0.95} = 1.645$ for the one-sided ordered alternative at $\alpha = 0.05$.

Results and Discussion

The results were similar between the equal and unequal sample sizes considered in this study. Therefore, only the results are given for equal sample sizes of 36. Results are given for each test statistic when the sample mean was used in calculating the sum of squared deviations for each subsample for the Moses technique and when the sample median was used.

Table 1: Type I Error Rates Percentage of rejection for $k=4$ populations; ($n_1 = n_2 = n_3 = n_4 = 36$; subgroup size $m=3$)

Distribution	Method	T ₁	T ₂	T ₃	T ₄
Normal	Mean	0.0476	0.0473	0.0488	0.0488
Normal	Median	0.0478	0.0473	0.0497	0.0508
Logistic	Mean	0.0524	0.0521	0.0528	0.0536
Logistic	Median	0.0524	0.0520	0.0541	0.0537
Laplace	Mean	0.0493	0.0503	0.0520	0.0530
Laplace	Median	0.0501	0.0504	0.0522	0.0529
Cauchy	Mean	0.0521	0.0522	0.0503	0.0507
Cauchy	Median	0.0521	0.0525	0.0509	0.0510
Exponential	Mean	0.0953	0.0947	0.0814	0.0806
Exponential	Median	0.0945	0.0934	0.0790	0.0792
Chi-sq df=5	Mean	0.0818	0.0822	0.0663	0.0671
Chi-sq df=5	Median	0.0823	0.0828	0.0663	0.0654
Chi-sq df=10	Mean	0.0691	0.0683	0.0611	0.0624
Chi-sq df=10	Median	0.0681	0.0678	0.0607	0.0610
Chi-sq df=15	Mean	0.0688	0.0691	0.0597	0.0607
Chi-sq df=15	Median	0.0685	0.0692	0.0619	0.0617

Table 1 presents the estimated Type 1 error rates for $k=4$ populations with equal sample sizes of 36 and subgroup size $m=3$. All four test statistics maintained nominal size under the symmetric distributions. For the Normal, Logistic, Laplace, and Cauchy distributions, estimated Type I error rates were close to 0.05 for both mean-based and median-based procedures. However, under the Exponential distribution, Type 1 error rates were substantially inflated, with values around 0.09 for T_1 and T_2 and around 0.08 for T_3 and T_4 . Similarly, under the Chi-square distribution with degrees of freedom 5, 10, and 15, Type 1 error rates exceeded the acceptable range of 0.045 to 0.055, with inflation decreasing as the degrees of freedom increased. These results indicate that the proposed tests are reliable under symmetric distributions but unreliable under strongly skewed distributions due to dependence between the location and scale component tests. As the skewness decreased, the Type 1 error rates became less inflated. The skewness value for the exponential is 2. The skewness values for the chi-square with 5, 10, and 15 degrees of freedom are approximately 1.13, 0.89, and 0.73.

Table 2: Location Alternatives

Distribution	Location Parameters	Method	T ₁	T ₂	T ₃	T ₄
Normal	(0,0.5,0.5,1)	Mean	0.8529	0.8493	0.3175	0.3167
Normal	(0,0.5,0.5,1)	Median	0.8512	0.8498	0.3191	0.3156
Normal	(0, 0, 1, 1)	Mean	0.9683	0.9713	0.4461	0.4449
Normal	(0, 0, 1, 1)	Median	0.9701	0.9722	0.4458	0.4445
Logistic	(0,0.5,0.5,1)	Mean	0.4947	0.4914	0.1776	0.1773
Logistic	(0,0.5,0.5,1)	Median	0.4920	0.4909	0.1777	0.1767
Logistic	(0, 0, 1, 1)	Mean	0.6846	0.6873	0.2387	0.2393
Logistic	(0, 0, 1, 1)	Median	0.6861	0.6904	0.2384	0.2390
Laplace	(0,0.5,0.5,1)	Mean	0.7556	0.7499	0.2712	0.2679
Laplace	(0,0.5,0.5,1)	Median	0.7541	0.7459	0.2690	0.2667
Laplace	(0, 0, 1, 1)	Mean	0.9088	0.9104	0.3719	0.3702
Laplace	(0, 0, 1, 1)	Median	0.9097	0.9111	0.3696	0.3738
Cauchy	(0,0.5,0.5,1)	Mean	0.4471	0.4417	0.1660	0.1639
Cauchy	(0,0.5,0.5,1)	Median	0.4457	0.4404	0.1644	0.1621
Cauchy	(0, 0, 1, 1)	Mean	0.6181	0.6208	0.2109	0.2135
Cauchy	(0, 0, 1, 1)	Median	0.6174	0.6206	0.2131	0.2136

Table 2 presents power results for pure location alternatives. Under all symmetric distributions, T₁ and T₂ showed substantially higher power than T₃ and T₄. For the Normal distribution with location parameters (0, 0, 1, 1), T₁ and T₂ achieved power around 0.97, while T₃ and T₄ achieved only around 0.45. Similar patterns were observed for the Logistic, Laplace, and Cauchy distributions, with T₁ and T₂ consistently outperforming T₃ and T₄. There was little difference in powers between mean-based and median-based procedures across all distributions.

Table 3 presents power results for pure scale alternatives. The pattern was reversed: T₃ and T₄ showed substantially higher powers than T₁ and T₂ across all symmetric distributions. For the Normal distribution with scale parameters (1, 1, 2, 2), T₃ and T₄ achieved power around 0.94, while T₁ and T₂ achieved only around 0.75. This pattern held consistently across the Logistic, Laplace, and Cauchy distributions. These results confirm that the weighted scale component in T₃ and T₄ provides an advantage when scale differences are present without location shifts.

Table 3: Scale Alternatives

Distribution	Scale Parameters	Method	T ₁	T ₂	T ₃	T ₄
Normal	(1,1,2,2)	Mean	0.7465	0.7505	0.9437	0.9470
Normal	(1,1,2,2)	Median	0.7416	0.7479	0.9401	0.9443
Normal	(1,1,1,2)	Mean	0.5212	0.5260	0.7536	0.7676
Normal	(1,1,1,2)	Median	0.5189	0.5221	0.7493	0.7618
Logistic	(1,1,2,2)	Mean	0.7102	0.7126	0.9174	0.9229
Logistic	(1,1,2,2)	Median	0.7037	0.7079	0.9148	0.9195
Logistic	(1,1,1,2)	Mean	0.4931	0.4953	0.7157	0.7270
Logistic	(1,1,1,2)	Median	0.4876	0.4915	0.7118	0.7224
Laplace	(1,1,2,2)	Mean	0.6310	0.6349	0.8572	0.8632
Laplace	(1,1,2,2)	Median	0.6255	0.6302	0.8517	0.8582
Laplace	(1,1,1,2)	Mean	0.4299	0.4304	0.6289	0.6374
Laplace	(1,1,1,2)	Median	0.4264	0.4274	0.6235	0.6294
Cauchy	(1,1,2,2)	Mean	0.3441	0.3464	0.5170	0.5232
Cauchy	(1,1,2,2)	Median	0.3391	0.3391	0.5070	0.5118
Cauchy	(1,1,1,2)	Mean	0.2392	0.2369	0.3438	0.3442
Cauchy	(1,1,1,2)	Median	0.2339	0.2309	0.3352	0.3360

Table 4: Location and Scale Alternatives

Distribution	Location Parameters	Scale Parameters	Method	T ₁	T ₂	T ₃	T ₄
Normal	(0,0,0.5,0.5)	(1,1,1.5,1.5)	Mean	0.9037	0.9060	0.8468	0.8517
Normal			Median	0.9022	0.9050	0.8434	0.8476
Normal	(0,0,0,0.50)	(1,1,1,2)	Mean	0.8253	0.8351	0.8670	0.8769
Normal			Median	0.8237	0.8328	0.8627	0.8728
Logistic	(0,0,0.5,0.5)	(1,1,1.5,1.5)	Mean	0.7198	0.7228	0.7240	0.7279
Logistic			Median	0.7199	0.7207	0.7175	0.7229
Logistic	(0,0,0,0.50)	(1,1,1,2)	Mean	0.6904	0.7002	0.7906	0.8018
Logistic			Median	0.6894	0.6979	0.7843	0.7947
Laplace	(0,0,0.5,0.5)	(1,1,1.5,1.5)	Mean	0.7114	0.7160	0.7152	0.7196
Laplace			Median	0.7113	0.7157	0.7175	0.7177
Laplace	(0,0,0,0.50)	(1,1,1,2)	Mean	0.7361	0.7442	0.7610	0.7696
Laplace			Median	0.7321	0.7381	0.7552	0.7650
Cauchy	(0,0,0.5,0.5)	(1,1,1.5,1.5)	Mean	0.4713	0.4755	0.3901	0.3950
Cauchy			Median	0.4689	0.4706	0.3848	0.3865
Cauchy	(0,0,0,0.50)	(1,1,1,2)	Mean	0.4243	0.4272	0.4412	0.4460
Cauchy			Median	0.4202	0.4201	0.4344	0.4362

Table 4 presents power results for combined location and scale alternatives. When both location and scale parameters changed together, the relative performance of the four statistics depended on the magnitude of each component. When location and scale changes were of comparable magnitude, all four statistics performed similarly. When the scale change dominated, T₃ and T₄ showed higher powers. When the location change dominated, T₁ and T₂ showed higher powers. Mean-based and median-based procedures produced similar estimated powers across all distributions. Figures A-D display the density distributions of Z(JT), Z(Moses), and T₁ under the normal, logistic, chi-square (df =

10), and exponential distributions, respectively, based on equal samples of size 36. Under the symmetric normal and logistic distributions, the three statistics exhibit very similar behavior. Under the skewed chi-square and exponential distributions, the combined statistic T₁ shows greater dispersion than the individual component statistics, with the effect being most pronounced for the exponential distribution. It is noted that the chi-square distribution with 10 degrees of freedom is less skewed. This behavior is consistent with the variance inflation observed in Table 5 and helps explain the elevated Type 1 error rates under skewed distributions.

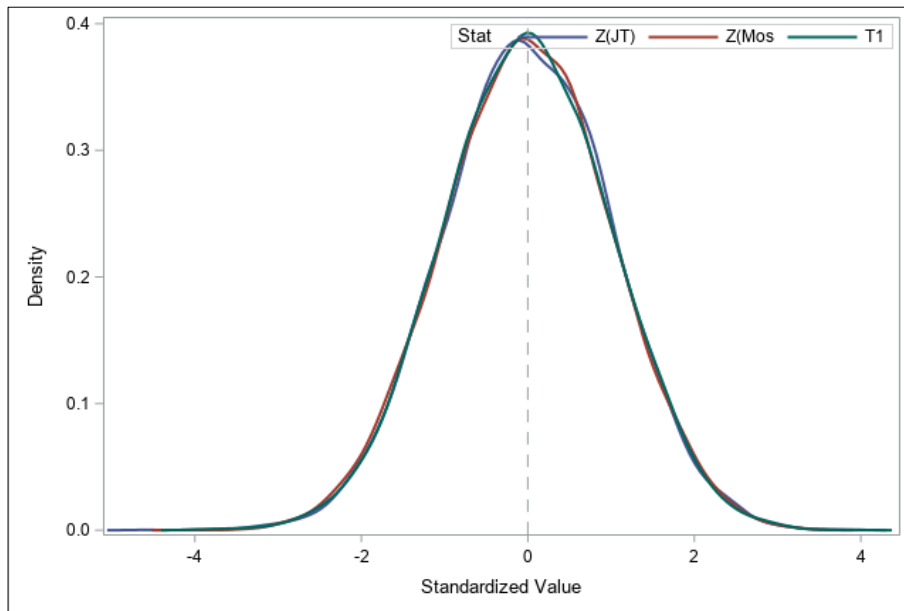


Fig A: Normal Distribution (H0) - Component Calibration Density Distributions of Standardized Test Statistics

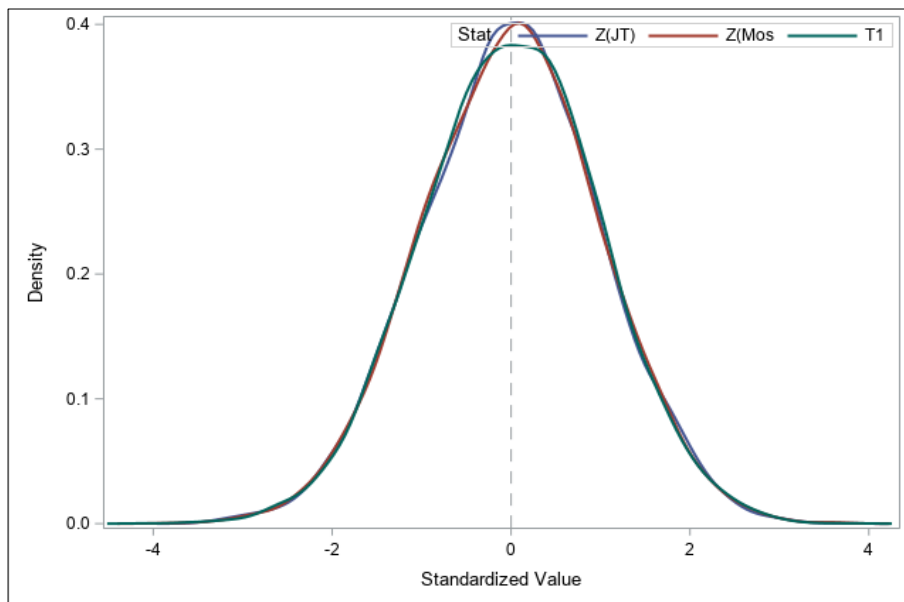


Fig B: Logistic Distribution (H0) - Component Calibration Density Distributions of Standardized Test Statistics

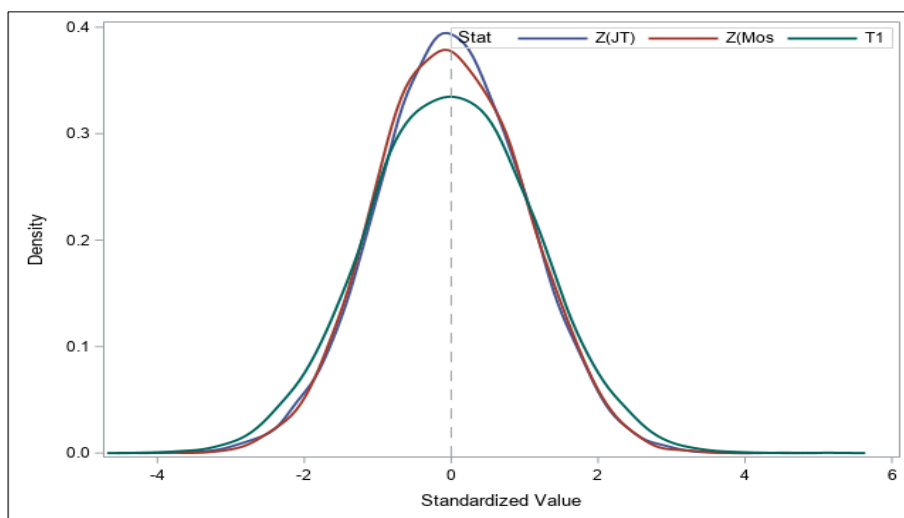


Fig C: chi Square Distribution $df = 10$ (H0) - Component Calibration Density Distributions of Standardized Test Statistics

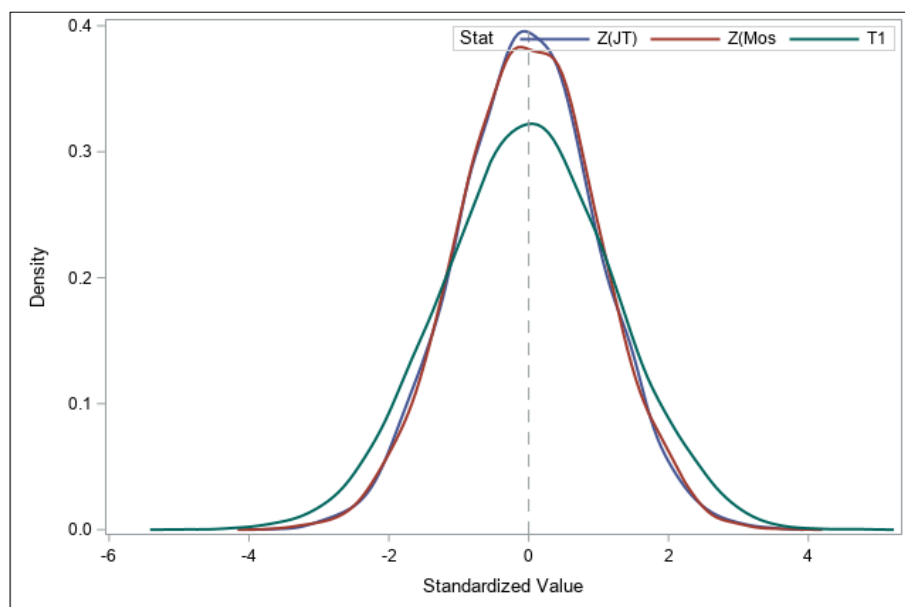


Fig D: Exponential Distribution (H0) - Component Calibration Density Distributions of Standardized Test Statistics

Table 5: Variance of standardized component statistics and combined test statistics under the exponential distribution ($H_0: \mu_1 = \mu_2 = \mu_3 = 1, \sigma^2 = 1$; 10,000 replications)

Statistic	Mean	Variance
Z(JT)	-0.031	0.982
Z(Moses)	-0.016	0.979
T ₁	-0.033	1.468
T ₂	-0.027	1.694
T ₃	-0.025	1.271
T ₄	-0.022	1.407

Note: Each component statistic individually maintains a variance close to one, whereas all combined statistics exhibit noticeable variance inflation due to induced dependence under skewness.

Conclusion

Four combined nonparametric test statistics with two variations each were compared for detecting nondecreasing ordered alternatives in both location and scale. Two of these were proposed in this research, T₃ and T₄, and two were existing tests. The sample median variation was also investigated in this research for all four test statistics. A Monte Carlo simulation study was conducted to evaluate Type 1 error control and power under six distributions for 3 and 4 populations with the results for 4 populations given in this paper.

The simulation results support the following recommendations. When the researcher suspects that location parameters differ and scale parameters are equal, either T₁ or T₂ is recommended. When scale parameters differ and location parameters are equal, either T₃ or T₄ is recommended. When both location and scale parameters differ, the choice depends on which component is expected to dominate; T₃ or T₄ are recommended when scale differences are expected to be larger, with T₄ having slightly higher estimated powers. There was little practical difference between mean-based and median-based procedures, so either may be used.

The tests perform well under symmetric distributions including the normal, logistic, laplace, and Cauchy. However, Type 1 error rates were substantially inflated under strongly skewed distributions such as the exponential and chi-square with small degrees of freedom. The tests should therefore not be used when the underlying distributions are strongly skewed. Future work will address the correction of Type 1

error inflation under skewed distributions by incorporating the covariance between the location and scale components.

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